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Introduction *to* Rocket Technology

Introduction to

ROCKET TECHNOLOGY

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Translated from the Russian by

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AUTHORS' PREFACE

Rocket technology today, is one of the most complex branches of the engineering sciences. In order to enter the field of rocket technology it is imperative to have a diversified preparation in a wide variety of engineering fields.

To fully understand rocket technology, it is necessary, aside from general engineering training, to have a good understanding of rocket motor and aircraft design, to know the chemistry of rocket propellants, theory of combustion, gas dynamics, and the fundamentals of heat exchange and heat transfer. It is also necessary to know the theory of structures, strength of materials, high-speed aerodynamics, ballistics, the theory of servomechanisms, the principles of navigation instruments, electronics, and radio technology. All subjects listed here under the heading of rocket technology are tightly interwoven, and in rocket design quite often exhibit unique and complex interaction, characteristic only of rocket technology.

At present there is a lack of text books, with the aid of which one could form a general idea of this subject. This is explained by the very newness of the subject, and also, the difficulties which arise in presenting the above mentioned diversified fields. The present work constitutes the first effort to resolve this problem to some extent.

The authors have undertaken the task of introducing the reader to the general subject of rocket technology, without demanding of him serious preparation, especially in the specialized fields of aero-gas-dynamics and thermodynamics. To understand the material in this book a general background in the elements of physics, chemistry, and the fundamentals of higher mathematics is all that is needed. In this way, the book is designed for the reader who has the background equivalent to two semesters of a technical college.

Inasmuch as the development of rocket technology has always been directed toward military application, the subject of rocket technology and the theory of reaction propulsion must, perforce, be associated with applied military problems. In the present work, this association is made of necessity and only in the description of types of construction and purpose of the rockets. This book does not touch on the subject of the military uses of rockets or general discussions of rocket artillery as a military science. Nor does this book dwell on many of the specialized subjects of rocket technology, having an independent character such as, radio control, guidance, telemetering and some others. Nevertheless, there is enough material in this book to enable the reader to form a first, sufficiently complete impression of rocket technology as a whole.

Chapters III, IV, V, and a large part of Chapter VI in this book were written by G. B. Siniarev. The rest of the chapters and sections were written by V. I. Feodosiev. He also edited the entire book.

THE AUTHORS

TRANSLATOR'S PREFACE

A little over a century ago (1851), Foucalt performed his famous pendulum experiment to prove to the skeptical that the earth rotates about an axis. A half century ago (1903), the Wright brothers flew at Kitty Hawk. Today, Cape Canaveral is a household word. Yet, the men and women who make this ever accelerating rate of scientific progress possible are considered as a negotiable commodity which can be obtained for an expenditure of so many billions annually. This is due in a large measure to the confusion between the terms "training" and "development." It is true that "manpower" can be bought, but not curiosity and imagination. The last two must be developed and are the main distinguishing characteristics between the scientist and the "trained manpower."

Rocket Technology and its extension, Space Technology, are composed of many highly specialized, but nonetheless interdependent, disciplines. Thus, to be effective, specialization must be built on an ever widening base of other sciences. The translation of this book was undertaken in the hope that it will serve as a foundation for this base, and as a catalyst for generation of curiosity and imagination.

This text in the original Russian used metric physical units throughout, as is the custom in the majority of countries outside the United States. In the translation it was decided to convert to the foot-pound-second system in all chapters not dealing with chemistry. Thus the systems of units used throughout the text are consistent with the unit systems currently (but, it is hoped, not for long) used in American colleges and universities.

The cooperation of the various branches of the Armed Services in supplying photographs for some illustrations is hereby gratefully acknowledged. To my wife I extend my sincere thanks for the patient typing of the manuscript and assistance in proofreading.

S. N. S.

Silver Spring, Maryland
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Introduction

When we speak of the principle of reactive motion we have in mind a motion imparted by a repelling force, i.e., a reaction due to a stream of particles being ejected from the apparatus.

It is not always possible to differentiate sharply between the reactive and nonreactive principles of motion. It must be kept in mind that in the broad sense of the word all methods of locomotion are based on the reactive principle; i.e., repulsion of a certain mass in a direction opposite to the direction of motion. A boat and a steamship move as a result of reaction against a mass of water, which is moved in a backward direction. The propeller of an airplane forces a mass of air backward, thereby creating thrust. An athlete, leaping up, simultaneously pushes down on the earth, although with an immeasurably smaller velocity than that with which he himself moves upward.

As we see, in all instances, the imparting of velocity to some mass is always connected with imparting a velocity to another mass in an opposite direction, so that in this sense all mechanical motion can be considered as based on the reactive principle. However, it is customary to consider the concept of the reactive principle in a narrower sense. The characteristic of reactive motion is the ejection of relatively small masses with relatively high velocity creating a direct reaction.

A boat, a ship and the propeller-driven airplane move as a result of indirect reaction. There, between the motor, which is the source of energy, and the moved mass of water or air, there is an intermediate mechanical element—the mover. In the case of a boat, the motor is the rower, and the oar is the mover. In the case of a ship, the mover is the screw; and for the airplane, it is a propeller. The athlete, leaping up, pushes away with the aid of his legs, which, in this case, act as the mover.

In the listed examples, it is the presence of the mover which is characteristic of indirect reaction. In those instances in which the mover is absent, the reaction created by the motor is called the direct reaction. This is the case, for instance, with a solid propellant rocket. Here the motor consists of a chamber in which gases are generated as a result of propellant combustion. Reaction is created directly by a stream of escaping gases. Between the motor and the ejected gas mass there are no intermediate mechanisms.

Speaking of reaction motors, a distinction should be made between the air-breathing motors, whose action depends on the surrounding medium,

and rocket motors which can, generally speaking, act independently of the surrounding medium.

Aircraft equipped with air-breathing reaction motors are called jet aircraft and, when guided by auto-pilot and used as missiles, pilotless aircraft. Air-breathing reaction motors use the fuel which is carried aboard the aircraft. Atmospheric oxygen is used as an oxidizer for this fuel. Therefore, the action of such a motor depends on the surrounding medium, and an aircraft equipped with a motor of this type can move through a vacuum only by inertia.

Apparatus equipped with rocket motors are known simply as rockets. Gases escaping from the rocket motor are formed exclusively from the fuel and oxidizer contained in the rocket. It is precisely this circumstance which makes the action of the rocket motor independent of the surrounding atmosphere. A rocket can travel in a vacuum (interplanetary space).

Air-breathing reaction motors are sometimes simply called reaction motors (jets) to differentiate them from rockets, although the term "rocket" is included in the broader term "reaction."

The advantage of the rocket motor consists in the fact that its action is independent of the surrounding medium. The disadvantage is that, with an equal initial weight, the rocket loaded with the necessary amount of fuel and oxidizer can act for a shorter time than a jet aircraft, which uses atmospheric oxygen for oxidizer and which only carries the fuel supply on board.

Let us now see what causes forced contemporary technology to turn to the principle of reaction motion, why this principle received its widest application in aviation, and why reaction technology is so rapidly developing at this particular time.

The crux of the matter is that the reaction principle is most advantageous at high speeds, characteristic of aviation, and that aviation is now realizing speeds at which the reaction motors are more effective than propellers.

At high velocities either through air or water, the resistance or drag greatly increases. Therefore, in order to maintain the higher velocity of an aircraft, with constant weight and configuration, it is imperative to have a motor of greater power.

Increase in the power of a conventional aircraft piston engine is accompanied by complexity of design and, more important, by an approximately proportional increase in its weight. At the same time, the increase in weight of the engine—and consequently of the aircraft—requires an increase in power in order to maintain the same speed. Therefore, it develops that without reducing the power loading, i.e., engine weight per unit of power (lb/hp), high speeds are unattainable, and the piston engine is ineffectual.

For attainment of high speeds, it therefore seems reasonable, first of all, to utilize a motor whose propeller is set in motion by a gas turbine, the so-called turboprop engine. Use of this engine results in a sharp reduction in the power loading as compared to the piston engine (approximately by a factor of two).

The next step toward attaining high speeds is the elimination of the "mover" as a source of increased weight of the engine, i.e., transition to the utilization of direct reaction and, consequently, the air-breathing reaction motor.

At low speeds, discarding of the mover is not justified, for at these speeds a mover of conventional design is quite effective. It is doubtful, therefore, that the reaction principle will find application in the automotive field. Its future is in high speed aeronatuics. For attainment of even higher speeds it is imperative to go beyond the boundaries of the atmosphere. Prolonged flight at speeds of about 1.24 miles/sec and more is very difficult, due to the intensive aerodynamic heating of the aircraft. An aircraft with an air-breathing reaction motor cannot enter into the rarefied atmosphere, where the frontal drag would be reduced. At very high altitudes the engine will not have enough air for proper operation.

Therefore, neither the air-breathing reaction motor nor the piston engine, whose operation in the absence of air is impossible, can be counted upon. The only motor suitable for such a flight is the rocket motor.

The rocket motor has one final characteristic. This is the only known motor insuring the eventual conquest of interplanetary space. Only the rocket opens for man the real opportunity in the future to overcome the force of gravity and depart for the distant and inviting travels in the Universe. This problem is not solved yet, but it is the order of the day, and contemporary science is already on the threshold of its solution.

The construction of the powerful present-day rockets was not achieved by mankind all at once. Many years of persistent effort in research among various branches of technology and natural sciences were required until sufficient knowledge was accumulated to permit the first noticeable progress in rocket technology.

The principle of reaction motion has been known for a long time. Some of the earliest rocket motors were gunpowder rockets used about 5000 years ago in China, first for entertainment and later for military purposes. M. G. Chernyshev expressed, from our point of view, a sound idea. The fact is that the invention of gunpowder rockets and their use in each country followed the appearance of gunpowder and that "the idea of a rocket was born independently and inescapably wherever the art of making gunpowder permeated, preceding the idea of firearms."

A man making gunpowder and inadvertently studying its properties

must, of necessity, have come across the phenomenon of burning powder in a partially closed vessel. He could not have ignored a sight outside the framework of every-day experience; the motion of the vessel in a direction opposite to the escaping gas and the torch of flame and smoke which accompanied burning. It is natural to suppose that it is precisely the unusual nature of this occurrence which must have served as an impetus to the creative activity of man leading to the invention of the rocket. Be that as it may, even though the name of the first inventor of the rocket, just as the name of the inventor of powder, remains unknown, we can rightfully suppose that the invention of the rocket in all countries accompanied either the invention or the import of gunpowder into these countries.

The Chinese are credited not only with the invention of the rocket but also with the first attempt to lift a man into the air with the aid of powder rockets.

There are many references attesting to the fact that in the tenth to thirteenth centuries the use of powder in rockets was widely known in Europe. There is also reliable evidence that rockets or, as they were known at that time, "fire arrows," were used about the same time for military purposes. In the following centuries, development of smooth-bore artillery and its major successes pushed rocket technology to a secondary level.

At the end of the eighteenth century, the interest in the use of powder rockets for military purposes was again aroused in Europe. The impetus for this were the Imperialistic Wars which were being conducted at that time by the British in India. In the course of these wars Hindus used military rockets. The simplicity of a rocket, the concentration of fire power, and the psychological effect on the infantry—all these qualities of the forgotten armament—forced the British to take another look at the rocket. As a result, the military rocket was accepted as armament in the British army within a few years and was later accepted by the armies of other European countries.

The man primarily responsible for the development of rocketry in the British army of that time was Colonel Congreve, who had perfected a gunpowder rocket which achieved a range of 4 miles.

The advantages of rocket armament consisted in its ability for rapid fire and for great concentration of fire. Lightweight launching platforms without heavy barrels and recoil installations make rocket artillery maneuverable and transportable, which is particularly advantageous under conditions of mountain warfare and the crossing of water obstacles.

Development of rockets in Europe continued until the '80's of the last century, with the advent of the breach-loading rifled artillery. Rockets could no longer compete with this weapon in the rapidity of fire, range, or fire power, and the rocket up to World War II is completely absent from the armament of armies of the whole world.

From available references, ancient Russian rockets were first described by Master of Gunnery Onisim Michailov in his "Manual of Infantry, Gunnery and other Arts relating to Military Science" (1607–1621). In describing Russian rockets in detail, he indicated that they were used not only for amusement but also as a military tool.

A great deal of serious thought had been devoted to the military application of powder rockets during the reign of Peter I. As early as 1680, by special decree a "rocket establishment" was created in Moscow, in whose work Peter himself took a very active part. During the reign of Peter I, a 1-lb signal rocket was developed and put into use which rose to altitudes of up to 0.6 miles. This signal rocket, called the "Model of 1717," remained as standard equipment to the end of the nineteenth century.

Notwithstanding a great deal of work which was devoted to rockets during the time of Peter and later, rockets did not have wide military application until the 20's of the nineteenth century. The basic reason for this was considerable success on the part of the Russian artillery in the fabrication and military application of artillery weapons. The beginning of the Franco-Russian War in 1812 created new demands on the artillery: increase in range, increase in mobility, and assurance of mass fire power. The best brains of the Russian military artillery, technologists, struggled with this problem. Further developing their country's technology in the field of artillery rockets, they independently created a successful design for military rockets (fused and incendiary) and light-weight launchers for them. Here, first of all, we have to take note of the work of General A. D. Zasiadko who, in the 1820's, made considerable contributions toward the development of his country's rocket technology.

He designed rockets of 2-, 2.9-, and 4-in. caliber with a range of 1.7 miles. Zasiadko also designed an improved launcher for rockets. (Instead of the previously used "horses," he proposed a wooden tripod and a metal tube rotating on the tripod in a horizontal and vertical plane.)

Note that the best British rockets of that time had the range of approximately $1\frac{1}{2}$ miles and were quite often launched from a heavy, massive gun. In France, the light-weight launchers, similar to the Russian launchers, were accepted as armament only in 1853.

Having successfully passed the tests, Zasiadko's rockets were approved by military experts and were accepted as armament in the Russian army.

Zasiadko does not only deserve credit for his work in designing new rockets. Military rockets existed in the Russian army before him. His basic contribution consisted in expending a great deal of effort to introduce this new armament into the army on a wide scale. General Zasiadko was primarily responsible for the use of rockets in large numbers in the Caucasus in 1825 during the war with Turkey, 1828–1829.

General C. I. Constantinov (1819–1871), an outstanding scientist and

artillerist, began his activities in the 1840's. He laid the cornerstone of scientific investigation and design of rockets, and organized mass production of powder rockets in Russia. Constantinov is deservedly considered the founder of Russian rocket artillery.

Being familiar with production and military application of rockets in Russia and abroad, Constantinov decided, first of all, to improve the technology of fabrication of powder rockets and to create rockets with greater range and smaller dispersion. Recognizing the importance of experimentation and accurate measurement in developing new designs, Constantinov constructed an electroballistic pendulum for rockets with the aid of which the magnitude of the reactive force could be measured at different times during the burning of the powder mixture. Thus was laid the foundation of rocket ballistics.

Further, Constantinov mechanized the process of rocket fabrication, having made it safe. He worked out the construction of powerful presses for loading the rockets, special drill presses for drilling holes in powder compounds, etc. He was the first in the world to apply automation processes, making wide use of electrical and acoustical regulators of his own design. Having basically improved the design of a military rocket and having standardized the manufacture of rockets, Constantinov increased the accuracy of the rocket and its range to $2\frac{1}{2}$ miles.

Constantinov designed a special rocket for training troops. He redesigned launching platforms, changed the configuration of the nose of the rocket (instead of the spherical form, the nose took on an ogival form), and continued to install rocket armament on shipboard and to defend forts by means of rockets. He was the first to show that the eccentricity of the reactive force was one of the major causes for the dispersion of missiles and reached the important conclusion that "at each moment of powder burning, the momentum imparted to the rocket is equal to the momentum of the escaping gases," i.e., he almost derived the formula for the velocity of the rocket (this problem later was successfully solved by Tsiolkovski).

Notwithstanding the great successes of the military rocket, in the second half of the nineteenth century, the Russian rocket, as well as its Western counterpart, lost its significance as the result of the successes of rifled artillery, and in the 1880's the rocket was discarded as a weapon in all countries. However, the idea of flight by means of reactive force remained alive, as evidenced by later proposals of various inventors.

The idea of flight by means of rockets received its solid scientific basis in the classic works of a talented Russian scientist, C. E. Tsiolkovski (1857–1935).

Tsiolkovski became interested in the principle of reactive motion in 1883, but then laid aside this problem and busied himself with the idea of

a metal aerostat. Tsiolkovski returned to reactive motion in 1896 and, in 1903, in a magazine called "Scientific Review," published his scientific work, "Investigation of Interplanetary Space by Means of Reactive Devices," in which he formulates his law for the maximum velocity of a rocket (see Chapter I).

It was in this article that the idea of the liquid rocket was first set forth, to be realized only after 30 years. Tsiolkovski worked out in detail a possible structural schematic of a liquid rocket and predicted in general terms the existing contemporary design. He wrote of a rocket loaded with liquid oxygen, using hydrogen as fuel; of utilization of fuel components for cooling the rocket motor; of the necessity for forced fuel feed by means of a pump; of the possibility of rocket stabilization by means of vanes located in the stream of the ejected gas; and, finally, of the automatic guidance of a rocket by means of instruments. In subsequent works he indicated a new fuel: energy derived from the splitting of the atom.

Tsiolkovski worked out in detail the conditions for the flight of the rocket in interplanetary space and the conditions of rocket escape from the earth. He showed the necessity for establishing intermediate stations for the realization of flight to other planets, proposing the idea of a rocket earth satellite.

In 1929 Tsiolkovski proposed construction of rocket trains, or multistage rockets. The works of Tsiolkovski were not immediately appreciated by his contemporaries. Not only that, they passed unnoticed.

Tsiolkovski's ideas first became the subject of conversation after a well-known popular writer, J. I. Perelman, issued a book in 1915, "Interplanetary Communication," in which, in popular form, he presented the basic ideas of Tsiolkovski. This book met with a great deal of success and had wide acceptance in our country (Russia) and abroad. Tsiolkovski was heard of again. His ideas were discussed and received wide acceptance. Tsiolkovski's priority on basic questions of rocket technology has been firmly established. Recognized foreign experts of rocket technology wrote of this. For instance, the well-known engineer Oberth (Germany) wrote in 1929, "You have lit the light and we shall work until the greatest dream of mankind is realized . . . I, naturally, will be the last one to challenge your leadership and your accomplishments in the rocket field."

Tsiolkovski's priority has been reiterated in a number of letters and addresses sent to the scientist on his 75th birthday. This, for instance, is what the German Interplanetary Society wrote to him: "The Society, from the day of its foundation, always considered you, dear Mr. Tsiolkovski, one of its spiritual leaders, and never passed up an opportunity to point out in print and by word your high achievement and your incontroversial Russian priority in the scientific pioneering of our great idea."

With the advent of the Soviet government, the works of Tsiolkovski received their deserved evaluation. The Soviet government gave all possible aid to the talented scientist. If Tsiolkovski during the forty-year period of his scientific activity before the great October Revolution wrote only 130 papers, during the 17 years after the revolution he wrote 450 papers.

A whole generation of students and enthusiastic followers were raised on the works of Tsiolkovski. The first place among these belongs to F. A. Tsander, a talented engineer and inventor.

F. A. Tsander (1887–1933) became interested in the problems of reactive technology before the revolution, but he was able to occupy himself seriously with this work only under the Soviet regime. In 1931 he worked out a motor with a thrust up to 11 lb, and in 1933 he worked out a more powerful second motor using alcohol and compressed oxygen. Tsander was a direct follower of C. E. Tsiolkovski and, like his teacher, devoted his entire life to the solution of problems of reactive technology. Tsander's proposal to utilize, as additional fuel, the metal parts of the structure which became unnecessary in interplanetary flight brought closer the technical accomplishment of the interplanetary rocket. The idea of a composite winged rocket belongs to Tsander.

Tsander's fundamental work, "The Problem of Flight by Means of Reaction Apparatus" (1932), unifies his original theoretical investigations in reactive technology.

The enumeration of our native workers in the field of rocket technology will be incomplete without mentioning the works of a talented self-taught mechanic, U. V. Kondratiuk, who, in the period from 1917 to 1925, contributed many original ideas to rocket technology. He was the first to propose using ozone instead of oxygen as an oxidizer for rocket motors. Unaware of the works of Tsiolkovski, he substantiated many of his conclusions and ideas and also, independently of Tsander, proposed combustion of metals in order to increase the effectivity of a rocket motor.

The questions raised by Tsiolkovski, Tsander, and Kondratiuk attracted considerable attention of the engineering community and, from about 1928–1929 on, new works devoted to the questions of rocket technology began to appear in print. For instance, in 1929 the first works of V. P. Glushko on liquid rocket motors appeared. Also, the first liquid rocket motors which successfully passed static tests in 1932 were designed by him. These motors used liquid oxygen, nitrogen tetroxide, and nitric acid as oxidizer, and alcohol and toluene as combustible.

Engineer M. K. Tihonravov developed several types of meteorological liquid rockets and in 1933 performed a series of successful launchings of these rockets. Works by V. P. Glushko, S. P. Korolev, U. A. Pobedonostcev,

and L. S. Dushkin on liquid rocket motors and the means of attaining high altitudes by means of liquid rockets were published in 1934–1935.

A rocket glider with a rocket motor with about 330-lb thrust of L. S. Dushkin design installed in it was developed under the guidance of S. P. Korolev in 1939. In February, 1940, pilot V. P. Fedorov performed a series of successful flights in the rocket glider.

During the Great War for the Fatherland (World War II), Soviet engineers developed new designs for rocket motors and types of rocket armament; in particular, an infantry minethrower which has received wide recognition.

Along with the above-listed works of the native scientists, starting with the second decade of the twentieth century there began to appear abroad the first studies of the problem of interplanetary flight and, later, of liquid rockets.

Among the Western scientists who have devoted their works to the above-mentioned questions, R. Eno-Peltri (France), whose first works appeared in print in 1913, and R. Goddard (U.S.A.), who started his work around 1915 and later designed several types of liquid meteorological (sounding) rockets, must be mentioned. Great contributions to the theory of rocket flight were made by H. Oberth (Germany) and E. Zenger(Austria).

A considerable step forward in the development of rocket technology was the design by the Germans during the Second World War of the guided liquid rocket, V-2.

At the present time rocket technology is on the threshold of the solution of the first problems of interplanetary flights. The long experience accumulated by designers, primarily Soviet designers, in the field of rocket technology is an assurance of the fact that in the nearest future the first problems of the conquest of interplanetary space will be solved by our science.

I. The Basic Relationships in the Theory of Reactive Motion

A. Thrust of a Rocket Motor

1. The Mescherski Equation

The basic relationships of reactive motion are derived from those laws of mechanics which define the motion of a rocket. One of these basic relationships is the equation of Mescherski—equation of motion for a body with changing mass.

Let a body with changing mass M have a motion of translation with velocity v. The mass of the body changes with respect to time because of continuous accumulation of particles which have a relative velocity w. In deriving the equation of motion of this mass let us start with Newton's law: Change in momentum is equal to the impulse acting on the system.

During time Δt the mass M is increased, with the relative velocity w by mass ΔM (Fig. 1.1).

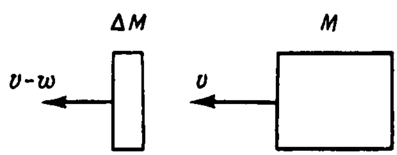


Fig. 1.1. Derivation of Mescherski equation.

Prior to the combining of masses the momentum of the system was:

$$Mv + \Delta M(v - w).$$

After the masses are combined the momentum of the system will be:

$$(M + \Delta M)(v + \Delta v).$$

According to Newton's law:

$$(M + \Delta M)(v + \Delta v) - Mv - \Delta M(v - w) = \Delta t \sum P_i$$

where ΣP_i is the summation of external forces acting on the system during the time Δt.

It is further obvious that:

$$M\Delta v + \Delta M w + \Delta M \Delta w = \Delta t \sum P_i.$$

Dividing both sides of the equation by Δt and differentiating with respect to time, we arrive at the equation of Mescherski:

$$M \frac{dv}{dt} = -\frac{dM}{dt} w + \sum P_i. \tag{1.1}$$

For a constant mass ($dM/dt = O$) the equation becomes the usual expression of Newton's second law:

$$M \frac{dv}{dt} = \sum P_i. \tag{1.2}$$

Comparing these two expressions, we see that the first may be written in the form of the second if the term $-(dM/dt)w$, which has the dimensions of force, shall be considered as a force applied to the body with the mass M. This force is called "reactive" and will be retarding (decreasing the velocity v) if $dM/dt > O$, i.e., if the mass of the system is increasing. If the mass is decreasing ($dM/dt < O$) the force $-(dM/dt)w$ will be accelerating. For a rocket $(dM/dt) < O$. The term $-(dM/dt)$ is expressed by m

$$-(dM/dt) = m \tag{1.3}$$

and is called the "mass rate of discharge."

Expression (1.1) takes on the form:

$$M\dot{v} = mw + \sum P_i \tag{1.4}$$

where $\dot{v} = (dv/dt)$.

2. Thrust

The equation of motion of a rocket is usually written in the form of Newton's law (1.2):

$$M\dot{v} = \sum P_i. \tag{1.5}$$

Here the mass M is considered as a function of time and determined by the mass rate of discharge. The forces comprising P_i include all forces acting on the rocket: force of gravity, force due to pressure distributed over the surface, and others. If equations (1.5) and (1.4) are compared, it becomes obvious that the term mw, found above, has to appear under the summation sign in one form or another.

Of the forces mentioned, the most significant is a force originating in the motor—the motive force. This force is called the thrust of the motor, or simply "thrust."

The thrust represents the axial resultant of pressure forces acting on the entire surface of the rocket. Included here, first of all, is the pressure p_r of

the gases ejected from the rocket motor, and acting on the internal surfaces. Aside from that, the thrust includes the axial component of force due to atmospheric pressure acting on the external surface of the rocket (Fig. 1.2).

Here it is necessary to underline the fact that the pressure considered above is that of the surrounding medium and not the actual pressure whose magnitude and distribution depend on the speed of flight and the aerodynamic configuration of the rocket. All additional forces arising from the motion of the rocket in atmosphere are considered as forces due to aerodynamic drag.

Regardless of the rocket and motor configuration, if the atmospheric pressure is considered separately, the axial component of the external static pressure is equal to $-pS_e$ where p is external static pressure.

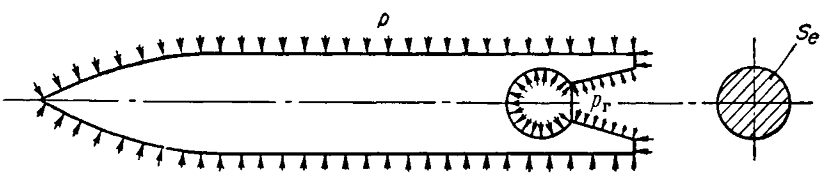


FIG. 1.2. Distribution of pressures along the surfaces of the rocket and formation of thrust.

S_e is the area of the nozzle exit plane (Fig. 1.2). The minus sign indicates that the force is directed backward and constitutes a retarding force.

Now we have to find the resultant force due to pressure on the internal surfaces of the rocket motor. To do this, let us consider separately the volume of gas contained between the internal surfaces of the motor and the nozzle exit plane (Fig. 1.3,a).

The forces due to gas pressure, p_r, acting on the internal surfaces of the motor and which yield the resultant P_i (Fig. 1.3,b), will act exactly in the same way on the released volume of gas yielding the same magnitude of the resultant but acting in an opposite direction.

Taking into account forces acting on the nozzle exit plane (Fig. 1.3,c) we find the resultant of the external forces on the released volume of gases in the form $P_1 - p_eS_e$

where S_e is the nozzle exit plane area as before.

P_e is the gas pressure at nozzle exit. (This pressure is not necessarily equal to the pressure of surrounding medium and may be either greater or smaller.)

The impulse of force $P_1 - p_eS_e$ during the time Δt is equal to the change in the momentum of gas:

$$(P_1 - p_eS_e)\Delta t = -\Delta Mw$$

where ΔM is the mass of gas exhausted during the time Δt with velocity w. Differentiating and taking into account the symbols of Eq. (1.3) we find the required quantity:

$$P_1 = mw + p_e S_e.$$

Adding to this the component of static pressure, found above, we arrive at the final expression for thrust:

$$P = mw + S_e(p_e - p). \tag{1.6}$$

It should be noted that this expression could have been derived directly from Mescherski equation (1.4). Suppose, for instance, that the rocket

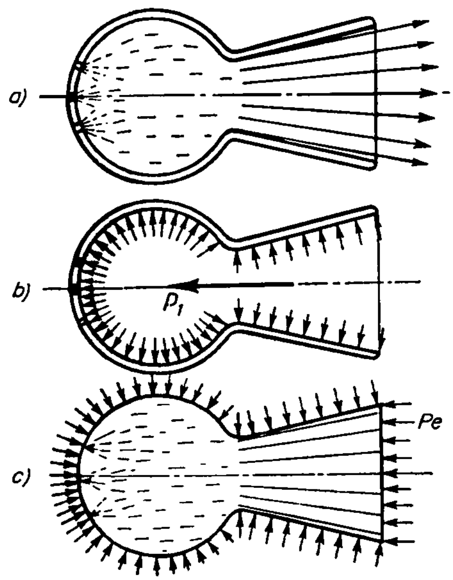


FIG. 1.3. Derivation of thrust formula.

(or only its motor) is immovably secured on a stand in a horizontal position (Fig. 1.4). Then, inasmuch as the rocket is stationary, $v = 0$. Among the forces acting on the rocket remain the forces of pressure $p_e S_e - pS_e$ and stand reaction R, which in this case is equal to the thrust P. This force is easily measured on a dynamometer. The equation (1.4) takes the form:

$$0 = mw + p_e S_e - pS_e - R$$

from which we find the expression derived above:

$$R = P = mw + S_e(p_e - p).$$

By this derivation, however, we do not show the system of pressure forces which leads to the existence of thrust.

Above it has been noted that the component of thrust $-pS_e$, derived from the static component of pressure of the surrounding medium, does not depend on the configuration of the rocket or its motor. Neither is the magnitude of the force due to gas pressure exerted on the inner surface of the motor dependent upon the characteristics of the rocket, and further (Chapter VI), we shall see that it is independent of the speed of flight or the condition of the surrounding medium.

Therefore, the thrust of a rocket motor can be considered as a characteristic of the motor only, independently of the rocket.

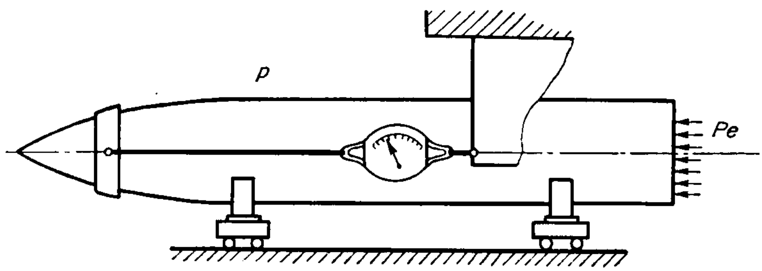


FIG. 1.4. Forces acting on a rocket fixed on a test stand.

Incidentally, the inconsistency of a widespread concept that the reactive motor pushes itself against the air by the jet is invalidated if we consider expression (1.6). Expression (1.6) shows that when the atmospheric pressure p drops, the thrust definitely does not diminish; on the contrary, it increases. Thrust P reaches its highest magnitude when $p = 0$, i.e., in a vacuum. The reactive motor pushes itself not against air but, in a manner of speaking, against the particles of gas flowing out of the combustion chamber. Because of this repulsion, gas acquires its velocity. The greater the velocity of flow, the greater is the repulsion received by the rocket with the same ejected mass.

Expression of thrust (1.6) can lead to another conclusion. Let p_0 represent atmospheric pressure at sea level and p_h, pressure at altitude h. Then the expression (1.6) can be rewritten in the form

$$P = mw + S_e(p_e - p_0) + S_e(p_0 - p_h)$$

or

$$P = P_0 + S_e(p_0 - p_h) \tag{1.7}$$

where P_0 is the thrust at sea level. This expression gives the relationship between thrust and altitude.

Sometimes the expression for thrust is written in the form

$$P = mw_{eh} \tag{1.8}$$

where w_{eh} is the so-called effective gas velocity.

By comparing expressions (1.6) and (1.8) it follows that the effective gas velocity depends on the magnitude of external pressure and the altitude

$$w_{eh} = w + \frac{S_e}{m}(p_e - p_h). \tag{1.9}$$

The expression of thrust in the form of Eq. (1.8) is ordinarily used in those instances when pressure p_h does not appreciably change during the trajectory and the magnitude of the effective gas velocity during constant combustion conditions can be considered as a constant.

3. The Specific Impulse

One of the basic indices of rocket motor effect veness is "specific impulse."

The specific impulse is the ratio of thrust to unit mass rate of flow $I = P/mg$ where the mass rate of flow mg is referred to sea level conditions (g is the acceleration due to gravity at sea level).

According to expression (1.6)

$$I = \frac{w}{g} + \frac{S_e}{mg}(P_e - P_h) = \frac{w_{eh}}{g}.$$

For a given motor, the specific impulse is a function of altitude or atmospheric pressure. The change in pressure from sea level to a complete vacuum changes the specific impulse by 12–15%.

Quantitatively, the specific impulse in lb sec/lb is

$$I \approx 0.031 w_{eh}$$

where w_{eh} is the effective gas velocity in ft/sec.

It follows that the specific impulse is determined by the gas velocity which, in turn, depends on the heat content of the fuel and, to some degree, on the design of the motor.

The last, i.e., the motor design, has a material, although secondary effect, on the specific impulse. Therefore, the specific impulse is usually considered as a characteristic of fuel. For instance, during the preliminary design and range calculations, for a rocket motor using alcohol and liquid oxygen as fuel, the specific impulse is taken to be 230–240 lb sec/lb as a first approximation, taking into account a motor of conventional design.

Motors using fuels of higher heat content yield a correspondingly higher specific impulse.

B. Tsiolkovski Equation for the Ideal Velocity of a Rocket

1. The Ideal Velocity of a Single Stage Rocket

Let us determine the velocity which a rocket can attain in the ideal case, i.e., when it moves not only through a vacuum, but also outside the influence of the earth's gravity.

In this case, expressions (1.5) and (1.8) yield

$$M\dot{v} = mw_{eh}.$$

In a vacuum, the effective gas velocity remains constant:

$$w_{eh} = w + \frac{S_e}{m} p_e = w_e.$$

Therefore, $M\dot{v} = mw_e$.

According to Eq. (1.3)

$$m = -(dM/dt)$$

substituting $M\dot{v} = -w_e(dM/dt)$ or $dv = -w_e(dM/M)$.

After integration we get

$$v = -w_e(\ln M - \ln C)$$

where C is a constant of integration.

When $v = 0$ the mass of the rocket is equal to the initial mass M_0 (the combined mass of the projectile and the propellant). Therefore

$$v = -w_e \ln (M/M_0). \tag{1.10}$$

This relationship was first derived by C. E. Tsiolkovski. Here M represents the variable mass of the rocket.

Relationship (M/M_0) is usually represented by μ ($\mu \leqq 1$). The reciprocal of μ is known as the Tsiolkovski number. The smaller the μ the greater is the velocity of the rocket.

However, μ cannot diminish without limit. When all the fuel is used up the mass M becomes equal to the mass of the empty ("expended") rocket M_b. Here, the greatest (final) velocity of the rocket is

$$v_b = -w_e \ln \mu_b. \tag{1.11}$$

This formula is often written in the form

$$v_b = w_e \ln \frac{1}{\mu_b} = w_e \ln \left(\frac{M_b + M_x}{M_b}\right)$$

where M_b is the mass of the rocket at burnout, and M_x is the mass of the expended fuel.

The magnitude of μ_b determines the quality of the design of the rocket. The smaller the μ_b the greater is the velocity that can be attained by the rocket with a given exhaust gas velocity, w_e, and the more perfect the design.

An idea of the real significance of μ_b can be obtained if it is recalled, as an example, that the initial weight of the V-2 rocket used in World War II, was 14.1 tons and the expended weight (less fuel) was 4.3 tons. Consequently, for this rocket $\mu_b \approx 0.3$. Taking into account that the design of this rocket is far from perfect and could be easily improved, the above number should be looked upon as a lower, easily achieved index of the quality of design. At the same time it is difficult to imagine the possibility of appreciably decreasing the magnitude of μ_b (even in the order of magnitude of 0.1) for single stage rockets, especially if it is remembered that the weight of the expended rocket includes the payload (instruments or rocket equipment).

So, it can be said that the magnitude of μ_b for single stage rockets of realistic design is within the limits 0.3 to 0.1.

The problem of increasing the design index (μ_b) by indirect means was investigated by C. E. Tsiolkovski, who first proposed the idea of a multistage rocket, disposing of fuel tanks after the expenditure of fuel (we shall take this question up later), and F. I. Tsander, who proposed the use of empty fuel tanks as fuel.

Greater velocity v_b can be attained, as follows from expression (1.11), not only by reducing the magnitude of μ_b, but also by increasing the effective gas velocity w_e.

If we turn again to the abovementioned V-2 rocket, whose motor used alcohol and liquid oxygen, we shall see that the exhaust gas velocity of combustion products from the motor nozzle has a magnitude approximately equal to 6,550 ft/sec. Therefore, according to expression (1.11), the ideal final velocity for this rocket is

$$v_b = -6{,}550 \ln 0.3 \approx 7{,}860 \text{ ft/sec.}$$

Actually, due to the acceleration of gravity and air resistance, the maximum velocity is reduced to 4,900 ft/sec (roughly speaking, a reduction of 3,000 ft/sec).

On the basis of potential heat release calculations of certain fuels (for instance, hydrogen with oxygen), it can be concluded that in the not too distant future exhaust gas velocities of the order of 9,850 ft/sec may be reached. Then, taking the upper limit (the lowest magnitude) of the index μ_b we get

$$v_b = -9{,}850 \ln 0.1 \approx 22{,}600 \text{ ft/sec.}$$

The ideal velocity of such an imaginary rocket approaches the so-called first cosmic velocity v_1, i.e., that velocity which will enable a rocket with an inactive motor to become an artificial satellite of the earth, rotating around the earth in a circular orbit (Fig. 1.5).

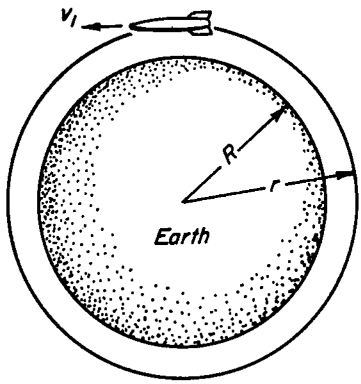


FIG. 1.5. Determination of orbital velocity.

This velocity is calculated by equating the force due to the earth's attraction (force of gravity) and the centrifugal force at flight radius r:

$$Mg_r = Mv_1^2/r$$

where g_r is the acceleration due to gravity at radius r. But

$$g_r = g(R^2/r^2)$$

and therefore

$$v_1 = \sqrt{gR^2/r}.$$

When the values of r are close to the radius of the earth, $R = 3{,}960$ miles,

$$v_1 = 26{,}000 \text{ ft/sec}.$$

Greater velocities may be attained with the aid of multistage rockets.

2. The Ideal Velocity of a Multistage Rocket

The basic requirement of a rocket, as of an aircraft, is to impart a given velocity to a given payload (equipment of the rocket, instruments, or a warhead).

The required supply of propellant depends on the magnitudes of a given payload and a given velocity. The greater the payload and the required velocity, the greater is the amount of fuel which must be aboard the rocket. On the other hand, the greater the fuel supply the greater is the weight of the rocket structure.

This happens, first of all, because the weight of fuel tanks increases with their volume and also because with the increase in the dimensions of the rocket greater demands are made on the structural integrity. The last condition leads to the need for strengthening the structure, thereby increasing its weight.

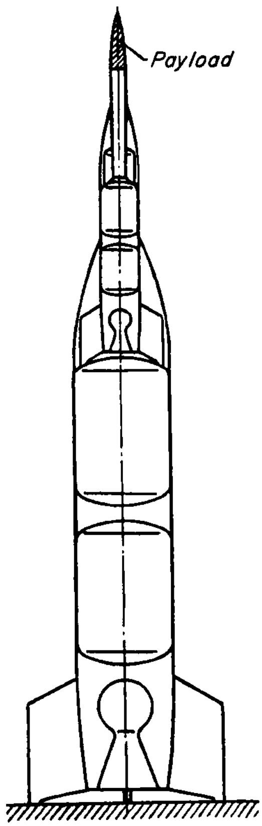


FIG. 1.6. Schematic of a three stage rocket.

It is precisely at this point that the shortcomings of single stage rockets becomes pronounced. The required velocity in these rockets is imparted not only to the payload but to the entire structure of the rocket, which leads to the expenditure of large amounts of extra energy. These expenditures are so great that, for instance, the cosmic velocity can never be achieved with a single stage rocket without radically increasing the gas velocity w_e.

Multistage rockets are partially free from the above drawback.

The term multistage rocket means a rocket composed of several rockets arranged, for instance, as shown in Fig. 1.6. Initially the first, the most

powerful, motor capable of lifting and accelerating the entire system to a certain speed is ignited. After that, when the main part of the fuel is expended, the starting motor (the motor of the first stage) together with the empty tanks can be dropped. The flight of the rocket continues further by means of the motor of the second stage, less powerful, but capable of imparting to the lightened rocket additional velocity.

Then, as the fuel of the second stage is burned, the motor of the third stage can be ignited and the second stage must be dropped from the rocket. The described process of the separation of stages can theoretically be continued further. In practice the number of stages is determined by the purpose of the rocket and the degree of structural difficulties which arise with the increase in the number of stages. As differentiated from a single stage rocket, the required velocity is imparted not to the entire structure but only to the payload and to the last stage. Lower velocities are imparted to the masses of the preceding stages.

Let us designate the relationship of the mass of the expended rocket of the first stage to the initial mass of the entire rocket by μ_1. Let us designate the relationship of the mass of the second stage, without fuel, to that mass of the rocket immediately after the separation of the burned out first stage by μ_2. By analogy the subsequent stages will be designated as $\mu_3, \mu_4, \ldots \mu_n$.

After the fuel of the first stage is expended the velocity of the rocket will be

$$v_1 = -w_{e_1} \ln \mu_1.$$

After the fuel of the second stage is expended, the above velocity will be increased by

$$v_2 = w_{e_2} \ln \mu_2.$$

Each succeeding stage will give an analogous addition to velocity. The total will be

$$v_b = -w_{e_1} \ln \mu_1 - w_{e_2} \ln \mu_2 \cdots - w_{e_n} \ln \mu_n.$$

If the gas velocity in each of the stages is the same ($w_{e_1} = w_{e_2} = \cdots w_{e_n} = w_e$), then

$$v_b = -w_e \ln \mu_1 \mu_2 \cdots \mu_n.$$

The magnitude of each μ is less then unity. With a sufficiently large number of stages (four or five) the product $\mu_1\mu_2 \ldots \mu_n$ can become sufficiently small and the magnitude of $\ln (\mu_1\mu_2 \ldots \mu_n)$ becomes large, which corresponds to large velocity.

This can be illustrated by the following simple calculation. Let us show, for instance, that with the aid of a three stage rocket with motors using existing fuels it is possible to reach cosmic velocities.

If for μ_1, μ_2, and μ_3 we take a realistic value such as 0.2 and if w_e is equal to 7,250 ft/sec, which for contemporary motors is not unusual, then we will find

$$v_b = -7{,}250 \ln 0.2^3 \approx 34{,}800 \text{ ft/sec.}$$

Substracting from this velocity 6,550 to 8,200 ft/sec as a further approximation in terms of losses due to aerodynamic drag and earth's gravitation, we shall get a number approximately equal to the first cosmic velocity—the velocity of the artificial earth satellite. It is necessary to point out that the practical construction of such a rocket would meet with unusually large technical difficulties. Suffice it to say that the starting weight of such a rocket would be 100 to 200 tons with the useful load approximately 22 to 33 lb.

Tsiolkovski formulas, not having taken into account the loss of velocity due to air resistance and earth gravitation, give only the upper limit of the rocket velocity. Nevertheless, in those instances in which the thrust of the motor is great compared to aerodynamic drag or in which the action of the rocket motor is short (as in artillery solid propellant projectiles), the Tsiolkovski formula is sufficiently accurate. Even for long range rockets with a long burning time and considerable losses in velocity due to earth's gravitation the Tsiolkovski formula is quite useful, since it enables us, as we have seen, with the aid of approximate calculations to evaluate the velocity capabilities of the rocket quickly.

3. The Influence of Earth's Gravity

Now let us see how the earth's gravity influences the motion of a rocket in the simplest case of vertical take-off.

In this case the expressions (1.5) and (1.8) for flight in a vacuum yield

$$M(dv/dt) = w_e m - Mg$$

where Mg is the weight of the rocket.

If during the action of the motor the distance of the rocket from the center of the earth changes insignificantly, the magnitude of g can be considered constant. Then the last expression can be easily integrated:

$$dv = -w_e(dM/M) - g dt$$

and

$$v = -w_e(\ln M - \ln C) - \mathrm{gt}$$

as in the case of the ideal flight, when $t = 0$, $v = 0$, and $M = M_0$. Therefore, $C = M_0$ and

$$v = -w_e \ln \mu - gt.$$

This expression could have been written immediately. We have only to keep in mind that, because of earth's gravitation, the ideal velocity v is reduced by an amount equal to the velocity attained by a freely falling body during the time t (in our case t is the action time of the motor).

Therefore, if, in the ideal case, the velocity of a rocket is independent of the regime (duration) of combustion and is determined only by the relationship of masses of the empty and the loaded rocket in the presence of gravitational force, the velocity imparted to the rocket depends on the rate of fuel consumption. The quicker the fuel is burned, the smaller will be the time t, and the greater will be the velocity v.

We can conceive of a case in which the velocity will be equal to zero. This will take place, for instance, if the fuel burns gradually at a slow rate of consumption and the motor develops thrust not exceeding the weight of the rocket.

One way or another we must strive to increase the thrust of the motor. However, the increase in thrust cannot be without limit. First of all, large thrusts give rise to great accelerations and consequently large inertial forces on the rocket. This reflects on its structural integrity and results in increased weight. Large inertial forces may affect the performance of the instruments. If the rocket is to carry living beings, the acceleration has to be limited further in order to insure their safety.

Furthermore, high rates of fuel discharge lead to increased dimensions of the motor and again result in the increased rocket weight.

Aside from all this, in the case of the long range rockets, whose trajectories, for the most part, are beyond the limits of the atmosphere, the rapid increase in velocity leads to considerable losses due to aerodynamic drag. It is more advantageous for such rockets to accelerate gradually in order that the lower, dense atmospheric regions be traversed at low velocity (low drag). The upper regions of the atmosphere can be traversed at greater velocities. It follows that, for a specific rocket, it is necessary to choose an optimum relationship between velocity losses due to earth's gravitation and aerodynamic drag and losses resulting from the relative weight of the structure, i.e., the number μ_b.

The velocity losses caused by earth's gravitation for an actual rocket can be evaluated by the example of the above-mentioned V-2 rocket, whose motor action time was approximately 64 sec. For the vertical take-off this rocket has a velocity loss, gt, equal to 2100 ft/sec, with a previously calculated ideal velocity of 7900 ft/sec. Since the rocket rises vertically only during the first 4 sec, and continues further along an inclined trajectory, the component of velocity loss along its axis due to earth's gravity is somewhat less.

C. Operating Efficiency of a Rocket Motor

Qualitatively, the operating efficiency of a rocket motor is analogous to the efficiency of any other heat engine and is determined by the ratio of useful work to the work expended.

The losses of a rocket motor consist of internal heat losses and losses resulting from the loss of kinetic energy in a gas jet.

The over-all thermodynamic efficiency of a rocket motor is the ratio of the kinetic energy of ejected gases to the thermal energy (heat content) of the fuel. Its magnitude is determined by the characteristics of the fuel and the combustion process and varies between the limits 0.3 to 0.5 (at best). The factors which influence the magnitude of the over-all thermodynamic efficiency shall be discussed later in the section devoted to processes in combustion chamber and nozzles. Here we shall discuss only the operating efficiency. This term is applied to the ratio of useful work of the gas jet to its kinetic energy. Let us derive an expression for the operating efficiency of a rocket motor.

Let a mass of fuel ΔM, which is ejected from the nozzle with the velocity w, burn during the time Δt. This mass has the kinetic energy

$$\Delta M w^2/2.$$

Until this fuel was burned it possessed the energy

$$\Delta M v^2/2$$

where v is the velocity of a rocket at a given moment.

Therefore, the mass leaving the rocket has absorbed an energy

$$\Delta M w^2/2 + \Delta M v^2/2.$$

The second term corresponds to the initial expenditure of energy. Before the fuel is burned we are compelled to carry it in the rocket, imparting to it, whether we wish to or not, a certain kinetic energy at the expense of other particles of burning fuel. Because of this the mass of fuel, from the standpoint of energy evaluation, becomes more "expensive."

Now let us see which part of this energy is the useful energy.

The mass ΔM ejected during the time Δt creates a thrust

$$\frac{\Delta M}{\Delta t} w$$

(component of thrust $S_a(p_a - p)$ is neglected), which in a distance $v\Delta t$ does the work

$$\frac{\Delta M}{\Delta t} wv\Delta t = \Delta M wv.$$

This work, then, is the useful work received from the over-all kinetic energy of mass ΔM. Therefore, the operating efficiency of a rocket motor is

$$\eta_e = \frac{\Delta M w v}{\Delta M w^2/2 + \Delta M v^2/2}$$

or

$$\eta_e = \frac{2v/w}{1 + v^2/w^2}.$$

The operating efficiency, as we see, is the function of flight velocity v (Fig. 1.7). When $v = w$ it has its greatest magnitude. This is understand-

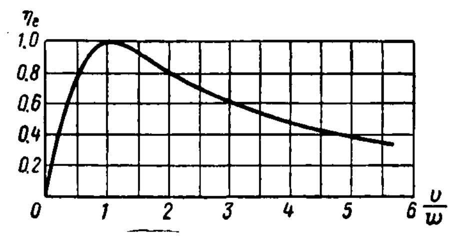


FIG. 1.7. The dependence of the over-all efficiency coefficient on the ratio of flight velocity to gas exhaust velocity.

able. When $v = w$ the velocity of ejected gas relative to the earth is equal to zero. The ejected particles have no velocity relative to the point of origin and the entire kinetic energy of the jet is converted into the energy of the rocket. We must not conclude, however, that the velocity of a proposed rocket must be based on the velocity of ejected gases. The point is that, from the energy standpoint, the evaluation of the quality of the engine by the operating efficiency is only a relative comparison useful in the solution of problems of a similar nature. At the present time the rocket is the only apparatus which enables us to attain velocities approaching interstellar velocities. The problem must be solved, and if there is no choice in the method of its solution, the operating efficiency becomes of secondary importance.

II. Types of Jet Propelled Aircraft and Their Basic Construction

A. Pilotless Aircraft and Their Purpose

In a wide sense, a rocket as an aircraft belongs, at the present time, to the general class of the so-called pilotless aircraft, i.e., aircraft which do not have a pilot on board to guide them. We say "at the present time" since we can conceive of a time when man will so develop rocket technology that he will, with confidence, utilize the rocket as a means of transportation for interplanetary travel.

In rocket technology, as in every rapidly developing new branch of technology, there is no stabilized, widely accepted terminology. Quite often similar types of apparatus have different names and, for certain types, accepted names have not been developed at all. It is not our intent to give here any kind of a general classification of the types of rockets. Such a classification at the present time would be very difficult and, however carefully it was compiled, it would contain a multitude of uncertainties, open to a purely academic discussion. Therefore, we shall further touch upon the question of rocket classification only to the extent that will be necessary for the continuity of description of the general scheme of aircraft.

The shape of the rocket, its dimensions, and construction are determined, in the final analysis, by the demands which are placed upon it. Therefore, the survey of existing types of rockets shall be made consistent with the purpose of the rocket. First, we will consider the missiles of the surface to surface artillery which can be subdivided into missiles of short, intermediate, and long range. Then we shall discuss the principle schemes of aircraft, anti-aircraft rockets, and glide bombs. In conclusion we will give a short description of the construction of certain special rockets.

Such a division is not the only one possible. Rockets can be divided according to the methods of aiming and according to the methods of guiding. Rockets can be classified according to the types of fuel used, according to construction details, according to types of motors, etc.

Depending upon the location of the launcher and the target area, military pilotless aircraft quite often are subdivided into the following classes: "surface to surface," "surface to air," "air to air," "sea to air," etc. The first word signifies the location of the launcher, i.e., ground, air, or water. (Translator's Note: In most cases water and land based missiles are classi-

fied as "surface" missiles.) Correspondingly, the second word indicates the location of the target.

This, however, can give rise to various uncertainties. For instance, it is not clear to which class a missile belongs when it is launched from an aircraft taking off from land or water and carrying the pilotless missile for some distance from the point of take-off. A new term is coming into use which covers the above-mentioned type of missiles: stand-off missiles. It is not clear how to classify a missile which can, with equal success, be launched from different launching sites at the same target or from the same launching site at different targets.

Regardless of the multitude of existing types and constructions of rockets and pilotless aircraft in general, there are certain elements of construction which are characteristic of them all.

Each missile has a motor installation which supplies thrust according to the reaction principle. The motor installation includes the motor (combustion chamber with the nozzle, and a cooling system), a method of supplying the fuel to the combustion chamber, and a system of combustion control.

It is a characteristic of all rockets that they have a supply of fuel which insures the functioning of the motor. In some cases the fuel supply is located within the combustion chamber itself, as in the case of the powder or solid propellant rocket.

In order to insure the correct flight path, it is necessary to introduce instruments for guiding the rocket in flight. Therefore, many rockets are equipped with a means of steering or stabilizing. This includes instruments which react to the attitude of the rocket in space and which may be located on board a rocket or on the ground (if the rocket is guided from the ground). Auxiliary servo and tracking installations are also included in guidance means. Finally, in the same category are the guiding controls; guiding mechanisms with air and gas vanes. If the rocket is not guided, then it is equipped with either an aerodynamic stabilization system such as empennage; or mechanical stabilization, by imparting a rapid rotation about the longitudinal axis.

Each missile must have a section assigned to the payload which must be delivered to target. If the rocket has a military application then this, as a rule, is the warhead. If the missile has an experimental goal then the payload is scientific instrumentation and radio transmitters.

All parts of the missile must be held together by the structure of the fuselage. In many cases the function of the fuselage is performed by the walls of the fuel tanks, and in the case of the solid propellant rockets by the walls of the combustion chamber. Finally, in speaking of rockets and pilotless aircraft, it is impossible to pass without mentioning a whole complex of ground equipment and above all launching installations.

For small solid propellant rockets without guidance, the launching installation consists of a simple arrangement of launching rails or guides, installed on self-propelled vehicles or airplanes. For long range liquid rockets, launching is accomplished from a launching pad with a complex system of auxiliary launching apparatus which is essential for the transportation and the elevation of the rocket on to the pad, supply and loading of the fuel, etc. The ground equipment also includes repair shops and equipment for checking the rocket on the launcher and for guiding and observing the rocket in flight.

The above-mentioned elements of missiles may be seen in the structural examples described in the present chapter. The following chapters will be devoted to a more detailed discussion of the various elements. Construction and performance of motors and the processes taking place within the motor will be discussed in Chapters III–IV. General information about forces acting on the rocket and about conditions in flight will be given in Chapters VII and VIII. Chapter IX will be devoted to the general principles of rocket guidance and stability in flight. Finally, Chapter X will deal with the description of ground equipment.

Let us now discuss in more detail types of rocket construction and pilotless aircraft.

B. Surface to Surface, Short Range Bombardment Missiles

Surface to surface rocket bombardment missiles can be divided into short range, intermediate, and long range missiles.

Targets within reach of the conventional artillery, i.e., up to 30 miles, are ordinarily bombarded by nonguided solid propellant rockets, which, because of their simplicity and mass concentration, can conduct an effective bombardment by areas. These are the missiles of the short range surface rocket artillery.

The Russian rocket used during the Second World War called "Katusha" was typical of this type of armament. A multitude of other solid propellant rockets used by other armies participating in the Second World War belonged to this type of armament. Fig. 2.1 shows a section through a typical solid propellant rocket. The rocket has a combustion chamber, *1*, containing the powder charge of the rocket. This charge oridnarily consists of several powder grains burning at the surface. The ignition of the charge is initiated by a special igniter, *2*. Gases forming as a result of the burning of the powder escape through the nozzle, *3*, resulting in a creation of thrust on the motor. To make sure that the powder grains are not displaced in a longitudinal direction during the combustion process and fragments of powder are not ejected through the nozzle there is a metal screen diaphragm installed at the rear of the combustion chamber.

The combustion chamber of all solid propellant missiles serves at the same time as a container for the entire fuel supply. From one point of view this is very convenient, since the structure of the rocket becomes extremely simple. From another point of view such a structural solution imposes certain restrictions on the action time of the rocket and its range. Greater range demands a greater quantity of fuel. If all of the fuel is contained in

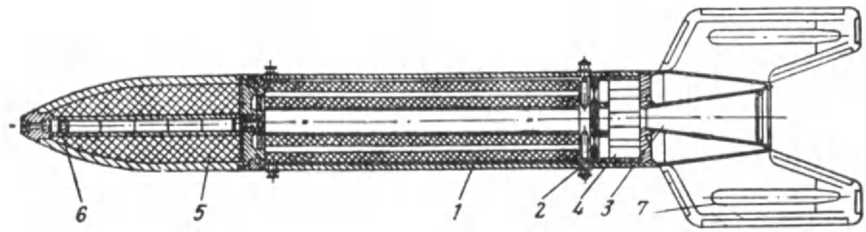


FIG. 2.1. The powder rocket missile: *1*, combustion chamber with the rocket charge; *2*, igniter; *3*, nozzle; *4*, diaphragm; *5*, warhead; *6*, detonator; and *7*, stabilizer.

the combustion chamber, then the volume of the chamber must be increased correspondingly. However, since the chamber is subjected to high pressures and temperature it is necessary to increase its strength, and consequently its weight. Therefore, for long range we must look for a different solution. The ways and means of doing this will be discussed later.

A rocket missile always has a warhead and a detonator, the construction of which depends upon the purpose of the missile. The rocket shown in Fig. 2.1 is designed as an anti-personnel missile.

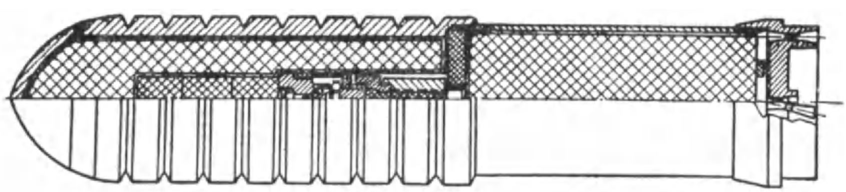


FIG. 2.2. Fragmentation shell.

The fragmentation effectiveness of the rocket must be augmented by lateral and transverse scoring of the warhead, as shown in Fig. 2.2. This is done to achieve a more even distribution of fragments during an explosion.

A delayed action, fused rocket is shown in Fig. 2.3. The warhead is contained in a strong shell enabling the shell to explode after penetrating the armor.

Fig. 2.4 shows the construction of three anti-tank rockets. The common characteristic is the shape of the warhead, which has a cavity at the head end. Such a charge is called the "shaped charge." At its explosion the

shock wave is concentrated in the shape of a narrow jet which has high armor piercing capability (Fig. 2.5).

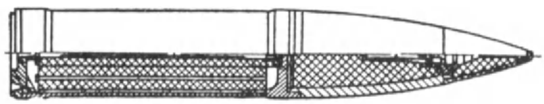


FIG. 2.3. Armor-piercing fragmentation shell.

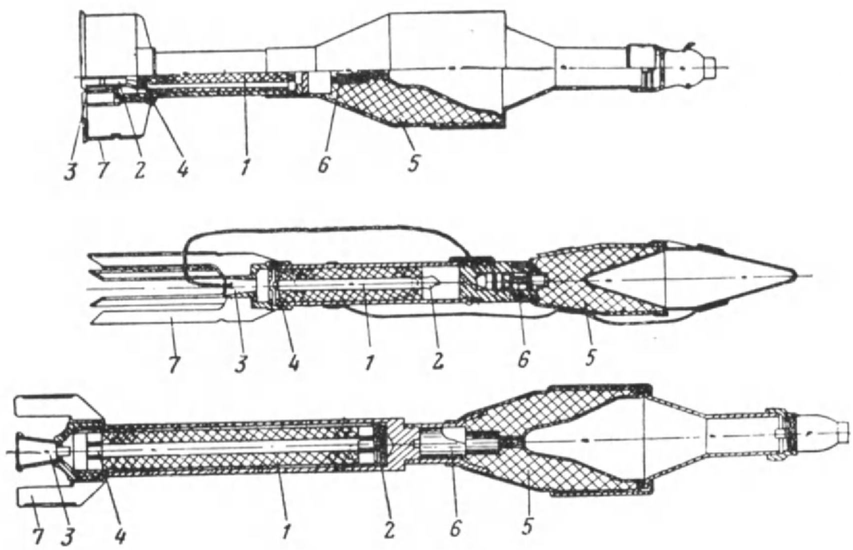


FIG. 2.4. Anti-tank rockets: *1*, combustion chamber with the rocket charge; *2*, igniter; *3*, nozzle; *4*, diaphragm; *5*, warhead; *6*, detonator; and *7*, stabilizer.

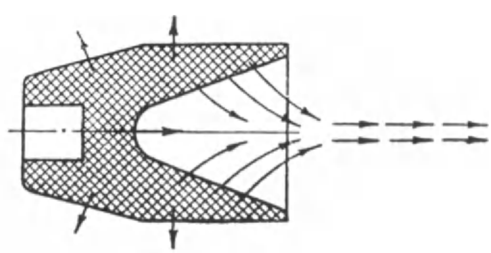


FIG. 2.5. Explosion of a shaped charge.

The warhead of the fragmentation-incendiary missile shown in Fig. 2.6 is equipped with a fragmentation charge. For a purely incendiary purpose the warhead of the rocket is loaded with an incendiary mixture (Fig. 2.7).

Let us point out a peculiarity of the rocket construction as shown in Fig. 2.6. The combustion chamber of this rocket is made of fire resistant ceramic wound on the outside with a layer of thin music wire. This con-

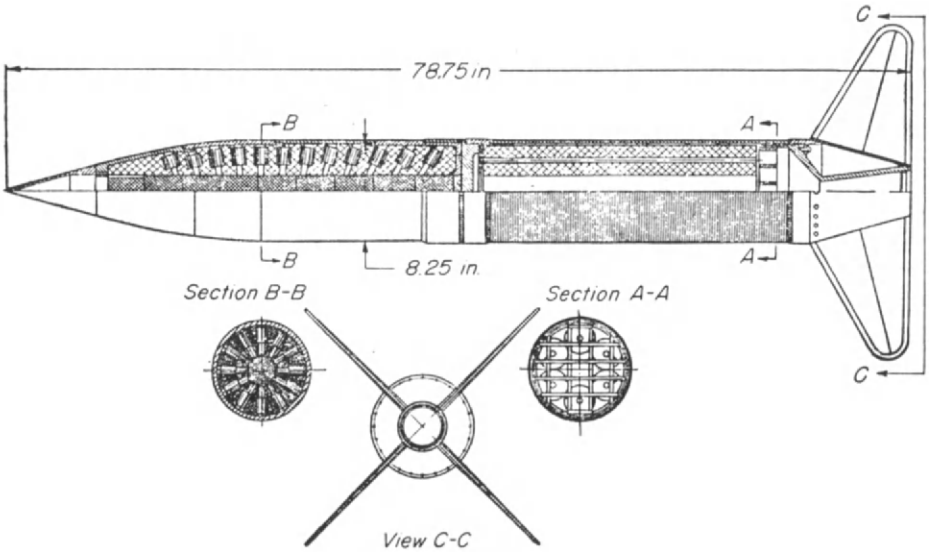


Fig. 2.6. Incendiary fragmentation rocket with the combustion chamber reinforced by thin wire.

struction has a certain weight advantage inasmuch as cold drawn wire has considerably greater strength than ordinary tubing. However, because of technical complications, high cost of the wire, and insufficient reliability, such combustion chambers did not find wide acceptance. Let us return to

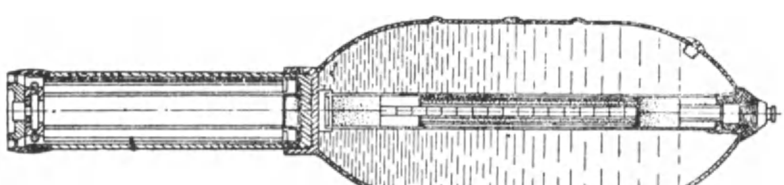


Fig. 2.7. Incendiary rocket.

the missiles shown in Figs. 2.1, 2.4, and 2.6. Each of these missiles is equipped with the empennage 7, necessary for stabilizing the rocket during flight.

This method of stabilization is not the only one possible. For short range powder rockets a common method of stabilization is by means of

imparting to the rocket rapid rotation about its longitudinal axis. In this case the rocket maintains its stability in flight like an artillery shell as a result of the gyroscopic effect. Rockets of this type are called spin-stabilized rockets. In this case the rocket is freed of empennage and the rapid rotation is imparted by the action of canted nozzles.

Fig. 2.8 shows an arrangement of a 380 mm spin-stabilized missile and a mine of a six-barreled German mine-thrower used during the Second World War. Both missiles have canted nozzles. It is interesting to note

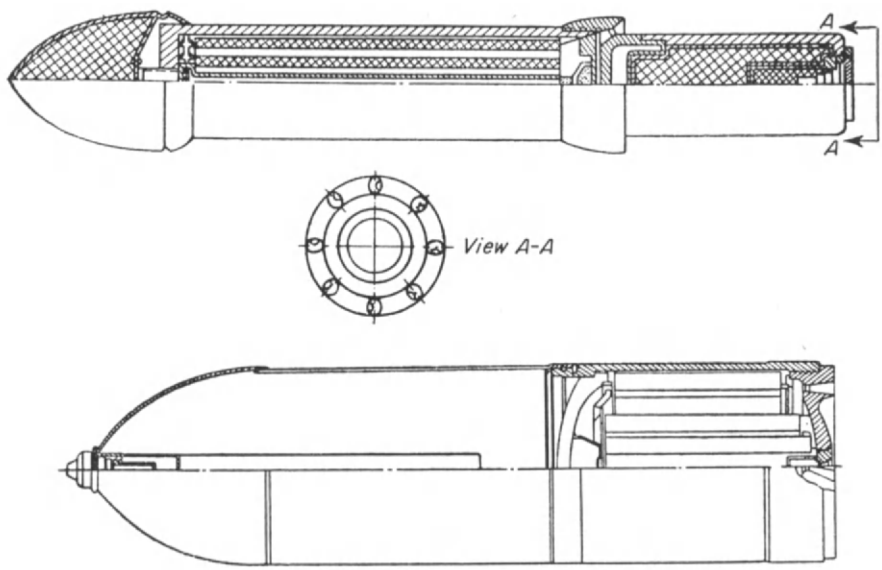


FIG. 2.8. Jet spin-stabilized shells.

that in the latter of the above-mentioned mines the escape of gases takes place through side nozzles arranged about half-way up the length of the missile and the warhead charge is located to the rear of the rocket charge, which should increase the explosive effect, since the explosion at impact takes place above ground level.

The missile shown in Fig. 2.2 is also in the catagory of spin-stabilized missiles.

All solid propellant missiles discussed above are unguided. Aiming is accomplished by pointing the launching installation, similar to the aiming of an artillery barrel. The choice of such an aiming system in this case is quite natural. At short range and firing at relatively large targets there is no necessity for using a guided rocket; for equipping it with expensive instruments and complicated launching installations and for the entire complex of ground equipment. Due to the low cost of the rocket the inaccuracies

can easily be compensated for by the quantity of missiles and this, in the final analysis, is more advantageous.

In some cases, however, it is sometimes desirable, even at short range, to introduce a guidance system for more accurate aiming of the rocket at the target and for controlling it during flight. This has a direct advantage in those cases in which the firing is done at a moving target (for instance, a tank), or against an object which requires not one but several direct hits for its destruction. In such cases, the inaccuracy of unguided rockets can hardly be compensated for by the quantity of rounds, aside from the fact that, in repelling a tank attack, there may not be enough time for a systematic coverage of an area.

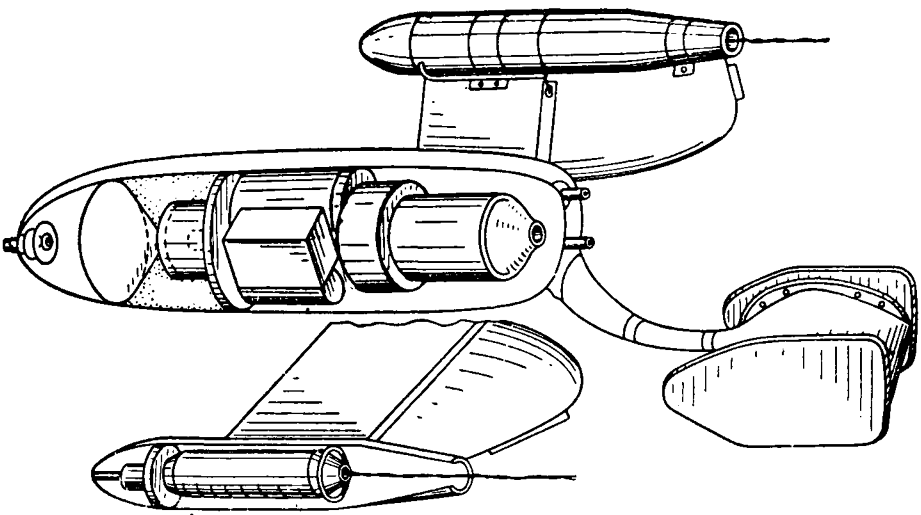


FIG. 2.9. Guided anti-tank missile.

Fig. 2.9 shows a schematic diagram of a guided solid propellant short range anti-tank rocket.

The signals to the rocket during flight are not transmitted by radio, but by two thin, rather strong, steel wires which unwind from a spool during the motion of the rocket. This type of communication, as opposed to the radio system, requires a somewhat simpler implementation and is more stable with regard to external accidental and intentional electronic jamming. At the same time, the system has a certain unreliability in that it does not exclude the possibility of the wires tearing as a result of tangling on the spool or being severed by a shell fragment. The wires may be torn by a shock wave as the result of an explosion.

The introduction of a guidance system for long range missiles is neces-

sary even when bombarding large areas. Long range unguided missiles are much too expensive to make up for their inaccuracy by numbers. Therefore, rockets whose range exceeds 30 miles are, as a rule, guided.

C. Long Range Bombardment Rockets

1. Methods of Increasing the Range of Solid Propellant Rockets

Missiles having a range of from 30 to 400–500 miles are called long range missiles or long range bombardment missiles.

It has been pointed out above that extending the range of solid propellant missiles is fraught with difficulties. The volume and weight of the combustion chamber increase with the increase in the amount of propellant carried. But that is not all. The lack of cooling of the walls of the combustion chamber and nozzle is of considerable importance. When the burning

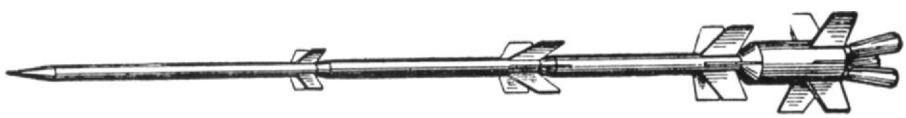


Fig. 2.10. Powder long range multistage rocket.

time is short, the walls don't have a chance to overheat, and therefore retain their strength. In increasing the range, the burning time necessarily is also increased, making the cooling of chamber walls unavoidable. However, the cooling of the solid propellant motor chamber is a complicated problem.

Within the limits imposed by the use of solid propellant, a certain increase in range can be obtained by means of multistage rockets. An example of such an arrangement is the four stage, unguided, solid-propellant rocket shown in Fig. 2.10.

The rocket is launched from inclined rails. When the fuel of the first stage is expended, the first (rear) motor is dropped, while the remaining three stages continue their flight. The motors of the second and third stage are then dropped in turn.

The trajectory of the rocket is shown in Fig. 2.11. The range is approximately 105 miles. The target is reached by the fourth stage, carrying about 88 lb of explosive in its nose. It is worthy of note that, for this payload and range, the total weight of the propellant is 1290 lb.

The characteristic of this rocket is shown by the graph of the increase in velocity (Fig. 2.12).

The ignition of the charge of each succeeding stage takes place at the end of burning of the preceding stage after some delay. The velocity increases while the motor is acting. When the thrust ends, the velocity gradually diminishes due to aerodynamic drag and begins to increase

again after ignition of the motor of the next stage. For that reason the graph of increasing velocity with respect to time is a broken curve (Fig. 2.12). Because the rocket is unguided, its long range dispersion must be quite large and its kill probability cannot be very large. It is not by chance,

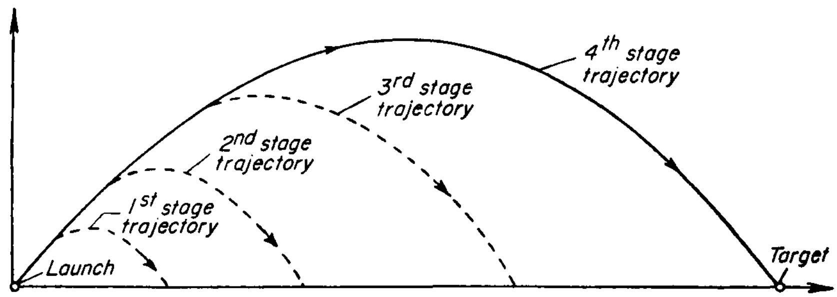


FIG. 2.11. Trajectory of a four stage rocket.

therefore, that such a rocket was not extensively employed during the Second World War and did not become a prototype of subsequent designs.

The range of a missile can be increased by combining gunnery with the jet propulsion principle.

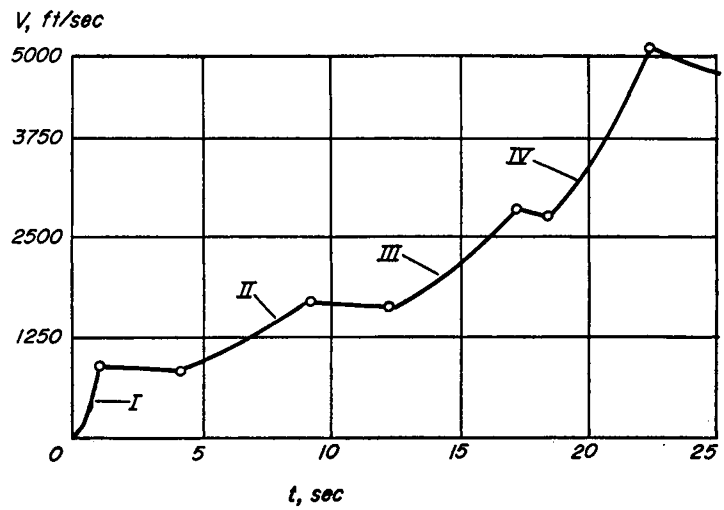


FIG. 2.12. Graph of velocity increase of a four stage rocket.

Fig. 2.13 shows the so-called rocket sustained projectile. The launching of such a missile takes place from an ordinary gun. Additional velocity of the missile is obtained from the combustion of the rocket charge which is located at the base of the missile.

Such a solution, however, cannot solve the entire problem of range and allows only a slight increase in range at the expense of accuracy. For long range, the dimensions of the missile must be large and the range therefore is limited by the gun caliber and its power.

Greater flight velocities and consequently longer range can be achieved by fuels having greater calorific content than powder. The development of contemporary rocket technology follows this road.

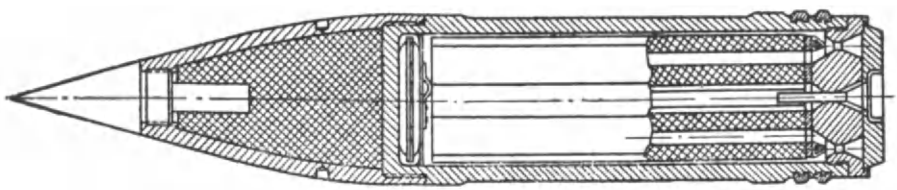


FIG. 2.13. Rocket sustained projectile.

All intermediate range missiles are activated by motors using fuels of maximum heat generating capacity: liquid alcohol, benzine, and kerosene, which are oxidized by nitrogen oxide, atmospheric oxygen, or liquid oxygen carried aboard the rocket.

For intermediate range rockets the powder is used only as a booster during launching, in order to impart to the missile the necessary initial velocity.

Fig. 2.14 shows a schematic of a jet propelled missile of 280 mm caliber, propelled with the aid of a powder motor during the initial part of its tra-

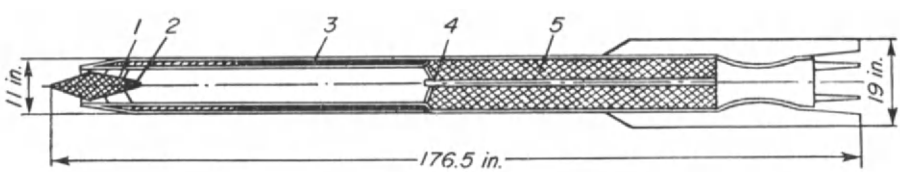


FIG. 2.14. Missile with a composite motor—powder and ram jet: *1*, liquid fuel; *2*, jets; *3*, warhead; *4*, cover with the ignitor; and *5*, powder charge.

jectory. When the missile has attained the necessary velocity, the sustaining ramjet* air breathing motor of the missile takes over; its fuel is benzine with the atmospheric oxygen as the oxidizer.

A missile of this type theoretically is capable of very long range. Specifically, the missile shown in Fig. 2.14 has a range of 320 miles.

An intermediate range missile, compared to a missile of short range, must have considerably greater dimensions. This is obvious because for

* See Chapter III for principles of operation.

greater range with the same warhead, more fuel is required. Aside from that, the warhead itself must increase in size with the increase in range. It is clear that it is senseless to use a rocket with a range of 320 miles with a warhead whose weight is from 22 to 44 lb.

The weight of all intermediate range missiles at the present time is measured in tons; and the weight of the warheads, in hundreds of pounds. Let us discuss some intermediate range missiles of the surface to surface type.

2. Pilotless Aircraft

The term pilotless aircraft denotes an expendable aircraft, similar to an airplane and carrying a warhead. A special characteristic of the pilotless aircraft is that the greater part of its trajectory is spent at constant velocity in an environment in which thrust is equal to air resistance. (For a rocket, the thrust on the motor is considerably greater then the resistance, and a rocket accelerates continuously so long as the motor is working.)

A pilotless missile is ordinarily equipped with an air breathing motor. This may be a pulsejet, a turbojet, or a ramjet. The fuel is carried aboard the aircraft and is oxidized by atmospheric oxygen.

A typical example of a pilotless missile is the V-1 used by the Germans during the Second World War. A general arrangement of this missile is shown in Fig. 2.15.

The power for the missile is supplied by the pulsing motor (pulsejet). The fuel (benzine) enters the combustion chamber, *1*, from the fuel tank, *2*, through nozzles, *3*. Benzine is forced from the fuel tank by compressed air contained in accumulators, *4*. The warhead, *5*, is located in the nose of the missile.

The pilotless missile is a guided missile. The direction of missile flight is controlled by the compass, *6*, interconnected with the gyroscope, *7*. The signals from this gyroscope are transmitted to the steering mechanism, *8*, which operates the vertical flipper, *9*. The altitude of flight is controlled by an aneroid, *10*, and the altitude is corrected by elevators, *11*, operated by the second steering mechanism, *12*.

A windmill, *13*, located on the nose of the missile, rotates with an angular velocity proportional to flight speed. The number of rotations made by the windmill is proportional to the distance flown. It is possible with this arrangement to determine the distance covered by the missile, with proper correction for drift. This is called a range computer. When the missile has covered a given distance the windmill generates a signal and the elevators point the missile down. At this time the pilotless missile is aimed at the target.

The existing pilotless missiles are usually not very large, about 26 to

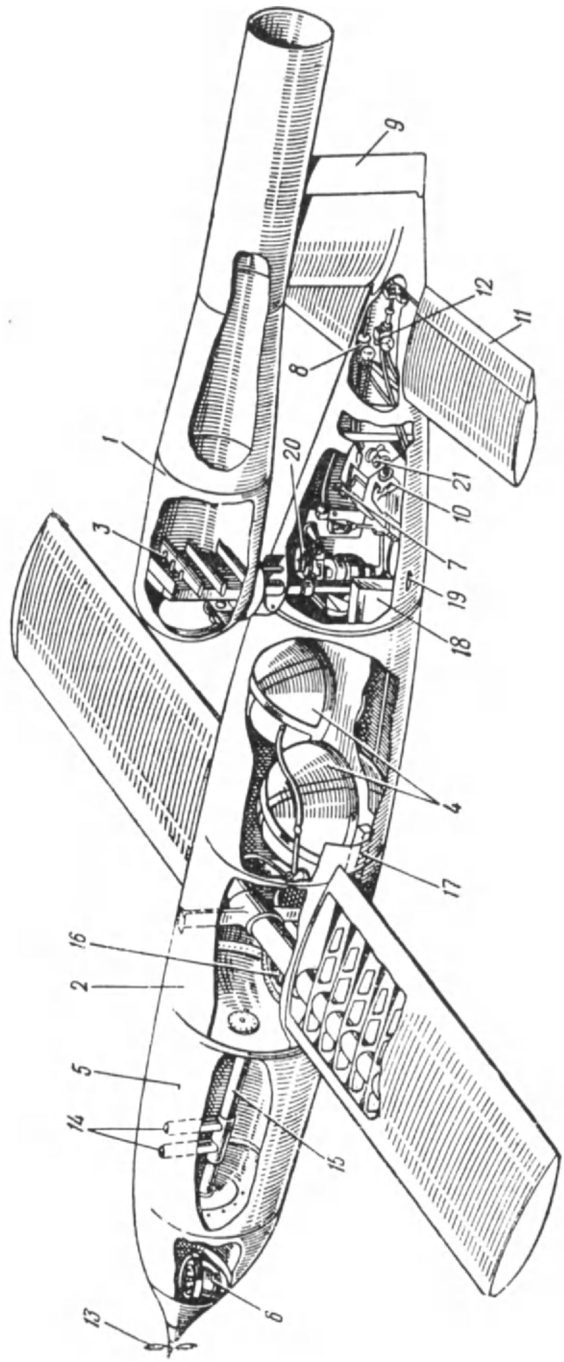


FIG. 2.15. General arrangement of pilotless missile V-1: *1*, combustion chamber; *2*, fuel tank; *3*, fuel jets; *4*, compressed air accumulators; *5*, warhead; *6*, magnetic compass; *7*, gyroscope; *8*, steering mechanism; *9*, vertical rudder; *10*, altitude control; *11*, elevators; *12*, altitude control mechanism; *13*, distance measuring windmill; *14*, explosive charges; *15*, the main explosive charge; *16*, fuel tank filter; *17*, launching rail; *18*, storage battery; *19*, launching contact; *20*, fuel regulator; and *21*, secondary gyroscope.

33 ft long with a wing span of from 20 to 33 ft. The maximum range is measured in hundreds of miles.

The structure of the pilotless aircraft differs substantially from the structure of an ordinary airplane.

These differences are attributable, first of all, to the absence of passengers aboard. Aside from that, the pilotless aircraft, as opposed to airplanes, are expendable. That is why the structure of the pilotless aircraft must be simple, less expensive, and meet somewhat lower standards of structural

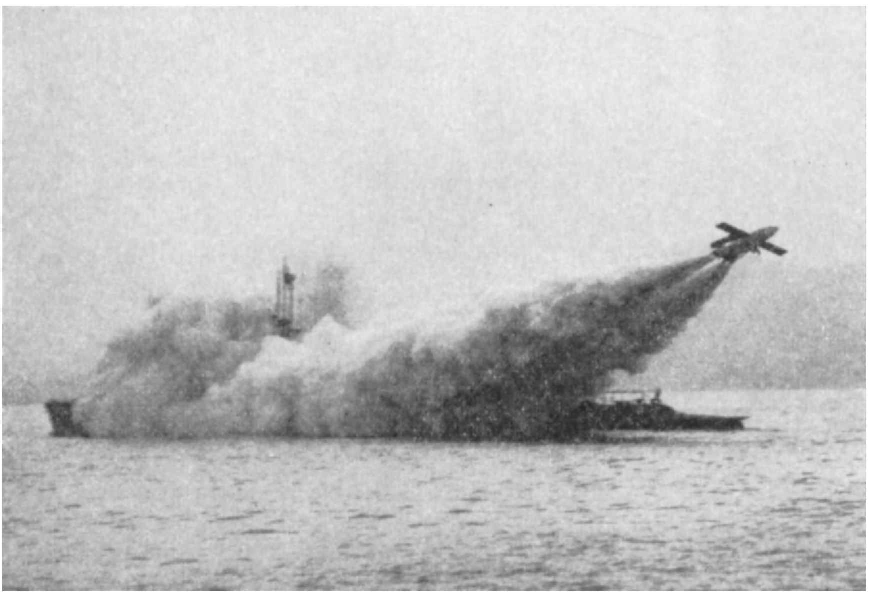


FIG. 2.16. Launching of a pilotless missile from a submarine by means of powder rocket boosters. (Official U.S. Navy photograph)

soundness. This last is explained, in part, by the fact that the fatigue characteristics of metals, so necessary a consideration in airplanes, do not have the same significance in expendable structure applications.

Ordinarily a pilotless aircraft cannot independently attain the sustaining speed at launching and therefore needs a booster.

During the Second World War the Germans used a cumbersone catapult whose energy was derived from the decomposition of hydrogen peroxide for launching pilotless missiles. The pilotless missile obtained the necessary velocity while on the guide-rails of this catapult and continued by means of the thrust of its ramjet motor.

A simpler method of accelerating pilotless aircraft is by means of auxiliary rockets (boosters) which are dropped after the missile has attained

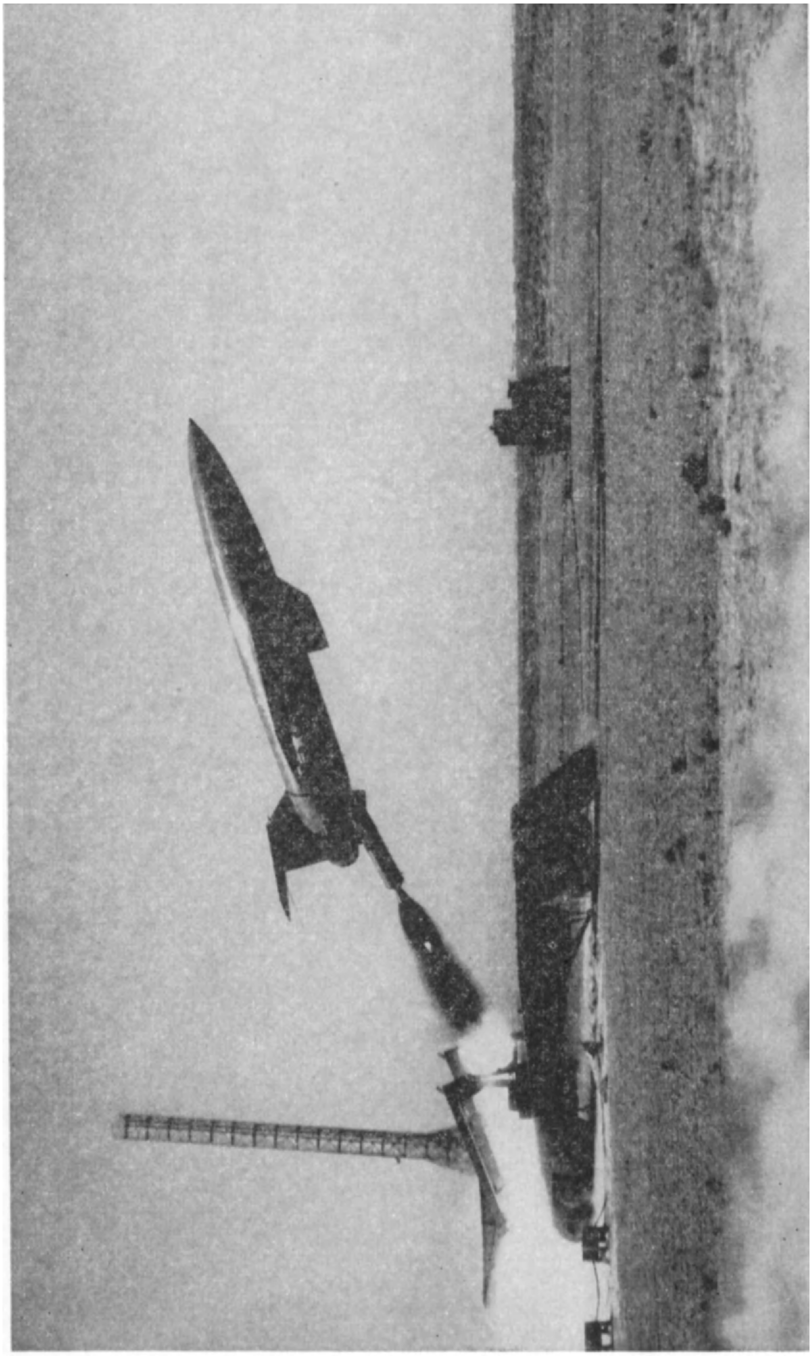


FIG. 2.17. Launching of a pilotless missile by means of powder rocket booster. (Matador launched from transportable launcher). (Official U.S. Air Force photograph)

the necessary velocity. Fig. 2.16 shows the launching of a pilotless missile from a submarine with the aid of boosters attached to the underside of the missile. Pilotless missiles can also be launched from a mother plane that will accelerate the missile to the necessary velocity.

Fig. 2.17 shows the launching of a pilotless missile with the booster in action. The sustaining motor of this pilotless missile is a turbojet. The air scoops for this missile are located in the lower part of the fuselage below the wings. The jet exits through a nozzle located in the tail and is clearly visible in Fig. 2.17.

The basic advantage of the pilotless missile as a weapon is its comparative simplicity and the small amount of fuel necessary for delivering a given load at a given range. For instance, to deliver 2200 lb of explosive to a distance 124 miles away by the missile shown in Fig. 2.15 it is necessary to use only 160 gallons of benzine (not counting, it is true, the expenditure of booster fuel). At the same time, the multistage rocket consisting of four stages described above, at a shorter range, and delivering only 88 lb of explosive, had to burn 1300 lb of powder.

The general disadvantage of pilotless aircraft is their comparatively slow speed (370–620 miles/hr), making them easily vulnerable to air defense countermeasures.

The second disadvantage is the difficulty of insuring a high degree of accuracy in hitting the target by pilotless aircraft. All the flight time of the pilotless missile, which is quite prolonged, is spent in comparatively dense regions of the atmosphere. The direction of flight is affected appreciably by random atmospheric effects whose prediction is impossible. Therefore, correcting for these in flight is often impossible.

3. Ballistic Intermediate Rockets

Pilotless missiles are not the only means of destroying targets located at long distances. This problem can be solved by utilizing ballistic intermediate liquid propellant rockets.

By ballistic we mean rockets whose flight trajectory, with the exception of that part which is traversed with an active motor, consists of a trajectory of a body in free flight. In this sense the above discussed powder rockets are also ballistic. However, the term "ballistic" became associated only with the larger rockets having a range of the order of hundreds of miles. An example of an intermediate ballistic rocket is the V-2 (another designation A-4), whose inboard profile is shown in Fig. 2.18.

This rocket used liquid fuel; ethyl alcohol and liquid oxygen. The heat generating capacity of such fuel is considerably higher than that of powder.

In the accompanying section of the V-2 it is easy to distinguish the basic components characteristic of all rockets.

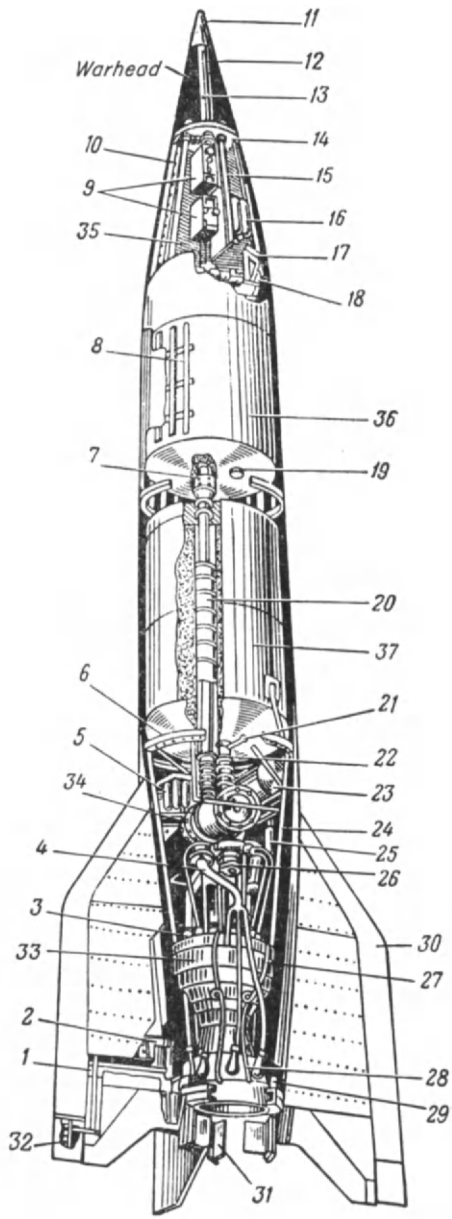


FIG. 2.18. The V-2 long range ballistic rocket: *1*, chain transmission to movable fins; *2*, electric motor; *3*, antechambers; *4*, alcohol feed line; *5*, air accumulators; *6*, aft bulkhead; *7*, solenoid valve for alcohol; 8, rocket structure; *9*, steering apparatus; *10*, piping from the alcohol tank to the head (explosive) section; *11*, nose section with the detonator; *12*, wire conduit; *13*, central tubing of the detonator; *14*, electrical squib; *15*, plywood bulkhead; *16*, nitrogen accumulators; *17*, forward bulkhead; *18*, gyroscopic apparatus; *19*, alcohol drain; 20, alcohol delivery piping to turbopump assembly; *21*, filler neck for liquid oxygen; *22*, sylphons (bellows); *23*, tank with hydrogen peroxide; *24*, motor mount frame; *25*, tank with potassium permanganate (steam generator is located in the rear); *26*, oxygen regulator; *27*, alcohol tubes for cooling; *28*, alcohol exit ports; *29*, steering mechanisms; *30*, stabilizers; *31*, jet vanes; *32*, movable fins; *33*, combustion chamber and nozzle; *34*, turbopump assembly; *35*, guidance section; *36*, alcohol tank; and *37*, liquid oxygen tank.

The motor installation is located in the aft of the rocket. It consists, basically, of a combustion chamber with nozzle *33*, and turbopump assembly whose purpose is to force feed the fuel into the combustion chamber.

The turbopump assembly consists of two centrifugal pumps connected to a turbine. The turbine is activated by the products of the decomposition of hydrogen peroxide (steam plus oxygen).

The products of the decomposition of hydrogen peroxide are formed in a special reactor (not shown in Fig. 2.18). Hydrogen peroxide is fed into the reactor from tank *23*, and decomposes in the presence of a catalyst, calcium permanganate, which is fed from tank *25*. The displacement of these compounds from their tanks is accomplished with the aid of compressed air contained in accumulators, *5*.

Included in the motor installation is a system of valves which regulate the operation of the motor, and a system of tubes which feed the fuel into the combustion chamber as well as into the cooling system of the motor.

The combustion chamber and all elements of the motor assembly are attached to the motor mount, *24*. The thrust on the motor of an intermediate ballistic rocket is measured in tens of tons and exceeds the initial weight of the rocket by approximately a factor of two. For the V-2 rocket, which is discussed here, the thrust at sea level is equal to 28.6 tons, and at altitudes beyond the limit of the atmosphere, it is 33 tons, while the initial weight of the rocket is 14.2 tons.

The action time of the motor is ordinarily 60-80 sec (for the rocket under discussion it is 64 sec).

Detailed examination of the motor, and its operation in particular, will be discussed in the next chapter.

The rocket has two tanks for fuel components (see Fig. 2.18): tank *36*, for alcohol; and tank *37*, for liquid oxygen. Before launching there is approximately 3.9 tons of alcohol and about 5½ tons of oxygen aboard the rocket. Alcohol is fed to the turbine pump through fuel line *20*, which passes through the oxygen tank.

The warhead of the rocket occupies the nose section and contains about 1.1 tons of explosives. The charge is detonated by the exploder, *11*, at rocket impact. In a rocket assigned to experimental purposes, the explosive is replaced by recording and transmitting instruments.

All basic components of a rocket are held together by means of a load carrying airframe, *8*, which consists of a rigid framework composed of longitudinal and transverse members (the first are called stringers and the second bulkheads), which are covered by sheet steel. The airframe is of welded construction. The motor mounting frame is fastened to the lower torsional bulkhead. The upper bulkhead mounts the instrument section, *35*. Such an arrangement is not the only one possible. There are known examples of construction where the walls of the fuel tanks serve as the airframe.

The rocket has four fins, *30*, which are attached to the tail section of

the airframe. As in the case of the airframe, the fins are of welded construction and consist of longitudinal and transverse members covered by sheet steel.

Ballistic rockets are guided and are equipped with automatic stabilization instruments which insure stable flight in a given direction. The direction of flight of the V-2 rocket is controlled by gyroscopic instruments located in instrument section *35*. The same instrument section houses other auxiliary guidance instruments located on the cross-member, *15*.

The actual steering is accomplished by jet vanes and air flippers.

The jet vanes, *31*, are located in the stream (jet) of the exhaust gases. When these vanes are turned, the gas stream is partially deflected from its axial direction and this creates a turning moment which turns the rocket in the desired direction. Since jet vanes do their work in exceptionally severe thermal environment they are fabricated from a material having the highest possible melting point—graphite.

The air flippers, *32*, have an auxiliary function. They are effective only when the rocket travels through sufficiently dense regions of the atmosphere and at high rocket velocity. During flight in a rarefied atmosphere, the rocket is guided by jet vanes.

The system of guidance and automatic stabilization of a ballistic rocket will be discussed in detail in Chapter IX.

Structurally, rockets of this type, in spite of their simplicity in principle, represent an extremely complicated mechanism. Suffice it to say that the rocket shown in Fig. 2.18 has approximately 30,000 details, some of which require a rather high degree of precision in fabrication.

The ballistic rocket is launched from a pad in a vertical direction which is maintained by means of external fins. When the rocket reaches sufficient velocity, it is directed at the target at an angle somewhat less than 45° to the horizon (Fig. 2.19).* At a given velocity and direction of flight, the motor is turned off, after which the rocket continues its flight as a projectile. The maximum velocity of the rocket equals approximately 4900 ft/sec.

The turning off of the motor of a ballistic rocket is done at altitudes at which the atmospheric air is practically nonexistent. From that moment on the rocket becomes unguided and its orientation in space depends upon the effect of random forces which may act upon it the moment the motor is turned off. In re-entering the comparatively dense regions of the atmosphere, the rocket, because of the presence of tail fins, is again oriented along the flight path and, toward the end of its flight, again travels with the nose forward, having at the time of target impact a velocity of 2300–2600 ft/sec.

* The reason for the choice of this angle will be discussed in Chapter VIII.

The kill effectiveness of the ballistic rocket on a target does not depend only on the presence of the warhead. The rocket at impact with the target possesses large kinetic energy. Aside from that, the fuel tanks of the rocket contain the remains of the fuel, which explodes on impact.

Compared to pilotless aircraft, rockets of the type described have a basic uncontradictabe advantage in the velocity at impact. It is difficult to destroy such a rocket in flight. Also, the ballistic rocket is influenced by random atmospheric effects to a lesser degree than the pilotless aircraft, inasmuch as the greater part of its trajectory takes place in the rarefied atmosphere (the greater the range of the rocket, the greater is the portion of its trajectory outside the atmosphere).

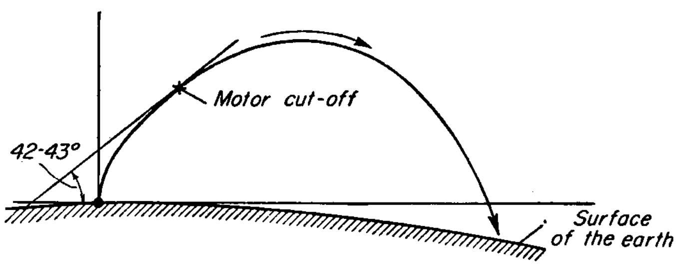


FIG. 2.19. The shape of a long range ballistic rocket trajectory.

However, for delivery of a given payload for a given distance, the ballistic rocket consumes an incomparably greater quantity of fuel than the pilotless missile.

A ballistic rocket represents a heavy and unwieldly apparatus necessitating complicated transportation and launching equipment. Because of this, it is difficult to adapt these rockets to shipboard use, whereas pilotless missiles, requiring less fuel and being readily transportable, can very easily be used as weapons aboard large warships.

As weapons, the ballistic rockets have one more disadvantage. If, a few seconds before impact, the rocket has been spotted by the enemy and the enemy has managed to make a few accurate calculations regarding its trajectory (and in principle this is possible), the trajectory of the rocket path becomes completely defined. It follows, then, that the launching point is also established. Since the launching point of a ballistic rocket represents a complicated aggregate of auxiliary equipment and does not have the freedom of maneuverability, it is obvious that it can be subjected to air strikes, or, in its turn, become the target of rocket bombardment.

At the same time, if we disregard the military goals in the use of a rocket, it can be said that the development of the ballistic rocket is a direct step into interplanetary space. In this sense, liquid rockets, not other types of pilotless aircraft, can yield the most fruitful scientific material.

4. Long Range Rockets

At the present time creating long range pilotless aircraft with a range exceeding 620 miles and extending from 3000 to 6000 miles is in the preliminary design stage. Judging by articles appearing in periodicals, we can conclude that a great deal of attention is devoted to the design of long range aircraft. It is very difficult to evaluate the achievements in this field. But the paths that must be followed for achieving long range are obvious.

In their time, the Germans concentrated a great deal of effort in working out designs for rockets with increased range. Specifically, a variation of the rocket A-4b was proposed. According to the proposal, the trajectory of this rocket was similar to the trajectory of an ordinary ballistic rocket but, further on, the rocket was supposed to glide in the atmosphere with

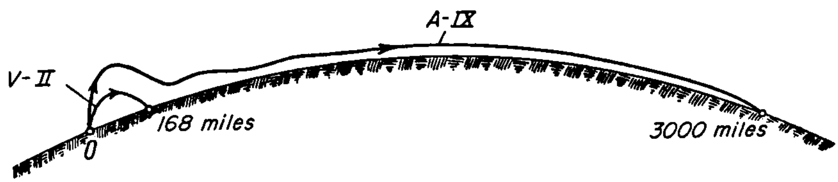


Fig. 2.20. Trajectory of a V-2 rocket and a proposed trajectory of a long range glide rocket A-9.

an estimated extension of its range to 340 miles. There was also a proposal made according to which the V-2 rocket was equipped with wings (rocket A-9) and had a booster (A-10). The rocket was expected to take off along a trajectory similar to that of the V-2 and attain a velocity at the end of the second stage of approximately 10,800 ft/sec. Further, according to the inventors, the rocket had to glide, extending its range to 3000 miles.

Fig. 2.20 compares the trajectories of the glide rocket A-9 and the V-2 rocket.

It is difficult to say what ranges could have been achieved in reality if such projects were actually completed. Undoubtedly, certain difficulties would have been encountered in the execution of these designs, especially in connection with aerodynamic heating, due to high velocities within the atmosphere.

In this connection one now hears of projected designs of long range pilotless aircraft using the liquid motor only as a booster. The booster is dropped and the missile, having a structure similar to that of an aircraft, whose sustaining motor is a ramjet, continues its flight at an altitude of approximately 12 miles. At velocities of hundreds of feet per second the problem of airframe heating would not play a major role. Such an aircraft

would require a comparatively small expenditure of fuel for delivery of payload to great distances.

The basic difficulty in bringing such proposals to realization is in designing guidance instrumentation of sufficient accuracy to guide the missile to its target. At long range this is the determining factor. One method of solving this problem which deserves special attention is automatic celestial navigation. It consists of designing instruments which would automatically orient the aircraft relative to celestial bodies (the sun, moon, and stars). Design of such instrumentation represents a somewhat complicated but not impossible solution. The working out of the principles of celestial navigation is at the present time the key problem in designing long range pilotless aircraft.

D. Anti-Aircraft Rockets

Anti-aircraft rockets, as the term implies. are a means of defense against aircraft. They are used for repelling enemy air attacks. The rockets can insure considerably greater effectivity, as compared to ordinary anti-aircraft artillery, as to range, velocity, and, in the case of guided rockets, accuracy. The greatest majority of anti-aircraft rocket types are intended to inflict direct damage on enemy aircraft, although auxiliary anti-aircraft rockets such as the barrier rocket shown in Fig. 2.21 exist. Immediately before an attack by enemy aircraft, a quantity of these rockets is launched in the vicinity of the defended objective. At the top of the trajectory each rocket ejects a wire truss, whose descent is retarded by a parachute. By these means a barrier is formed. Since the parachutes descent slowly, the barrier is effective for a period of a few minutes, and in case of necessity can be supplemented by launching additional rockets.

Anti-aircraft rockets are divided into guided and unguided rockets. In many respects the firing of unguided rockets is analogous to firing anti-aircraft guns. The accuracy of these rockets is not very great and is compensated for by the number of launched rounds.

The advantages of rocket missiles compared to artillery shells consist, in this case, of greater range and greater firing power. The crux of the matter is that the high altitudes which have become commonplace in aviation today make it necessary to fire large caliber shells, requiring, in turn, guns of greater power, which, due to the large number of guns needed to defend large targets, presents a serious difficulty. The launching installations of unguided anti-aircraft rockets are invariably considerably simpler, lighter, and more transportable.

Fig. 2.22 shows a variation of a powder, single stage, unguided anti-aircraft rocket. Its construction is self-explanatory. Fig. 2.23 shows a liquid propelled variation of the same rocket. Components of fuel, *1* and *2*, are

forced from the tanks and are fed into the combustion chamber by the pressure of powder generated gases which are formed by the burning of powder grain, *3*, located behind the warhead, *4*.

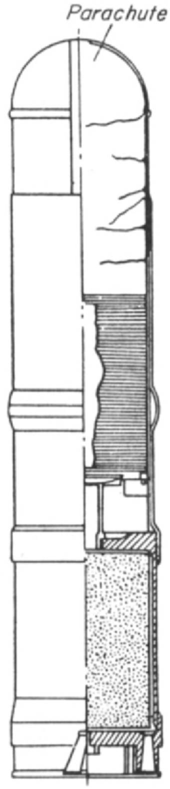


FIG. 2.21. Anti-aircraft barrier rocket (86 mm caliber).

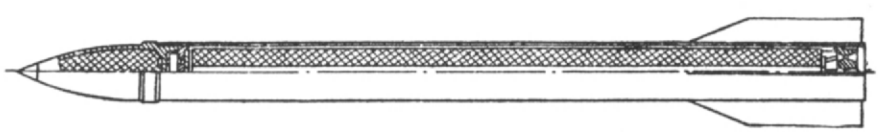


FIG. 2.22. An unguided anti-aircraft powder rocket.

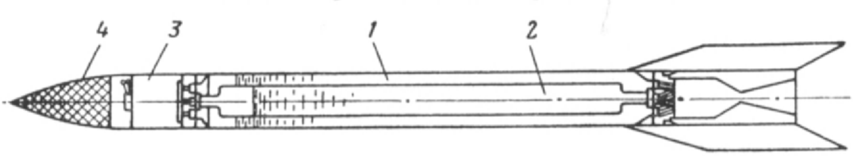


FIG. 2.23. Liquid fueled anti-aircraft unguided rocket: *1*, combustible tank; *2*, oxidizer tank; *3*, powder pressure accumulator; *4*, warhead.

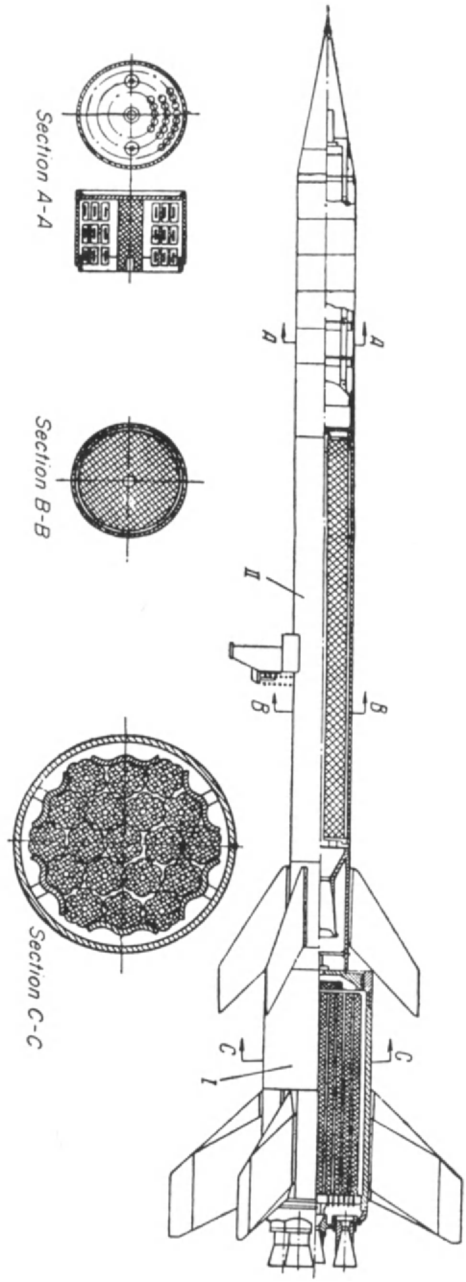


FIG. 2.24. A two stage, unguided anti-aircraft powder rocket.

In order to increase the range of the unguided anti-aircraft rockets, these rockets are often made in two stages. Fig. 2.24 shows a powder, two stage, unguided anti-aircraft rocket. The first stage, *I*, has a propulsive charge in the form of several power grains; the second stage, *II*, in the form of a single powder grain. The warhead (section A-A) contains the fragmentation-incendiary charge for increasing the damage effectivity of the rocket.

A somewhat unique rocket construction is shown in Fig. 2.25. This is the unguided mother rocket, which carries as its second stage several small rockets. These, in their further action, cover a given area. The result is somewhat similar to shooting with "rocket bird-shot" (Fig. 2.26).

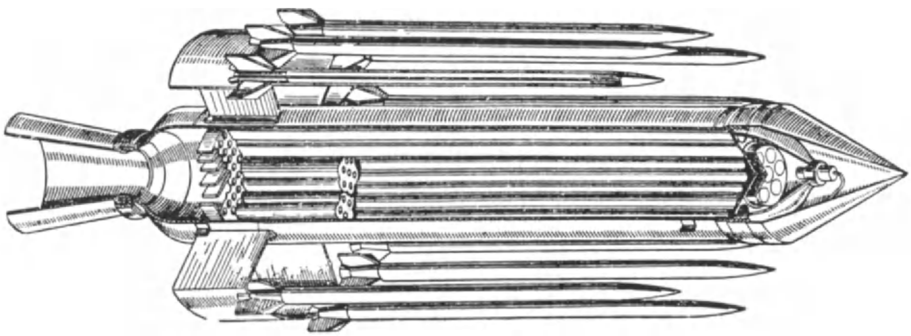


FIG. 2.25. A powder anti-aircraft rocket having a group of smaller rockets as a second stage.

The unguided anti-aircraft rockets, even though they have certain advantages in comparison with anti-aircraft artillery, cannot, in view of their lack of accuracy (especially at high altitudes), solve the problem of an effective air defense. It is there that the guided anti-aircraft missiles are called upon to perform a basic function.

According to the type of guidance, the guided rockets can be divided into rockets guided by commands from the ground and rockets equipped with a homing system. Transmission and receiving of commands from the ground is done by means of radio. One of the principles employed in the homing system is the transmission to and reception of reflected radio signals from a target. The rocket can also home by means of receiving heat or sound waves emitted by the target.

The division of guided rockets into those homing and those guided from the ground is purely arbitrary, inasmuch as it is possible to use both methods simultaneously. Up to a certain point the rocket may be guided from the ground and, at the end of its trajectory, when the target is near and can be "seen" by the rocket, the homing system can take over.

The action of the rocket at the target consists of exploding its warhead

at a distance which will insure the greatest kill effectivity by fragmentation. Inasmuch as fragments, besides the velocity of dispersion, also have the velocity of translation of the rocket as a whole, detonation obviously should take place before the rocket reaches its target. The time of explosion is determined by a special ranging device which measures the distance to target (proximity fuse).

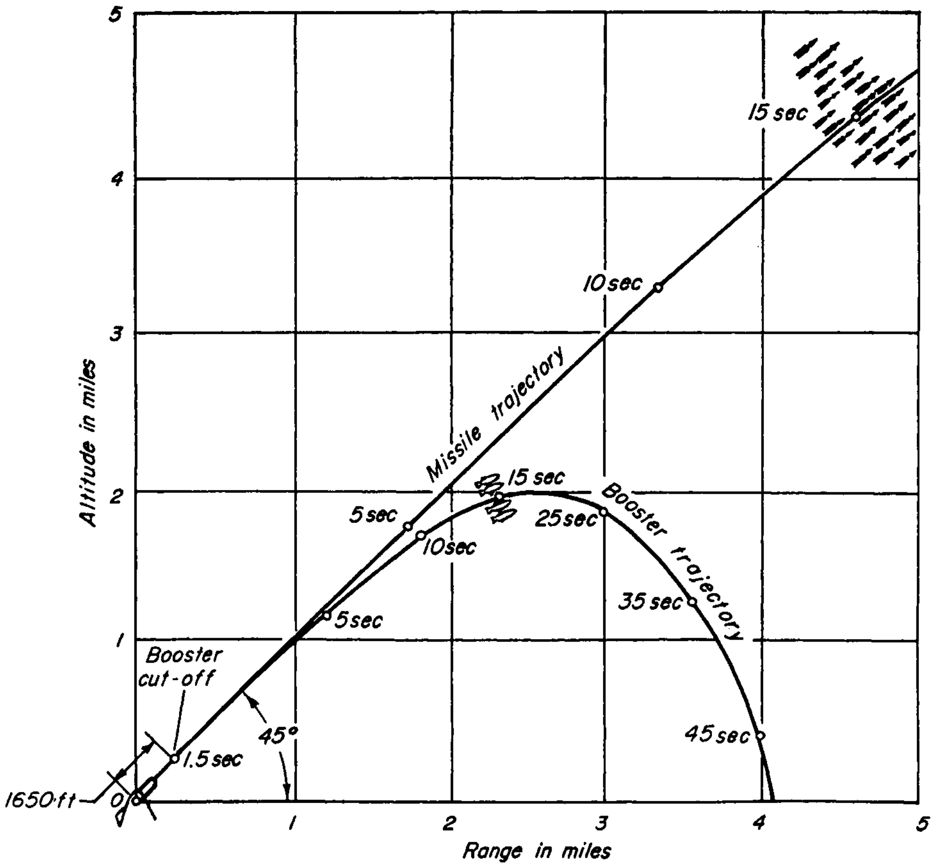


FIG. 2.26. Trajectory of an anti-aircraft mother rocket carrying small rockets.

Fig. 2.27 shows the construction of a ground controlled powder rocket having a booster.

The guidance instruments, *1*, are located in the nose. The rocket is controlled by four air rudders located at the nose. Aircraft with this control arrangement are known as the "canard type." Four fins are installed on the booster, *3*. The booster is dropped 1.5 sec after the launching of the

rocket and the motor of the rocket itself takes over. It is worthy of note that the warhead, *4*, is located behind the fuel supply, *5*, and that the powder-generated gas stream escapes through side nozzles, *6*. The rocket

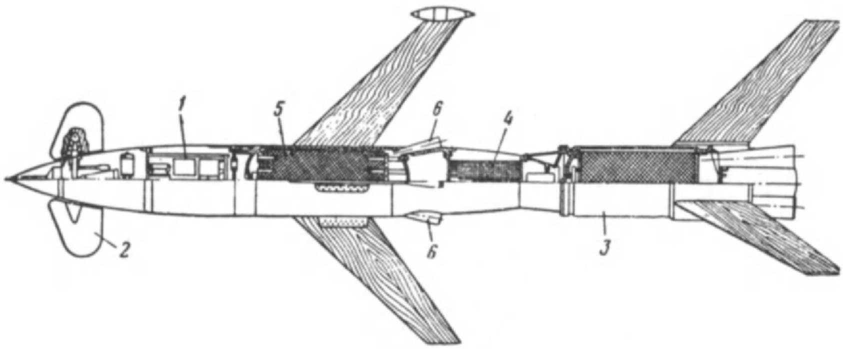


FIG. 2.27. A guided anti-aircraft powder rocket: *1*, guidance instrumentation; *2*, steering fins; *3*, booster chamber; *4*, warhead; *5*, sustaining charge; and *6*, rocket nozzles.

has six wings equally spaced around the periphery. Two of these wings have tip flippers which prevent the rotation of (roll stabilize) the rocket about its longitudinal axis.

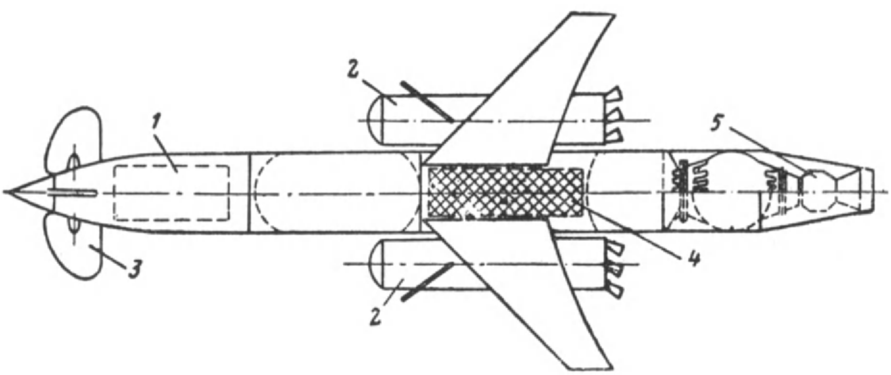


FIG. 2.28. A liquid fuel variation of an anti-aircraft guided rocket: *1*, guidance instrumentation; *2*, booster rockets; *3*, steering fins; *4*, warhead; and *5*, motor.

In flight the rocket is guided toward its target by radio. The trajectory is observed during the entire flight: in case of good visibility, with the aid of an optical theodolite system; and during poor visibility, by means of radio. Arming of the detonator is done by a radio signal.

Fig. 2.28 shows a liquid propelled variation of the same rocket and Fig. 2.29 shows a powder rocket in flight.

Construction typical of intermediate range ballistic rockets can be applied to anti-aircraft rockets, as was done, for instance, in the rocket whose cross section is shown in Fig. 2.31. In this rocket, the fuel is fed from

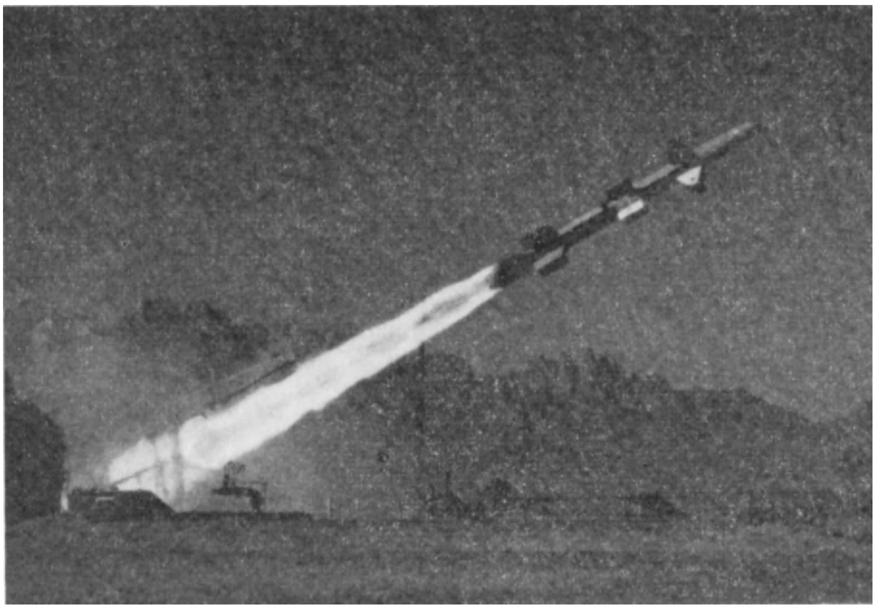


FIG. 2.29. Launching of a Talos anti-aircraft guided missile. (Official U.S. Navy photograph)

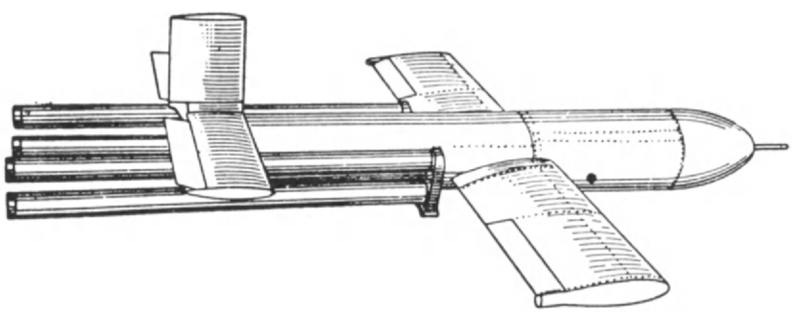


FIG. 2.30. Anti-aircraft rocket with powder boosters.

the tanks into the combustion chamber by compressed air contained in spherical accumulators.

There is considerable variation in types and construction of anti-aircraft guided rockets. Several examples are presented here in accompanying figures. Figs. 2.30 and 2.32 show anti-aircraft short range rockets intended for defense against enemy aircraft.

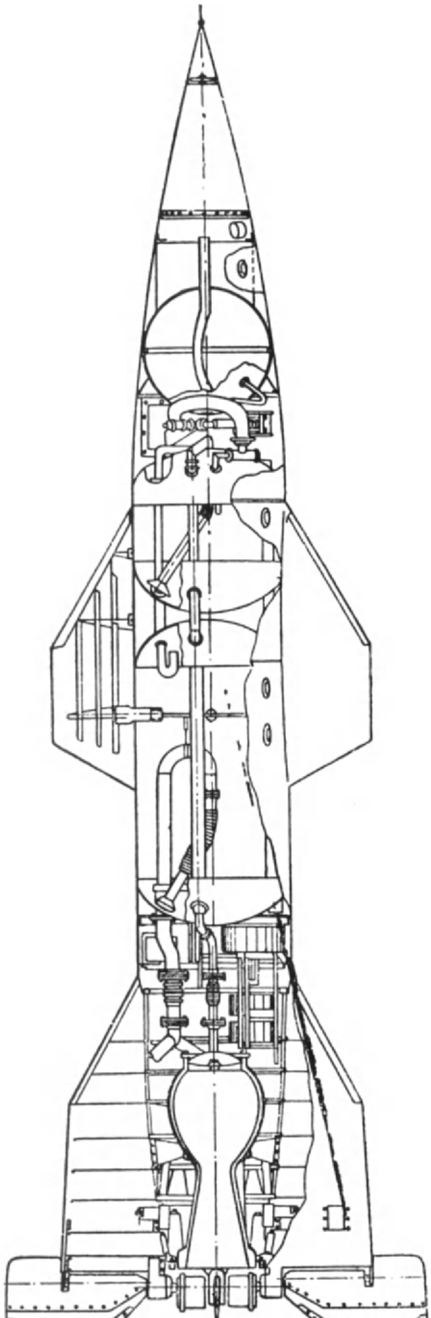


FIG. 2.31. Cross section of a liquid propelled anti-aircraft guided rocket.

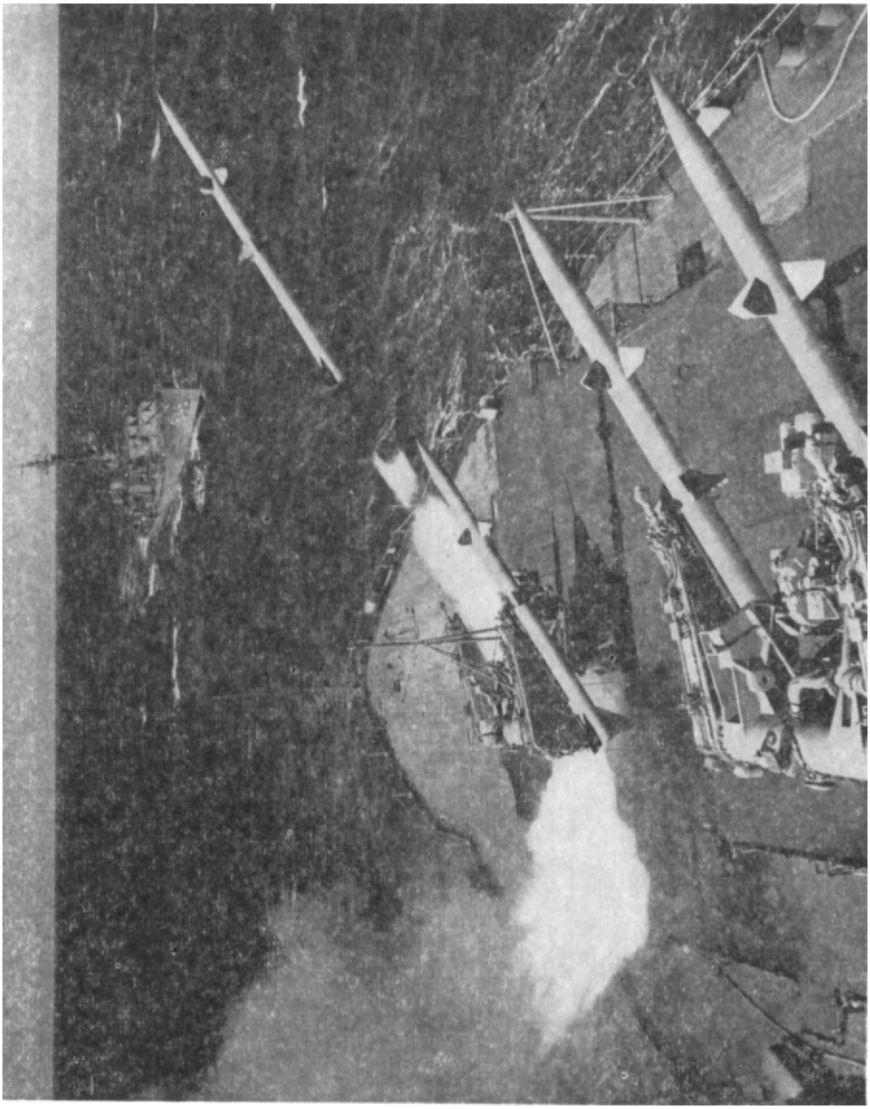


FIG. 2.32. Terrier anti-aircraft missile launched from the deck of a guided missile ship. (Official U.S. Navy photograph)

E. Other Types of Jet Propelled Aircraft

1. Aircraft Rockets

Aircraft rockets may be used against ground and air targets. "Air to surface" missiles are related to aircraft bombs and for the most part are intended for destruction of the target by direct hits. They can be guided or unguided.

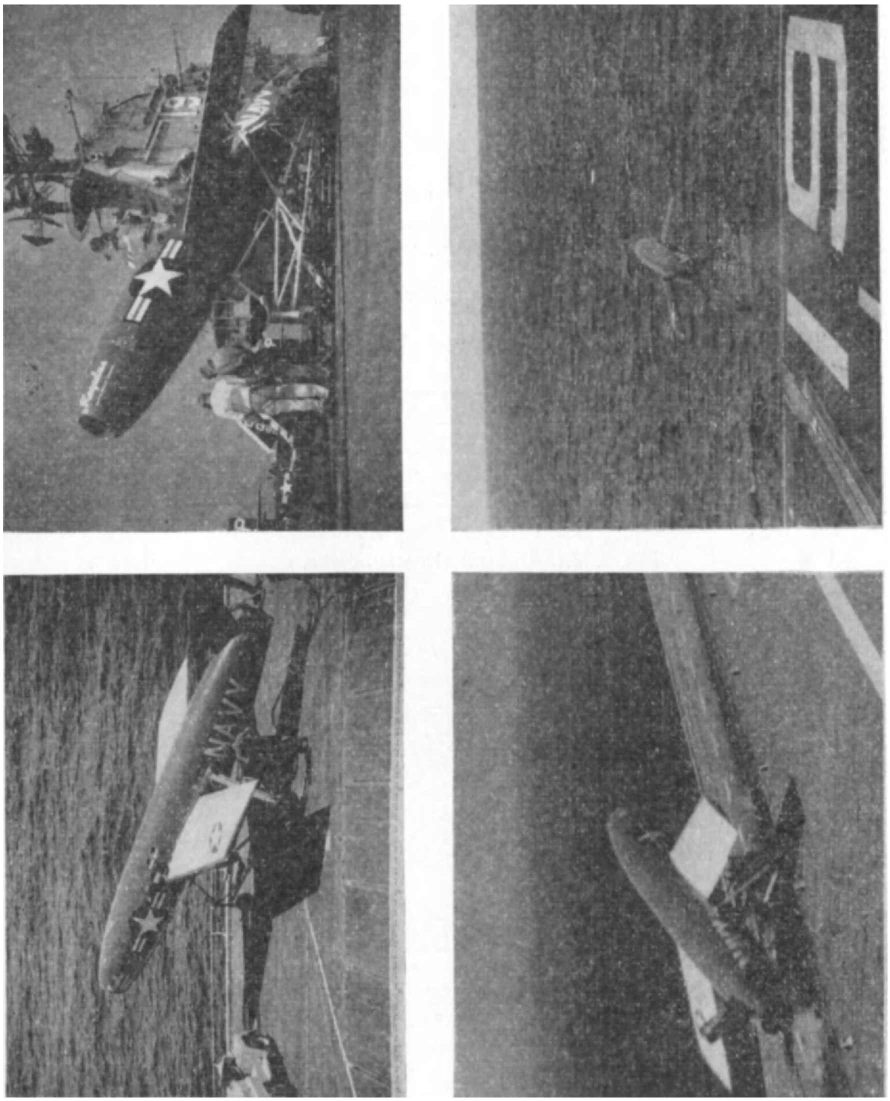


FIG. 2.33. U.S. Navy Guided Missile Regulus launched by a steam catapult from the deck of the aircraft carrier USS Hancock. (Official U.S. Navy photograph)

In order to increase their velocity, missiles of this type are equipped with rocket motors. On one hand, additional velocity is necessary for increasing penetration capability when directed against targets such as, for instance, armored ships. On the other hand, in the case of the guided missile, the additional velocity is needed to increase the effectivity of guidance and range, so that an airplane will have the ability to hit the target without approaching it too closely.

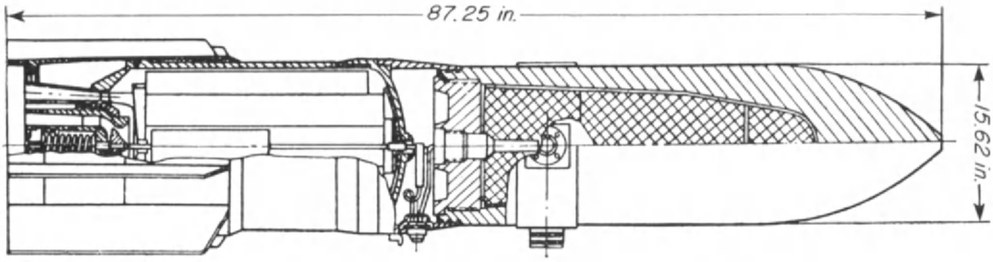


FIG. 2.34. Transverse section of an aircraft armor piercing bomb.

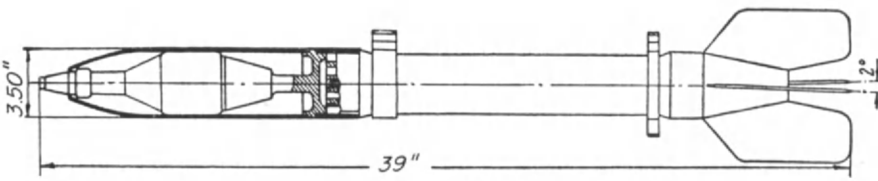


FIG. 2.35. Anti-tank aircraft missile.

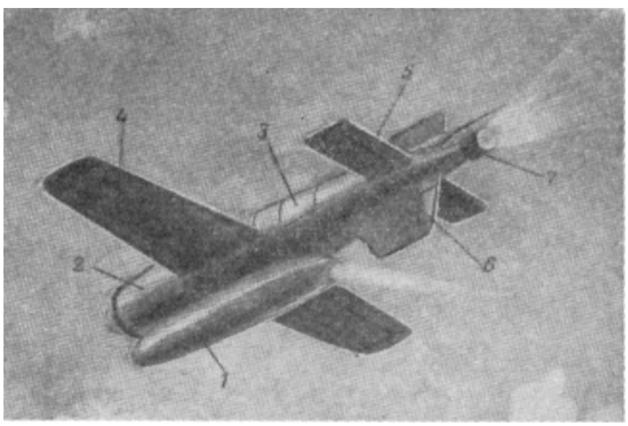


FIG. 2.36. An aerial torpedo: *1*, motor; *2*, warhead; *3*, aft section of the air frame containing steering apparatus; *4*, ailerons; *5*, elevators; *6*, rudder; and *7*, tracer.

Fig. 2.34 shows a cross section of a 2200 lb armor-piercing aircraft bomb. It can easily be seen that a comparatively small rocket charge cannot insure long range for such a massive bomb, and serves only as an accelerator which increases its penetrating ability.

Fig. 2.35 shows an anti-tank aircraft rocket containing a shaped charge.

Common missiles of the "air to surface" type are the guided aircraft bombs and aerial torpedoes intended for damaging warships and large

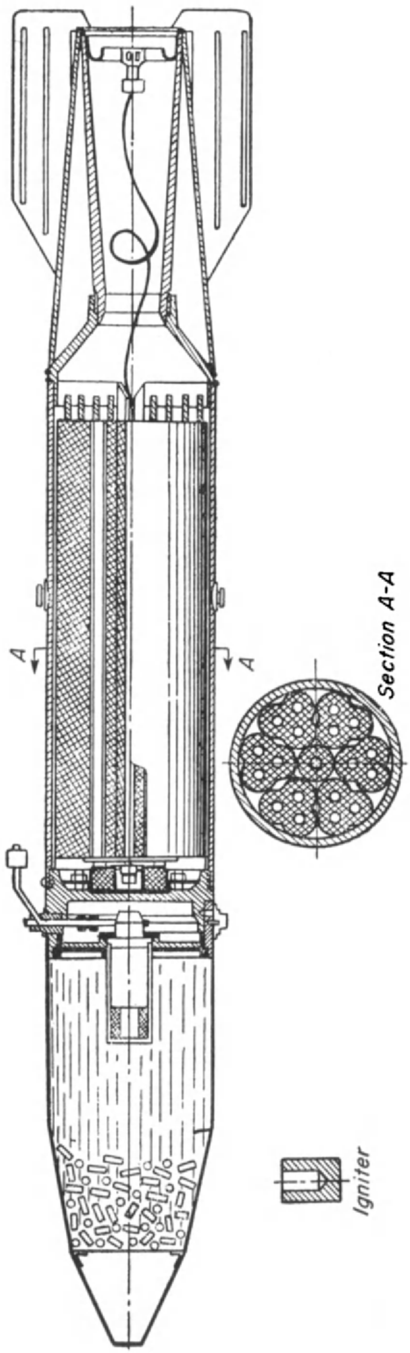


FIG. 2.37. Aircraft fragmentation rocket.

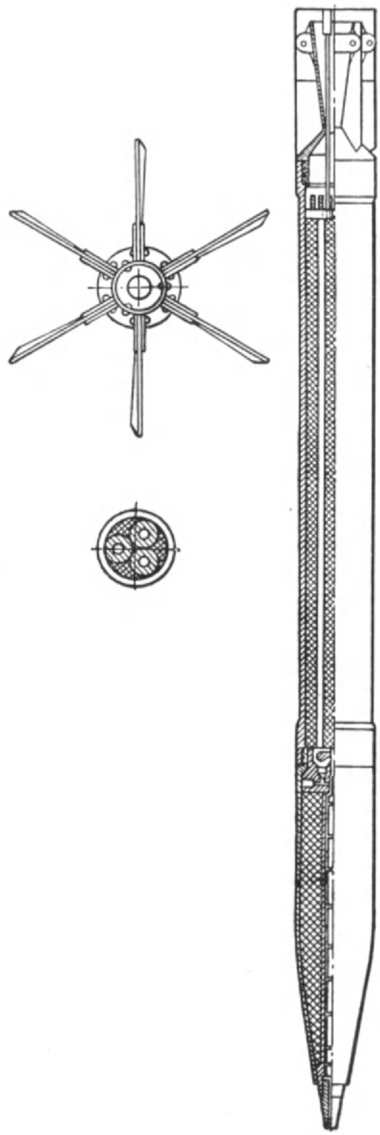


FIG. 2.38. Aircraft missile with unfolding fins.

ground objectives. Here, as in the case of anti-aircraft missiles, either a homing system or guidance to the target from the mother bomber is possible.

Fig. 2.36 shows the over-all view of one of the guided missiles (torpedoes) of the Second World War. This missile is equipped with a rocket motor, *1*, using hydrogen peroxide as fuel. The warhead, *2*, is located in the nose. The guidance instruments are located at the rear of the airframe, *3*. The missile is equipped with ailerons, *4*, elevators, *5*, and rudder, *6*. Tracer, *7*, (a rear light making the torpedo visible at night to the bombardier in the bomber) is also visible in Fig. 2.36

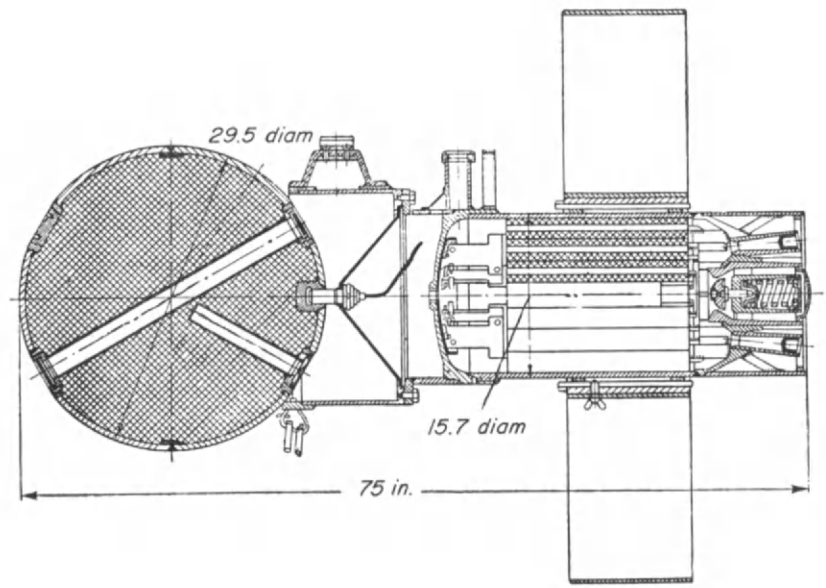


Fig. 2.39. A marine skip bomb.

Rockets for firing from one aircraft at another do not differ in principle from the short range bombardment missiles.

The destruction of an enemy aircraft is the result of a direct hit or of the fragmentation action of the missile, which is exploded by means of a proximity fuse. Fig. 2.37 shows an aircraft missile of the fragmentation type used by pursuit planes against bombers. During the explosion of the warhead the incendiary fragments disperse within a cone of 30° included angle.

Fig. 2.38 shows the arrangement of an aircraft missile with unfolding fins. While the rocket is within the launching tube, the fins are folded. As the rocket emerges from the tube, the fins unfold.

Aircraft rocket weapons open up wide possibilities in increasing the firing power of aircraft. The application of large caliber artillery to aviation is impossible because of the great weight of a barrel installation and of recoil forces during firing. Launching installations for firing rockets are extremely simple and light, and the rocket, at launching, has no recoil.

2. Some Other Types of Rockets

Above, we have discussed projectiles of surface rocket artillery and aircraft projectiles. Naval rocket projectiles and pilotless aircraft also exist. The method of transportation is reflected in certain structural characteristics peculiar to these weapons. These weapons must be capable of compact stowage aboard ship. They must use fuel which permits long storage. The launching installations must also have certain characteristics.

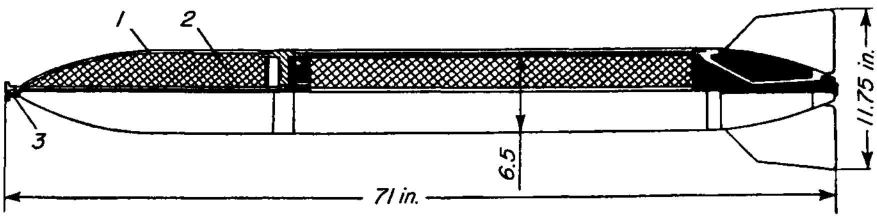


Fig. 2.40. Underwater rocket missile: *1*, warhead; *2*, tube for exhausting gases forward, thereby reducing friction forces between the body and water; and *3*, port for powder gas escape.

Firing at water borne targets does not differ basically from firing at land based targets. However, there are certain exceptions.

Fig. 2.39 shows a spherical aircraft skip bomb. This projectile, equipped with a rocket motor, is launched from an aircraft at the target ship, intentionally short of the target. When the rocket fuel is burned out the motor separates from the warhead. Subsequently, the spherical charge, equipped with contact and depth sensitive detonators, ricochets along the surface of the water, making up to 12 skips and, if correctly aimed, hits the ship.

It is also possible to employ rockets for underwater firing. Fig. 2.40 shows an underwater projectile for launching from a submarine against the submerged part of a ship's hull.

In concluding the review of existing types of rockets, let us pause and examine the so-called meteorological (Translator's note: or sounding) rockets. The purpose of these rockets is to carry to a specific altitude various recording instruments to investigate atmospheric properties and physical environments beyond the atmosphere. Altitudes of hundreds of miles have been attained by sounding rockets, and cannot at present be attained by any other means. The sounding rocket differs little from a military rocket

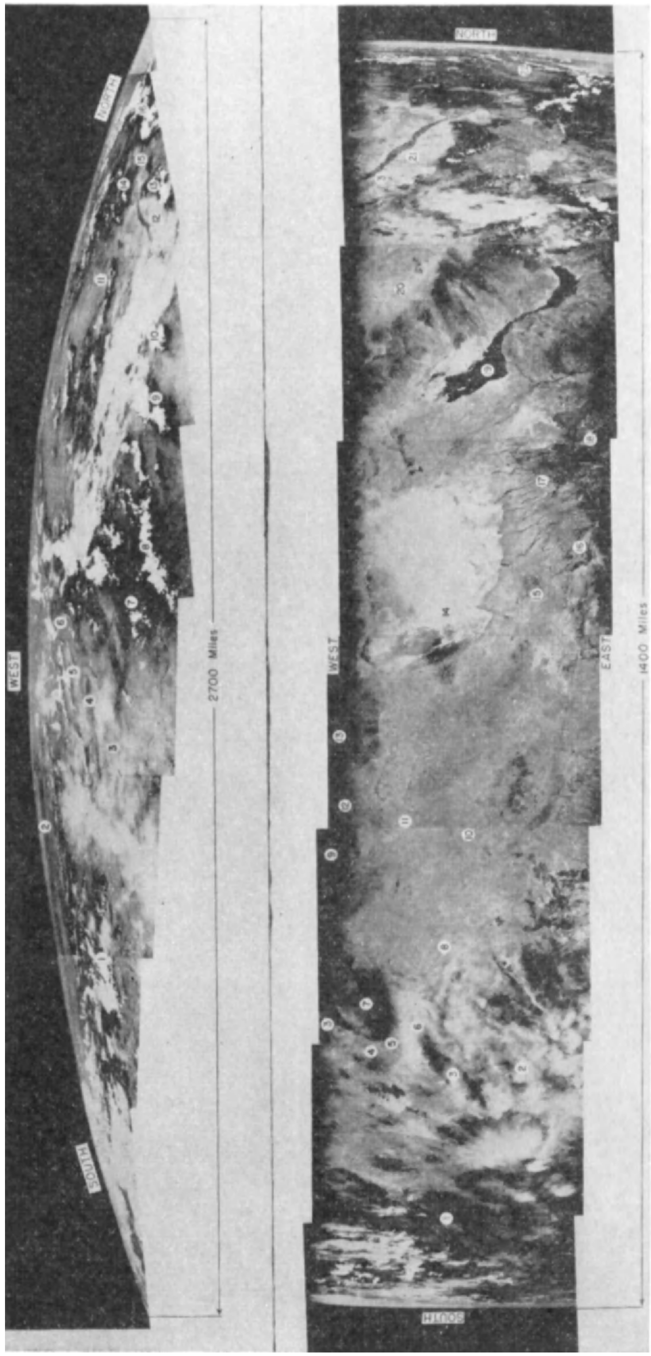


FIG. 2.41. Photograph of the earth from an altitude of 50 miles. (Official U.S. Navy photograph)

of intermediate range. The difference consists, primarily, in installation of observation instrumentation, which replaces the warhead. Sounding rockets have simplified guidance, inasmuch as the dispersion and accuracy of the flight path are not of prime importance. At the same time, sounding rockets require the solving of the problem of salvaging the instruments during their descent. Because of great heights and correspondingly large descent velocities, this is somewhat difficult to do.

Fig. 2.41 is a photograph of the earth's surface from an altitude of 50 miles taken with the aid of a sounding rocket. At the right mountainous terrain is seen. To the left is cloud overcast. The curvature of the earth is well-defined.

III. Types of Reaction Motors, Their Construction and Operational Characteristics

A. Conversion of Energy and Types of Existing Reaction Motors

1. Elements of Energy Conversion

In order to create thrust on the reaction motor it is necessary to impart velocity to the mass of ejected combustion products. This velocity is the result of converting the chemical energy of the fuel into kinetic energy of the gas stream.

Let us discuss the consecutive processes which take place in a reaction motor (Fig. 3.1).

The fuel of the great majority of reaction motors consists of two parts: oxidizer and combustible. These parts are called components of fuel.

The oxidizer contains large quantities of oxygen or some other oxidizing element. The combustible consists basically of combustible elements capable, during the process of chemical reaction with the oxidizing elements, of yielding large quantities of heat. Therefore, the reactive motor fuels are the carriers of chemical energy.

Characteristics of fuels will be discussed in more detail in the following chapter.

Fuel combustion takes place in the combustion chamber of the motor (Fig. 3.2). But before that, the fuel must pass through several preliminary stages.

First of all, the fuel must be compressed to a pressure greater than that in the chamber. Otherwise it will be unable to enter the chamber. Compression and feeding of the fuel into the motor are accomplished by a feeding system including tanks, compressor assembly (for instance, compressed air accumulators, turbopump, etc.), and also a system of valves and piping which insure reliability and ability to control the motor.

In feeding the fuel into the chamber, the fuel must be conditioned in order to promote easier combustion. This conditioning consists of mixing the combustible and the oxygen in the most homogeneous vapor mixture possible. In order to get such a mixture, the combustible is atomized into small droplets by a series of nozzles at the head of the motor, *3* (see Fig. 3.2). The droplets, in traversing the chamber, vaporize. After vaporization, both components of the fuel mix.

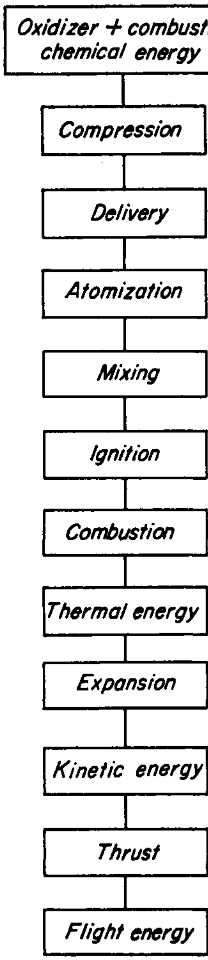


FIG. 3.1. Schematic of energy transformation in direct reaction rocket motors. The kinetic energy is that of the directed motion of the combustion products; the flight energy is the work done to displace the reaction device.

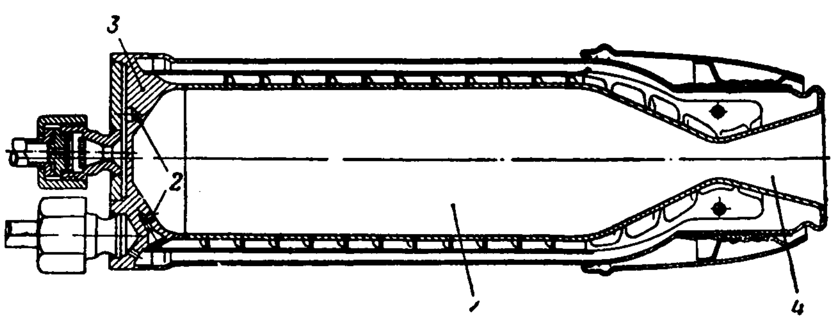


FIG. 3.2. The chamber of a rocket motor: *1*, the combustion chamber proper; *2*, fuel jets; *3*, head; and *4*, nozzle.

Ignition and combustion of the fuel in the chamber are the next necessary stages in the working process of the motor. During combustion, the chemical energy is converted into thermal energy.

In reaction motors the conversion of thermal energy into kinetic energy is the result of the expansion of the gaseous products of combustion as they move along the nozzle. The gas stream, directed through the nozzle, leads, as we have seen, to the creation of thrust.

In imparting translation to the motor assembly, and any payload associated with it, the thrust does work.

2. Basic Types of Reaction Motors

The construction of the motor and its basic components is primarily determined by the type of fuel used. Accordingly, all existing reaction motors can be divided into two groups: air breathing reaction motors and rocket motors.

The distinguishing characteristic of air breathing motors is their use of atmospheric oxygen as oxidizer.

The advantage of these motors is an appreciably decreased requirement of fuel aboard the vehicle (in contemporary rocket vehicles the weight of

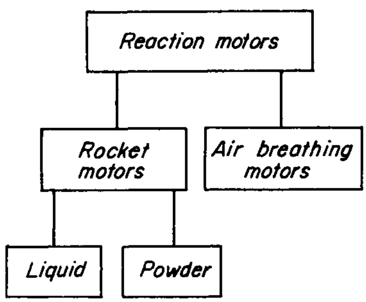


Fig. 3.3. The schematic division of reaction motors into basic groups

the oxidizer constitutes from 60 to 80% of the weight of the fuel). The need for oxidizer tanks and a mechanism for feeding the oxidizer into the combustion chamber is obviated.

On the other hand, the use of atmospheric oxygen confines the air breathing vehicle to operating altitudes below those at which the rarefied atmosphere cannot supply the necessary oxygen.

The second group contains motors which utilize fuels carried aboard the vehicle in their entirety. Such motors are called rockets. Their basic advantage, as we have already noted, consists in their independence of the environment. Rocket motors can be used at all altitudes and, specifically, in interplanetary space.

Rocket motors are divided into liquid fuel motors and solid propellant fuel motors.

At the present time the only solid propellant fuel used in rockets is powder. Therefore, motors using solid propellant are simply called powder motors. A schematic, showing the division of motors into their basic groups, is shown in Fig. 3.3.

Let us delve further into the discussion of air breathing and liquid propelled rocket motors. We will not discuss the construction of powder motors further. Their construction is so simple that it is easily understood without explanation (see examples of powder rockets in the preceeding chapter).

B. Air Breathing Motors

1. Turbojet Motors

The widest application of turbojet motors is in contemporary high speed aircraft.

The construction of one of the types of turbojet motors presently used is shown in Fig. 3.4.

The air enters through air scoops, *15* and *17*, and is directed at the vanes of the double sided compressor wheel, *4*. The vanes of the compressor force the air at higher pressure into the compressor diffuser, *6*, where its velocity is reduced and the pressure correspondingly increased.

The compressed air from the diffuser is directed to the combustion chamber, *7*, through manifold, *5*. The fuel (kerosene) is fed into the chamber through nozzles, *14*. The air turbulator, *13*, imparts a rotation to the air around the fuel injection nozzle in order to insure the proper mixture of air with fuel, and to get greater stability in the completion of the combustion process.

The combustion of fuel takes place only in the first section of the flame tube, *12* (zone of combustion), and not in the entire volume of the combustion chamber. In order to have exhaust gases at sufficiently low temperatures an excess of air is fed into the combustion chamber. This air, having entered the flame tube through the side ports, is mixed with the products of combustion. This, naturally, lowers the temperature at the exit of the combustion chamber.

Hot gases at a temperature of about 900°C pass through the stator vanes, *9*, of the turbine, *10*. Here, they are directed with increased velocity against the vanes of the turbine, thereby turning it.

The turbine of the turbojet is used almost exclusively for turning the air compressor. Only a small part of the power delivered by the turbine

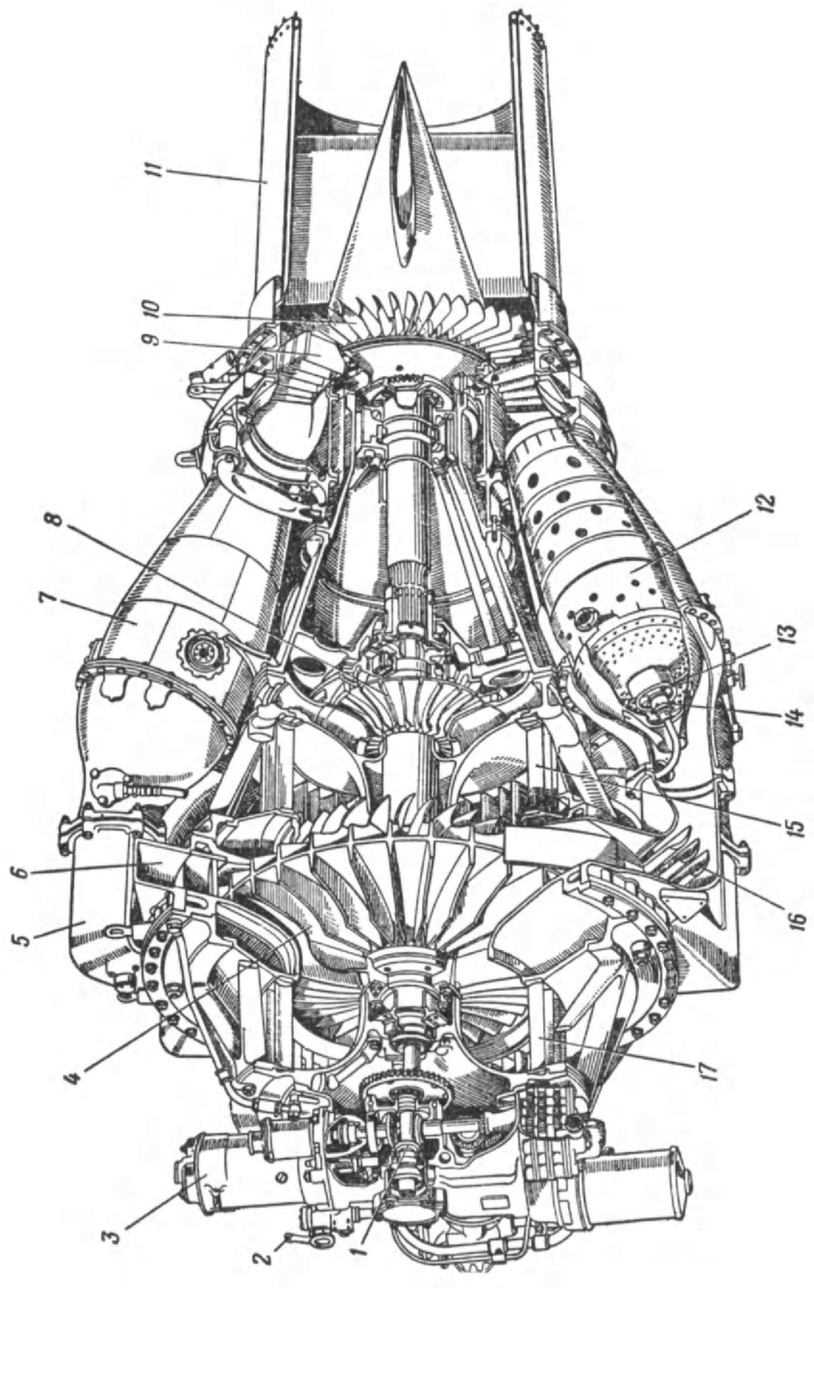


FIG. 3.4. Construction of a turbojet motor with a centrifugal compressor. *1*, auxiliary component shaft; *2*, fuel regulating lever; *3*, electric starter; *4*, compressor wheel; *5*, manifold; *6*, compressor diffuser; *7*, combustion chamber; *8*, cooling fan; *9*, turbine stator; *10*, turbine; *11*, motor nozzle; *12*, flame holder; *13*, air turbulator; *14*, fuel jets; *15*, aft air intake; *16*, section through an air duct; and *17*, forward air intake.

to the shaft, *1*, is utilized for operating auxiliary assemblies: fuel and oil feed pumps, electric generators, regulators, etc.

The turbine exhaust gases have a pressure higher than atmospheric pressure. In the motor nozzle, *11*, the gases expand and are ejected from the nozzle at high velocity. This velocity, due to the processes which take place in the motor, substantially exceeds the velocity of the vehicle, which results in thrust.

The cooling of heated parts is accomplished by a cooling fan, *8*, which forces air along the shaft toward the turbine disk.

Starting of the motor is accomplished by an electric self-starter, *3*, which increases the rotation of the motor until it becomes self-sustaining.

An axial compressor motor has a more widespread application than the above-described centrifugal compressor motor.

A section of a turbojet with an axial compressor is shown in Fig. 3.5.

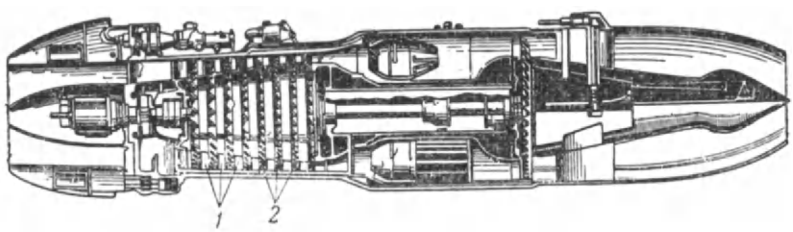


FIG. 3.5. Turbojet motor with an axial compressor: *1*, stator vanes; and *2*, turbine wheel with buckets.

The compressor for compressing air to the necessary pressure consists of a series of stages, each of which consists of a stator, *1*, and a turbine wheel, *2*. Aside from the differences in the compressor, the remaining elements of the motor, i.e., the combustion chamber, turbine, and nozzle, remain the same, in principle, for both types of motors.

At the present time, turbojets of the most diversified types are being built with thrusts of from 2 to 6 and more tons. Motors of 4 ton thrust have a diameter of 3 to 5 ft and a length of from 10 to 16 ft. The power absorbed by the compressor and, therefore, the power of the turbine, constitutes 15,000–20,000 hp. In one second, such a motor "inhales" up to 176 lb of air. At sea level this corresponds to 2460 ft^3 of air. The fuel economy is 1.98–3.3 lb per lb of thrust per hr.

At flight velocities above 500–560 mi/hr the turbojet is more effiicent then a propeller driven aircraft. At lower, but still sufficiently high, velocities, the turbo-propeller (turboprop) motor is the more desirable. In this motor the turbine rotates not only the compressor but the propeller as well, which generates the basic thrust of the motor. The gases are only

slightly accelerated in the nozzle, resulting in a very small addition to the thrust of the turboprop installation.

2. Ducted Reaction Motors

The basic characteristic of the above-discussed turbojet motors is the presence of a compressor which insures the compression of air fed into the combustion chamber. Air compression is necessary for the propagation of the combustion process.

The turbojet motors can be used not only on subsonic aircraft, but on aircraft which appreciably exceed the speed of sound. However, at high flight speeds of 1,100 to 1,250 mi/hr and above, the necessity of mechanically compressing the air falls by the wayside, since the necessary pressure can be obtained from the forward velocity of the aircraft itself. Therefore, an air breathing motor can dispense with the necessity of having a turbine and a compressor. This considerably simplifies the construction of the motor. A motor which compresses air by the motion of the aircraft is called a ducted air breathing reaction motor.

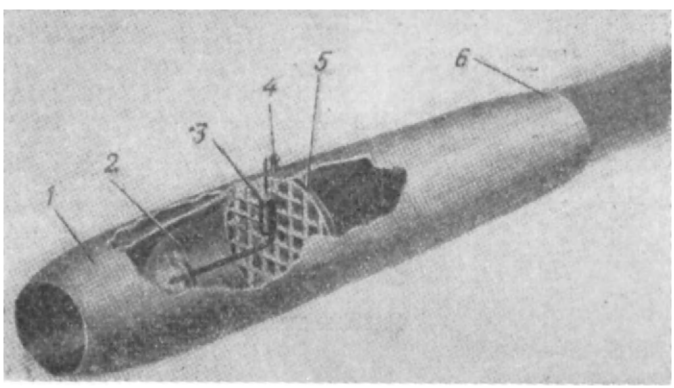


Fig. 3.6. Schematic arrangement of a ducted jet motor: *1*, diffuser; *2*, fuel jets; *3*, igniter; *4*, fuel supply; *5*, flame holder; and *6*, jet nozzle.

Ducted air breathing motors can be employed even at subsonic velocities. In this case, however, due to the insufficient velocity head and low compression, the operating efficiency of the ducted engine is very low. The optimum velocities for ducted air breathing motors must be in the neighborhood of 1,850 to 3,100 mi/hr.

A schematic of the ducted subsonic motor is shown in Fig. 3.6.

The body of the motor consists of a tube of a variable cross section. The air enters the diffuser, *1*, where its velocity is reduced and pressure increased. In order to improve mixing, the air is made turbulent before

entering the combustion chamber by means of a special grid (not shown in Fig. 3.6). The kerosene is injected into the chamber through nozzles, *2*. Here combustion takes place and the heated air, together with the products of combustion, escapes at high velocity through nozzle, *6*, resulting in thrust on the motor.

The air stream enters the combustion chamber at high velocity (above 330 ft/sec). At these extreme velocities it is possible to eject the flame from the motor and terminate combustion. At the same time it is undesirable to decrease the velocity in the combustion chamber, since that will result in considerable increase in the transverse dimensions of the motor. In order to prevent flame-out, the combustion chamber is equipped with flame holders, *5*. Flame holders consist of nonstreamlined profiles which form vortices, reverse flow, and low velocities. The flame is maintained in these zones and is continually fed by a flow of fuel mixture. The initial ignition is performed by an igniter, *3*.

At first acquaintance with the principles of ducted air breathing motors, there arise perplexing questions about the mechanism which gives rise to the thrust on the motor.

From the discussion above it is clear that, inasmuch as the air mass upon entering the motor has a velocity less than the velocity of ejected gases, there is a change in the velocity of gases. Therefore, as in the case of a rocket motor, this give rise to thrust—reversed force directed in the direction of flight.

From the point of view of mechanics, the thrust on the motor can be considered as the resultant of the excess of pressure acting on the internal surface of the motor. In order to yield thrust the shape of the duct must be such that pressure forces would yield a resultant directed in the direction of flight. In other words, it is necessary for the forward (entrance) opening of the motor to be smaller than the exit.*

The advantage of the ducted air breathing motor, in comparison to other motors, is its extreme simplicity. The construction of the ducted motor does not include any rotating or translationally moving parts. The fuel system can also do without any mechanisms and can use compressed air. A big disadvantage of the ducted motor operating along the described lines is that it does not have a starting thrust. For a ducted motor to develop appreciable thrust it must be accelerated to a considerable velocity. This acceleration is usually accomplished by means of powder or liquid fuel propelled rocket motors, which must become a part of the vehicle assembly. Therefore, the ducted motor quite often is referred to as a sustaining motor.

* Further discussion of this question can be found in a book by N. V. Inozemcev and V. S. Zouiev, "Aircraft Gas Turbine Motors," Oborongiz, 1949.

We must once more emphasize that a construction of the ducted missile with the simplest of diffusers, as shown in Fig. 3.6, is useful only at relatively low flight velocities. At higher, supersonic velocities, in order to obtain more complete conversion of the ram pressure into thrust, it is

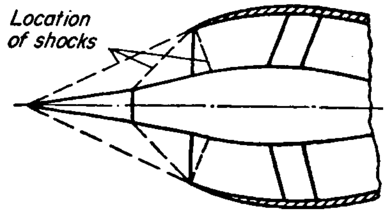


FIG. 3.7. Schematic of a multishock diffuser.

necessary to employ diffusers of complex configuration, with conical inner bodies, the so-called multishock diffusers, which create a system of shocks (oblique and normal) at the entrance to the motor. A schematic of one of these diffusers is shown in Fig. 3.7.

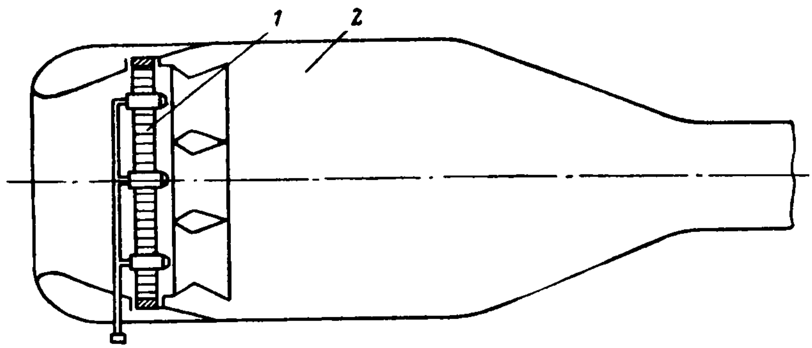
3. Pulse Jet Motors

The desire to utilize the extremely simple construction of a ducted motor and adapt it to slower speeds led to the design of a different type of compressionless motor; specifically, the pulse jet.

Since at low flight velocities it is not possible to create high pressure in the combustion chamber, pulse jets utilize a form of check valve at the entrance to the chamber in order to get increased pressures.

The air enters the motor (Fig. 3.8) by deflecting the spring plates (or reeds) of the check valves, *1*, and fills the combustion chamber, *2*, at the same time displacing, through the exhaust tube, the products of combustion of the preceeding cycle. Concurrent with the filling of the chamber with air, easily vaporized fuel, benzine, is injected into the chamber through jets. The mixture of benzine vapor and air is ignited by the heat of the gases remaining in the tube and burns rapidly. The rise in pressure in the combustion chamber forces the check valves to close while the products of combustion flow through the exhaust tube; at first with high velocity and subsequently with constantly decreasing velocity. As the gases flow out of the chamber, the pressure decreases. The gas column, filling the long exhaust tube, possesses considerable inertia. Therefore, the gases continue to move in the initial direction, even after the pressure in the combustion chamber becomes equal to atmospheric pressure. Due to the inertial motion of the gases, a certain degree of rarefication is created in the combustion chamber.

As the result of external ram pressure, and the created low pressure in the combustion chamber, the check valves open and allow new air to enter the combustion chamber again, thereby filling and simultaneously scavenging it. The working cycle of the motor is repeated in this manner. The cycle frequency depends on the geometric parameters of the motor and equals approximately 3000 cycles a minute.



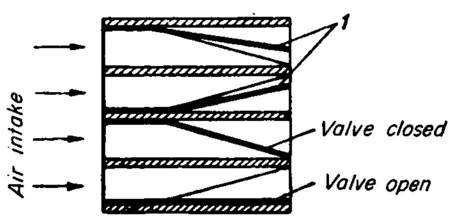


FIG. 3.8. The arrangement of a pulse jet motor: *1*, reed check valves; and *2*, combustion chamber.

It is important to note that the pulse jet, due to the presence of the long exhaust tube, can create thrust at launching. The low pressure, which results from the inertial motion of gases along the tube, is sufficient for opening the valves and inhaling the necessary quantity of air for the repetition of the cycle.

The pulse jet motor uses 3 to 4 lb of fuel per lb of thrust per hr.

Therefore, the basic advantage of a pulse jet motor is its structural simplicity as compared to the turbojet, and greater thrust and economy at lower speeds as compared to ducted motors. This assured the possibility of mass application of these motors during World War II, on such aircraft as the pilotless missile V-1 (see Chapter II). The velocities of vehicles equipped with pulse jets are relatively low (up to 620 mi/hr).

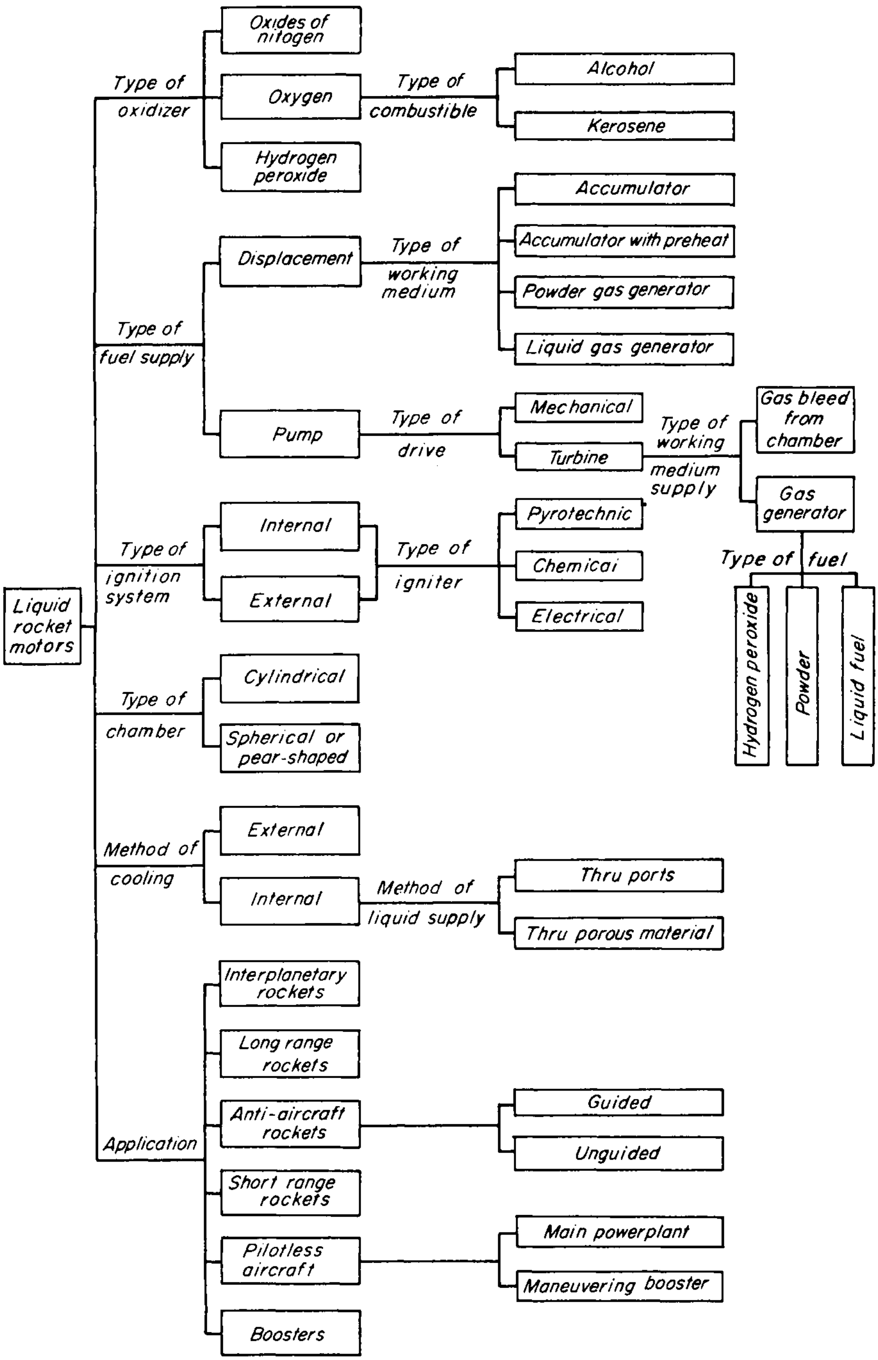


FIG. 3.9. Classification of liquid propelled rocket motors.

C. Rocket Motors Using Liquid Fuel

1. Types of Existing Liquid Fueled Rocket Motors

In classifying liquid fueled rocket motors (Fig. 3.9) it is necessary, first of all, to consider the kind of fuel used in the motor.

Since the characteristics of various fuels are determined primarily by the kind of oxidizer used it is necessary, first, to subdivide the liquid fueled rocket motors according to the kind of oxidizer that is used (nitrogen oxide, liquid oxygen, and hydrogen peroxide), and, after that, according to the kind of fuel (kerosene, alcohol, etc.).

The second distinguishing characteristic of the construction of liquid fueled rockets is the method of fuel feed. There are two primary methods: displacement, and pump system of feeds. The first is the simpler of the two and is used with motors of comparatively small rockets. The second is used with motors of long range rockets with thrusts of 4000 lb or more, and also with motors of long range aircraft.

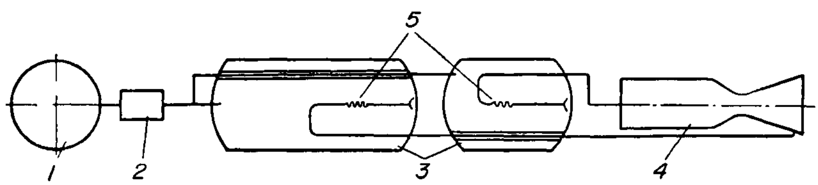


Fig. 3.10. Schematic of accumulator fuel supply: *1*, accumulator with compressed gas; *2*, pressure reducer; *3*, tanks with fuel; *4*, motor chamber; and *5*, flexible joints of fuel intakes.

In the displacement system, the fuel components are fed into the combustion chamber with the aid of compressed gas, which enters the fuel tanks under high pressure. It is important that the gas pressure exceed the pressure in the combustion chamber. The excess pressure is necessary in order to compensate for losses in the fuel ducts (this includes piping, valves, jets, coolers, and others). In this manner a high pressure is established in fuel tanks using this system of delivery. In order to withstand the pressure, the fuel tanks must have considerable thickness. This leads to heavy construction and this, generally, is the deficiency of all displacement systems of fuel delivery.

The simplest type of displacement delivery, used for a long time in rocket motors, is the accumulator delivery (Fig. 3.10). In this system, the high pressure for fuel displacement out of the fuel tanks is obtained from highly compressed air (3550 to 4250 lb/in.^{2}), or some other gas contained in the accumulator, *1*. The accumulator ordinarily is made in the form of

a sphere because the weight of such an accumulator for a given volume is least as compared to the weight of accumulators of any other shape.

The necessary accessory in the accumulator delivery is the pressure reducing valve, *2*. The pressure reducer is an instrument which insures constant gas pressure in fuel tanks, *3*, and, consequently, an even fuel delivery into the combustion chamber, *4*, which is necessary for preserving the con-

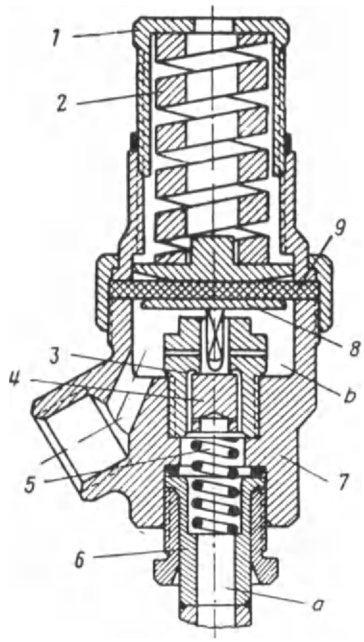


FIG. 3.11. Construction of a pressure reducer: *a*, high pressure cavity; *b*, low pressure cavity; *1*, hollow nut; *2*, mainspring; *3*, valve seat; *4*, valve; *5*, spring; *6*, fitting; *7*, valve body; *8*, plate; and *9*, diaphragm.

stant operating conditions in the motor. Without a pressure reducer, the pressure in the fuel tanks would at first rise sharply, and then, as the fuel was used up and gases expanded, fall.

The construction of a pressure reducing valve is shown in Fig. 3.11.

The accumulator air, at high pressure, enters the pressure reducing valve fitting, *6*. The air flows through the clearance between the walls of the valve seat, *3*, and valve body, *4*, and approaches the throttling section of the pressure reducer, which is formed by the gap between the valve seat and the valve.

After throttling through the gap, the gas enters the low pressure void, *b*, and, subsequently, the piping leading to the fuel tank.

The degree of pressure drop is determined by the regulator valve opening. The more the valve is displaced from the seat the less the pressure drop is. The displacement of the valve during the working cycle takes place as a result of the forces acting upon it. These forces are as follows: (a) force due to pressure of the gas flowing into the valve; (b) the force of the spring, *5;* (c) the pressure force on the valve due to the gas which has passed through the clearance; and (d) the force of the mainspring of the regulator, *2*, which through the membrane, *9*, and the disk, *8*, acts on the valve. The first two forces tend to close the valve and increase the amount of throttling of the gas; the second two forces act in an opposite direction. The force at the mainspring, *2*, is opposed by the force of the gas pressure exiting from the regulator and acting on the flexible membrane, *9*. This force reduces the opening of the valve and increases the throttling effect.

The pressure reducing valve is a regulating mechanism. If, for instance, the gas pressure at the exit of the pressure reducer drops, the forces acting on the membrane and the spring cause the valve to increase the opening. Because of this, the throttling decreases and the pressure, at the exit of the pressure reducer, increases. The nut, *1*, serves to regulate the tension of the mainspring of the pressure reducer. By increasing the tension on this spring we are opening the valve to a greater extent, and the pressure at the exit will increase. The spring, *5*, has an auxiliary function. By reducing the tension on spring *2*, it closes the valve, and the pressure reducer functions as an ordinary vent.

As can be seen from the description and Fig. 3.10, the accumulator system of fuel delivery is very simple. That is why it is used in many motors. However, the accumulator delivery, aside from deficiencies which are common to all displacement deliveries, has one more major deficiency, which is that the weight of gas necessary for the displacement of the entire fuel supply is quite large. For instance, in order to feed 35.3 ft^3 of fuel, at a fuel tank pressure of 500–575 $lb/in.^2$ it is necessary to have 110 lb of air. Also, in order to contain this air aboard the rocket it is necessary to have an accumulator of 5.3 ft^3 capacity even if the initial pressure in the accumulator is only 4260 psi. Such an accumulator (spherical in shape) made from quality steel will weigh about 330 lb with a diameter of 27 in.

It can be seen, therefore, that the accumulator delivery results in an appreciable increase in weight of the motor installation. This disadvantage can be partially overcome if the air flowing into the fuel tanks is preheated by burning a small amount of fuel in it. In this case the specific weight of gas flowing into the tank, and consequently the amount of air necessary for displacing a given volume of fuel, is reduced. The dimensions are also reduced, along with the weight of the accumulator necessary for containing the air aboard the rocket.

Even more advantageous from the point of view of weight is the system of fuel delivery which utilizes the combustion products of a powder charge to displace the fuel. Recently, slow burning powder charges suitable for this purpose became available (see page 144). These powder charges generate an equal weight of combustion products per unit time, thereby accomplishing even displacement of fuel from the tanks.

Due to the low specific weight of the hot gases displacing the fuel, in order to deliver 35.3 ft^3 of fuel it is necessary to have 26–33 lb of powder (taking into account the cooling of gases on the way to the tank and in the tank itself). Powder in a solid state has a sufficiently high specific weight, about 93.5 lb/ft^3. Therefore, the dimensions of the chamber in which it will burn will be comparatively small, about 0.43 to 0.53 ft^3. Aside from that, the combustion of the powder can take place at relatively low pressures, only slightly exceeding the pressures in the tanks. Therefore, the chamber walls where the powder charge burns need not be very thick and the weight of the chamber will be only about 55 lb. By using the combustion products of the powder charge for fuel displacement it is possible to save (as compared to the accumulator delivery) for each 35.3 ft^3 of displaced fuel about 350 lb in weight.

In some liquid fuel rocket motors, the products of combustion of the liquid fuel are used for the displacement of fuel, In other words, a small liquid fuel motor is used for displacement of fuel. Such a system of fuel delivery is called a liquid pressure accumulator system and its weight is approximately equal to the weight of a delivery system using a powder charge for generating displacing gases.

The application of fuel delivery by means of powder and liquid accumulators is complicated by high temperatures of combustion and difficulties in obtaining a stable, uniform combustion of the liquid fuel, and, to an even greater degree, of the powder charge.

Generally speaking, other methods can be proposed for obtaining high pressure gases necessary for the displacement of fuel. Therefore, a further classification of motors with displacement delivery systems should be made according to the type of working medium used for displacement (cold compressed gas, preheated compressed gas, powder gases, etc.).

The tank weight of the displacement systems, used for small and medium sized rockets with relatively low pressures in the combustion chamber, is not very high. But in motors of the large rockets, especially if they have high pressure in the combustion chamber, the weight of the tanks increases to prohibitively large proportions and the pump system of fuel delivery becomes more attractive.

This system, as can be seen from its name, uses special pumps for delivery of fuel components from the fuel tanks into the combustion cham-

ber of the motor. The advantage of the pump system over the displacement system lies in the fact that it is not necessary to have high pressure in the motor fuel tanks. Some low pressure of 3–4 psi is maintained in the fuel tanks of a motor with pump delivery in order to assure reliable flow of liquid to the pumps. The resulting thickness of tank walls is no greater than is required by structural considerations.

Liquid rocket motors ordinarily use pumps of the centrifugal type which operate along the same lines as the centrifugal compressor of the turbojet.

The centrifugal pump is convenient in that while having small dimensions it can supply large quantities of fuel components at high pressures. This is accomplished by high peripheral speeds of the impeller and a correspondingly high number of revolutions (from 5000 to 25000 rpm).

However, in order to rotate the fuel pump it is necessary to have some sort of a motor. The most convenient motor for use in conjunction with centrifugal pumps is the turbine, which is capable of imparting high rotational velocity to the pump. In this way, a single mechanism, often having a common shaft and comprising a turbine and the necessary number of pumps, is evolved. Such a machine is called a turbopump (Fig. 3.12). It is necessary to have either hot gas or steam (the working medium) under high pressure to operate the turbine. The working medium can be obtained in various ways. The most convenient and widespread method is the decomposition of a highly concentrated hydrogen peroxide in a special reactor aboard the rocket. The heated mixture of oxygen and steam, formed by decomposition of hydrogen peroxide, is capable of operating a turbine of the required power output.

In the classification chart (Fig. 3.9) various methods of obtaining the working medium for the turbine are noted.

From the above short description, it is seen that the pump fuel delivery is more complicated than the displacement system. However, as was already mentioned, it is preferred for use in conjunction with motor installations of considerable power output.

The method of fuel ignition for starting the motor is a very important consideration in liquid propelled rockets. The method and implementation of the ignition system depends almost exclusively on the purpose for which the motor is designed.

First of all, the ignition can be either external or internal. External ignition consists of introducing an igniting flame into the combustion chamber externally, through the nozzle. Such a system, specifically, is used in starting motors of ballistic rockets. For internal ignition there are devices located in the head of the motor.

Further classification of ignition systems is according to the characteristics of flame initiators. Powder charges ignited by electrical squibs and

in turn igniting the liquid fuel are an example of an initiator which may be used. Such a system of ignition is called pyrotechnic.

Sometimes the ignition of liquid fuels is accomplished by an electric spark. However, such a method is not sufficiently reliable because in large motors the power of the electric spark may be insufficient, and there may

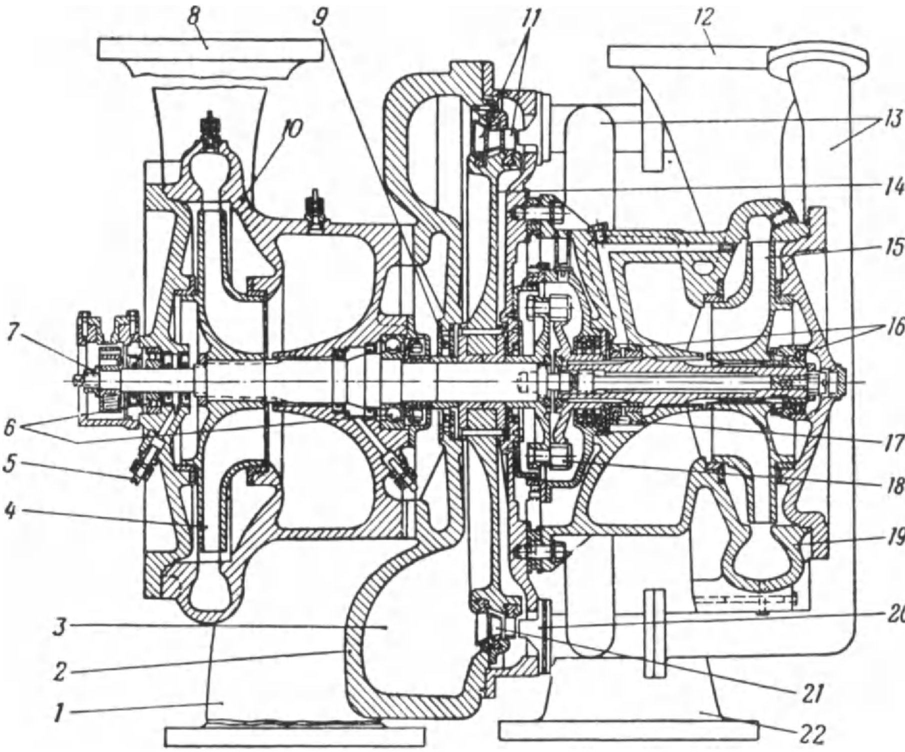


FIG. 3.12. Turbopump assembly of a V-2 rocket motor: *1*, alcohol intake; *2*, turbine housing; *3*, turbine outlet manifold; *4*, alcohol pump impeller; *5*, drain fitting for leaked alcohol; *6*, ball bearings; *7*, governor; *8*, alcohol intake flange; *9*, turbine shaft packing; *10*, alcohol pump housing; *11*, turbine blades; *12*, oxygen intake flange; *13*, steam inlet pipes; *14*, turbine wheel; *15*, oxygen pump impeller; *16*, oxygen pump bearings; *17*, shaft packing; *18*, coupling; *19*, oxygen pump housing; *20*, turbine nozzle; *21*, turbine stator; and *22*, oxygen intake.

be a delay in ignition. Therefore, a purely electrical ignition is used only for small motors. Motors of a larger size use a more complex system in which a small amount of auxiliary fuel is ignited by an electric spark plug and the small torch thus formed ignites the main fuel.

The types of liquid fueled rocket motors, and especially their construction, are determined, to a considerable degree, by the purpose of the power

plant. In this way we can differentiate among interplanetary rocket motors and rockets of the earth satellites, long range ballistic rocket motors, motors of guided and unguided anti-aircraft rockets, liquid fuel rockets—boosters, and liquid fuel rocket sustainers.

The motors of interplanetary rockets (earth—interplanetary space and earth satellite rockets) which have not yet been constructed must have large absolute thrust, calculated in hundreds of tons; very high specific thrust (approximately 320 lb sec/lb); and small weight. The motors of contemporary long range ballistic rockets are prototypes of such power plants. These motors have a thrust of several tons and a specific thrust equal to 230–240 lb sec/lb. In order to obtain the indicated specific thrust in the motors of ballistic rockets, fuels with the greatest possible chemical energy content are utilized, specifically, oxygen plus alcohol and oxygen plus kerosene.

In order for the rocket to have low final weight (low value of μ_b), the motor must be light. Therefore, long range rocket motors use the pump fuel delivery. Undoubtedly, the increase in rocket range will make it mandatory to increase the absolute thrust considerably, simultaneously increasing the specific thrust, and decreasing the relative weight of the motor.

Because the development of liquid fuel rocket motors is in most cases connected with the development of new types of weapons, we have no exact information on the latest long range rocket motors. However, information published so far concerning the construction of test centers leads us to believe that motors having thrusts of 88–110 tons are either made or will shortly be made. It is also conceivable that further increase in thrust on the motor installation will be achieved by utilizing clusters of simultaneously operating motors. Therefore, on the basis of developed motors with thrusts of 55–110 tons it will be possible to construct motors with thrusts of several hundred tons.

A very important objective in utilizing liquid fuel rocket motors is the guided anti-aircraft rockets of the surface–air type. Such rockets have motors with thrusts of from 2.2 to 8.8 tons with burning time up to 50 sec: the fuel feed system is ordinarily the displacement type. Liquid fuel rocket motors for unguided anti-aircraft rockets and short range rockets with a range of up to 62 miles have approximately the same parameters: thrust of from 1.1 to 3.3 tons and burning time of 10 to 20 sec. They must be simple to fabricate, since the application of unguided anti-aircraft rockets is of a mass character.

A characteristic of anti-aircraft rockets is the necessity for storing them for some length of time in a loaded condition. This forces certain requirements on the choice of fuel materials and the construction of the motor itself.

The liquid fuel rocket motors may be used in aircraft either as sustaining motors or as auxiliary boosters.

The sustaining aircraft liquid fuel rocket motors have a relatively complicated construction which allows regulation of thrust between the limits of 2,200 to 22,000 lb, a turbopump feed for fuel components, and a repetitive starting system. The auxiliary boosters also have a pump feed, but they are operated from the main aircraft power plant; they must be capable of repetitive starting during flight.

Finally, the last group of rocket motors consists of launching boosters for various flying vehicles such as heavy aircraft vehicles with ducted motors, missiles, etc.

Notwithstanding the fact that rocket technology has been developing intensely only during the last 20 years, at the present time a multitude of examples of liquid fuel rocket motors differing from each other by the magnitude of developed thrust, fuels used, systems of fuel delivery, and basic construction, already exists.

Let us go on now to an investigation of the structures of some specific motors.

2. The Construction of the Liquid Fuel Rocket Motor of an Intermediate Range Rocket

The power plants of the ballistic intermediate range rockets are of the more complicated and highly developed types of liquid fuel rocket motors. As an example of such motors let us discuss the construction of the V-2 rocket power plant.

This motor operates on fuel consisting of liquid oxygen (oxidizer) and a water solution of ethyl alcohol, 75% concentration by weight (the combustible). The thrust on the motor is about 29 tons (at sea level) at a burning rate of approximately 270 lb/sec. The specific impulse developed by the motor is 208 lb sec/lb, and the velocity of the exhaust gases from the nozzle is 6,600 ft/sec.

The over-all view of the motor installed in the rocket is shown in Fig. 2.18, and the external view of the motor without tanks in Fig. 3.13. The basic schematic of the motor and its components is shown in Fig. 3.14.

As is easily seen from these figures, the basic parts of the motor are the spherical combustion chamber, *4*, and the nozzle, *1*. The products of combustion expand in the nozzle of the motor to an exit pressure of 11.4 psi, and in doing so gain high velocity.

The diameter of the combustion chamber at its widest part equals 36½ in. The throat diameter of the nozzle is 15.7 in., and the diameter of the nozzle exit is 28.7 in. The weight of the combustion chamber and the nozzle is 925 lb.

The head of the combustion chamber contains 18 similar ante-chambers, *3*. Each ante-chamber serves as a housing for jets, through which the fuel and oxidizer enter the combustion chamber in an atomized state. The ports which atomize the oxidizer are located in the jet head to which oxidizer tubing, *5*, is connected. The fuel jets are placed on the side walls of the ante-chamber. The fuel enters the jets through the upper cavity of the combustion chamber head from the cooling jacket of the motor, having passed through the main alcohol valve, *6*.

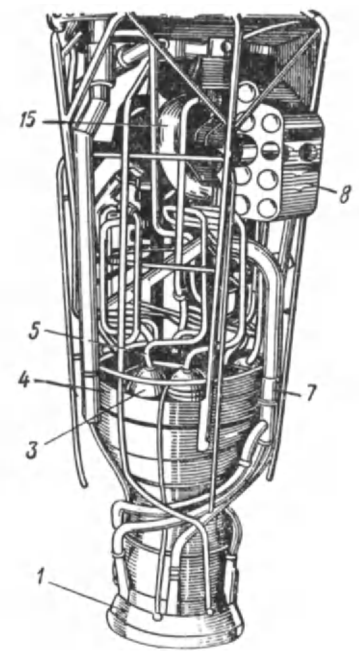


FIG. 3.13. External view of the V-2 rocket motor. Numbering is the same as in Fig. 3.14.

The cooling of the motor is performed in the following manner: The main volume of alcohol, before it gets to the jets, is forced through tubing, *7*, into the cooling jacket formed by the double walls of the combustion chamber and nozzle. Passing through this jacket at considerable speed, the alcohol absorbs the heat from the internal walls of the chamber in the nozzle and, in so doing, cools them. Part of the alcohol is diverted by a system of piping and through ports, *2*, to the inner surface of the motor wall and, evaporating on it, creates a vapor curtain.

Compression and delivery of fuel into the motor is performed by centrifugal pumps of oxidizer and combustible, *16* and *14*. The steam turbine,

15, operates the fuel pumps. The turbine and the two pumps, connected by a common shaft, constitute the turbopump assembly. The turbopump assembly delivers 465 hp and weighs 350 lb.

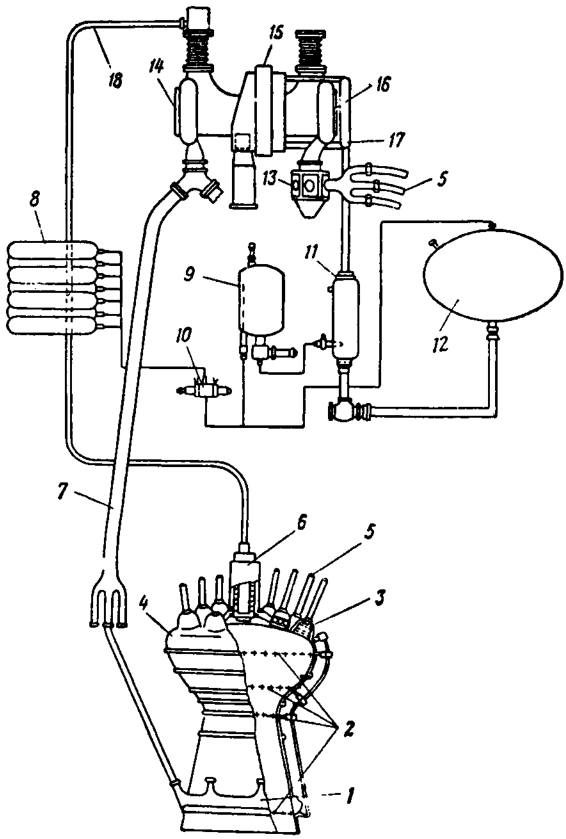


FIG. 3.14. The basic schematic of a V-2 motor: *1*, nozzle; *2*, delivery system of combustible for internal cooling; *3*, ante-chamber; *4*, combustion chamber; *5*, liquid oxygen supply to the ante-chambers; *6*, main valve for the combustible; *7*, tube for combustible supply to the cooling jacket; *8*, high pressure accumulators; *9*, catalyst tank; *10*, air pressure reducer; *11*, reactor; *12*, hydrogen peroxide tank; *13*, oxidizer main valve; *14*, combustible pump; *15*, turbine; *16*, oxidizer pump; *17*, steam line to the turbine; and *18*, combustible by-pass line to the pump during motor cut-off.

The tube, *18*, connecting the main fuel valve, *6*, and the pump, is necessary for bleeding the air when the system is filled with fuel, and for the prevention of a hydraulic hammer when the motor is stopped. The oxidizer, fed by the pump, *16*, passes through the main oxidizer valve, *13*, and then is led by tubes, *5*, into the ante-chamber.

The steam necessary for operating the turbine is generated in the steam generator by decomposition of concentrated hydrogen peroxide, H_2O_2. During the decomposition of hydrogen peroxide, sufficient heat is generated to raise the temperature of the products of decomposition to 750°–930°F (depending upon the dilution of hydrogen peroxide in water). A reliable and rapid decomposition of hydrogen peroxide takes place in the presence of a catalyst. A concentrated solution of potassium permanganate, $KMnO_4$, is such a catalyst.

The steam generator consists of the hydrogen peroxide tank, *12*, and a smaller tank, with a solution of potassium permanganate, *9*. These substances are fed into the reactor, *11*, by displacing them from their tanks by compressed air. The supply of compressed air is contained in accumulators, *8*. Before entering the tanks of the water vapor generator, the air pressure is dropped in the reducer valve, *10*.

The steam generator yields 4.4 lb/sec of steam, which is sufficient to insure the necessary power output of the turbine. The steam generated in the reactor, *11*, is led to the turbine, *15*, by tubes, *17*.

The weight of the complete assembly of the steam generator is 327 lb.

The thrust developed by the motor is transmitted to the air frame of the rocket by means of a thrust frame. The frame of the motor weighs 123 lb. Therefore, the total weight of the motor is 2,050 lb, or less than 88 lb per lb of thrust developed by the motor. The start, operation, and stopping of the motor take place in the following manner.

Before the start of the rocket, and after the testing of all the instruments directly on the launching pad, the loading of the rocket with the components necessary for its operation takes place. These components include combustible, oxidizer, hydrogen peroxide, solution of potassium permanganate, and compressed air.

At the time of start, flares, which are mounted on a small rotating platform within the combustion chamber, are ignited. The flares are mounted on the platform in such a way that the thrust developed by them causes the platform to rotate. Due to this rotation the heated products of the flare combustion fill the combustion chamber evenly and preheat it. After the necessary preheating (2 to 3 sec after beginning of start), a magnesium strip, located in the most inaccessible preheat spot in the chamber, burns out. This serves as a signal for the continuation of start. By means of an electropneumatic system, the main fuel valve, *6*, and the main oxidizer valve, *13*, are opened by 0.080–0.110 in. The alcohol and oxygen begin to enter the combustion chamber in comparatively small quantities due to the pressure of their respective liquid columns (gravity feed). The fuel entering the motor is ignited by hot powder gases and the combustion becomes more intense. Fuel combustion is stabilized during the first 2–3

sec. After that, the quantity of fuel may be increased. For this purpose, and with the aid of the electropneumatic system, the steam generator and turbopump assembly are introduced into the working cycle. The air from accumulators, *8*, begins to flow through the pressure reducer, *10*, into tanks, *12* and *9*, displacing therefrom hydrogen peroxide and potassium permanganate into the reactor, *11*. The generated steam begins to rotate the turbine, and in 1–2 sec the fuel consumption reaches its nominal value of 276 lb/sec, and the rocket begins its flight.

During flight, it is necessary to have sufficient pressure in the fuel and oxidizer tanks to insure a steady flow of components to the pumps. For this purpose the atmospheric air is fed into the fuel tanks at a pressure equal to the velocity head (ram pressure), and the oxidizer tank is supplied with a small amount of oxygen vaporized by exhaust gases of the turbine in the heat exchanger.

The stopping of the motor takes place in two stages. First, when the electropneumatic system is exhausted, the flow of hydrogen peroxide into the reactor is reduced. Because of this, less steam finds its way to the turbopump assembly and the turbine reduces its rpm. The amount of fuel supplied by the pump diminishes, and the motor thrust is reduced to about 30% of nominal. The motor is now working at what is known as the final stage. It continues working at the final stage until the rocket reaches the required velocity (see Chapter IX). At this time the flow of hydrogen peroxide into the reactor ceases entirely and the main fuel and oxidizer valves are closed. The motor stops completely.

3. A Liquid Fuel Rocket Motor With Accumulator Feed

We have seen the construction and operation of a liquid fuel rocket motor with a turbopump fuel delivery. Now let us examine the construction and operation of a motor having an accumulator feed.

The schematic of a motor intended for the anti-aircraft guided missile is shown in Fig. 3.15.

The motor utilizes self-igniting fuel components. The motor thrust is 8.8 tons and the fuel consumption is 92.5 lb/sec. The motor operates in the following manner.

At the start, the squib, *2*, is exploded. Due to the action of powder gases, the piston, *3*, ruptures the membrane, *4*, as a result of which the air in the high pressure accumulator, *1*, passes through the pressure reducer, *5*, where the pressure is lowered to the working level. Simultaneous with the rupture of the membrane, *4*, the low pressure explosive valve, *6*, is opened. By rupturing membranes, *7*, the air passes through the tubes, *8* and *12*, into fuel and oxidizer tanks, *11* and *15*. The fuel is evacuated from the tanks by means of scoop tubes, *10* and *14*, flexibly mounted on joints, *9* and *13*,

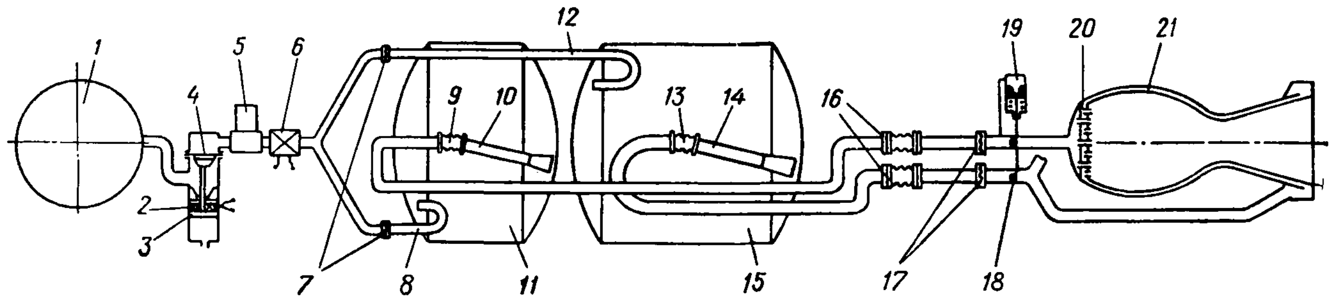


FIG. 3.15. Liquid fuel motor with accumulator delivery: *1*, compressed air accumulator; *2*, powder charge; *3*, piston; *4*, high pressure diaphragm valve; *5*, gas pressure reducer; *6*, low pressure explosive valve; *7*, rupture membrane; *8*, air supply duct to the combustible tank; *9*, flexible joint of the combustible scoop; *10*, combustible scoop; *11*, combustible tank; *12*, air supply duct to oxidizer tank; *13*, flexible joint of the oxidizer scoop; *14*, oxidizer scoop; *15*, oxidizer tank; *16*, sylphon (bellows) joints; *17*, diaphragms in combustible and oxidizer lines; *18*, throttles, baffles; *19*, servo-piston throttle control; *20*, motor head; and *21*, combustion chamber.

which insure that the deflection of the scoops will follow the change in liquid level caused by rocket maneuvers.

Under the action of air pressure, the fuel ruptures membranes, *17*, in the piping of the fuel components and begins to flow into the motor. The throttling vanes, *18*, are located in the piping to insure smooth starts. When a motor is started they are in a partially open position. After membranes, *17*, are ruptured the fuel enters the cylinder of the servo-piston, *19*, which, under fuel pressure, is slowly displaced, opening vanes, *18*. This insures smooth increase in fuel flow and a gradual attainment of operating conditions. In a subsequent stage of motor operation the vanes remain open.

The combustible enters the head of the motor directly, whereas the oxidizer first passes the cooling jacket of the motor. The combustible and the oxidizer mix, self-ignite, and burn in the combustion chamber, *21*. In contrast to the previously described motor of the V-2, the present method of start takes place faster, as is necessary for an anti-aircraft rocket. The motor schematic itself, with the displacement feed, is also much simpler than the motor of a ballistic rocket.

4. A Liquid Fuel Rocket Motor of Interceptor Aircraft

In conclusion, let us discuss the liquid fuel rocket motor of the fighter-interceptor aircraft. This motor (Fig. 3.16) is interesting in that its thrust is adjusted in flight, by means of changing its fuel consumption, through a wide range: 440 to 3,300 lb.

The motor uses self-igniting fuel components. The combustible consists of a mixture of hydrocarbons—hydrazine hydrate and methyl alcohol (methanol), and the oxidizer is hydrogen peroxide.

The motor has three groups of fuel jets which are turned on or off, depending on the necessity for increasing or decreasing the force. Therefore, the adjustment or regulation of the thrust of a motor is accomplished in steps.

During start, the turbopump assembly (including the hydrogen peroxide pump, *3*) is rotated by an electric motor, *4*. Hydrogen peroxide, supplied by the pump, is directed to the hydrogen peroxide flow regulator, and by corresponding rotation of the sleeve, *9*, lifts the regulator valve, *12*, and passes through the manifold surrounding valve, *6*, into the reactor, *1*. As opposed to the motor of the ballistic rocket, this motor uses a solid catalyst, which, in the shape of a slug, *2*, is placed in the reactor. The water vapor generated in the reactor continues to accelerate the turbine; at a certain rotational velocity of the turbopump assembly, the electric motor, by means of a free wheeling device, is disengaged from the shaft of the turbopump assembly and comes to a stop. The turbopump assembly continues to work while coasting. The number of revolutions during coasting is determined by spring tension, *11*, of the regulator, *12*.

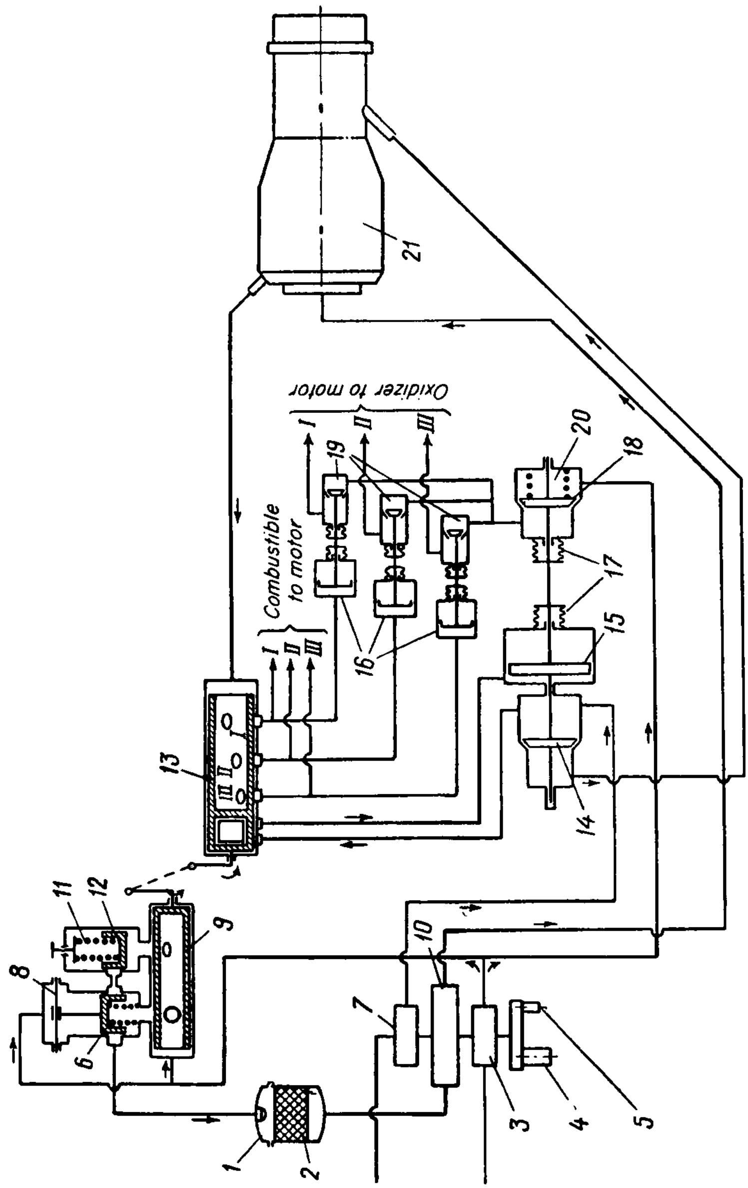


FIG. 3.16. Schematic of an interceptor aircraft motor: *1*, steam generator reactor; *2*, slug of a solid catalyst; *3*, hydrogen peroxide pump, electric motor; *5*, tachometer; *6*, hydrogen peroxide regulator valve; *7*, combustible pump; *8*, diaphragm; *9*, slide valve of the hydrogen peroxide regulator; *10*, turbine; *11*, coasting spring regulator for turbo-pump assembly; *12*, coasting regulator valve; *13*, fuel regulator slide valve; *14*, combustible valve; *15*, servo-piston of the combustible and oxidizer main valves; *16*, servopiston of the hydrogen peroxide valves; *17*, sylphon (bellows) seals; *18*, hydrogen peroxide main valve; *19*, hydrogen peroxide supply valve; *20*, chamber of the coupled main fuel valve; and *21*, combustion chamber.

During the coasting operation, the pressure from the hydrogen peroxide pump, *3*, is transmitted into the chamber of the interconnected main fuel valve, *20*, resulting in a closing of both the hydrogen peroxide valve, *18*, and the fuel valve, *14*. This does away with the leakage of fuel into the motor combustion chamber. In order to bring the motor to operating con-

ditions, the pilot turns the sleeve, *9*, also turning the sleeve, *13*, which is paired with it. Sleeve *13* allows fuel into the cylinder of the servo-piston, *15*, through a port in the left chamber. The piston translates to the right, thereby allowing hydrogen peroxide to enter the three valves, *19*. During this time the combustible is able to pass through the cooling jacket and enter the inner cavity of sleeve *13*.

Further rotation of sleeve *13* allows the sleeve port, *I*, to pass fuel into the fuel line, *I*, to the motor. Simultaneously, the fuel flows along line *I* to the servo-piston, *16*, of the oxidizer valve, *19*, and opens it, allowing hydrogen peroxide to enter line *I* leading to the motor. In this manner the group of fuel jets, *I*, begins to operate, self-ignition of the fuel components takes place, and the motor operates in regime *I*.

At the same time, due to the rotation of sleeve *9*, the hydrogen peroxide, through corresponding ports, gains access to the valve, *6*, and lifting it, passes in large quantities into the reactor of the water vapor generator, *1*. As is seen in Fig. 3.16 the pressure of hydrogen peroxide, which depends on the number of revolutions of turbopump assembly, is applied to membrane *8*. The last, deflecting under this pressure, tends to close valve *6*, and in so doing reduce the supply of hydrogen peroxide. Due to this arrangement, the excessive supply of hydrogen peroxide and disintegration of the turbopump assembly are prevented.

In order to increase thrust further it is necessary to continue rotating sleeve *13*, which, through ports *II* and then *III*, introduces the flow of fuel and oxidizer into fuel jet groups *II* and *III*. During this operation, the hydrogen peroxide regulator maintains the number of revolutions of the turbopump assembly at such a value that the pressure of the fuel feed remains constant.

The stopping of the motor is accomplished by turning the sleeve valves in an opposite direction; while in flight the turbopump assembly does not need to stop but may continue to idle.

In order to prevent the accumulation of fuel components in the combustion chamber from leaks in the fuel system while idling, a certain amount of water vapor, which has formed in the turbine, is directed into the motor chamber and there decomposes the hydrogen peroxide which might have leaked into it and accomplishes the scavenging of the motor chamber.

The interconnected main fuel valve, sleeve valve *13*, and the three oxidizer valves (*I*, *II*, and *III*) are physically located in the same block. In order to prevent the possibility of interaction between fuel components and their ignition, all connections which pass from the cavity of one component into the cavity of another are hermetically sealed by bellows, *17*.

We have discussed briefly the construction and operation of only three different liquid fuel rocket motors. We might have dwelled on the construc-

tion of a "cold" motor, operating on the decomposition products of hydrogen peroxide; described the operation of a motor with powder pressure accumulators; the action of an aircraft booster; etc. However, this is not necessary, inasmuch as the described three schematics are, to a certain extent, typical, and can serve as a basis for understanding the main construction characteristics of any one of the liquid fuel propelled rocket motors.

In conclusion, it must be noted that all liquid fuel rocket motors are characterized by a relatively high degree of automation in the starting and stopping operation of the motor.

For instance, in an anti-aircraft rocket motor the entire starting cycle is accomplished automatically, after supplying an electrical pulse to the squibs, *2* and *6* (Fig. 3.15). It is true that this motor does not stop, and continues to work until all the fuel components have been exhausted from the tanks. In an aircraft motor the control is also reduced to a minimum of required operations performed by the pilot. In discussing the motor of an intermediate range rocket we did not discuss, in detail, the consecutive operations necessary for starting. But even in this case only two or three commands are given during starting. All further interaction of a comparatively complicated electropneumatic apparatus is done automatically.

The trend to automate the operation of the motor to the limit is explained by the basic peculiarity of rockets, and particularly of liquid fueled rocket motors; specifically, the short time of their action, measured in seconds and minutes. During this short time it is necessary to perform all the operations essential for the reliable combustion of fuel; increasing fuel feed to nominal consumption, maintaining this fuel consumption constant, and changing it to correspond with the programmed performance of the motor. Finally, during this same time, it is necessary to stop the motor at the right moment.

We must not forget that the fuel entering the combustion chamber in a condition suitable for combustion represents a typical explosive mixture. Taking into account the high fuel flow rates, it can be easily understood that the slightest disturbance in the proper operation of the fuel feed system or delay in ignition time, which can lead to the accumulation of fuel in the chamber, can result in an explosion or a considerable fluctuation and increase in pressure which can also result in the destruction of the motor. We can expect the same result if there is accidental interruption in combustion or if the fuel has been fed into the chamber after the stopping of the motor. The ignition of fuel in this case, from the heated structure of the motor, can lead to irregular combustion and consequent explosion.

From what has been said above, it is clear that the fuel system must operate very smoothly. This is achieved by automation to the limit and wide

use of interlocks in the design of the fuel system. By interlocks we mean a design in which each succeeding operation in the fuel system cannot take place until the previous one has been successfully completed. Automation of fuel systems in contemporary liquid fuel rocket motors has reached such perfection that all operations for starting, reaching operating conditions, and stopping the motor are performed after only one command signal has been issued to the motor.

It is clear that such a high degree of automation is also necessary because in most cases the liquid fuel rocket motor is installed on pilotless aircraft.

IV. Rocket Motor Fuels

A. The Chemical Energy of Rocket Motor Fuels

1. The Sources of Energy for Rocket Motors

A rocket motor, like any other motor, requires some sort of energy source for its work.

The only source of energy presently utilized in rocket motors is chemical energy. During the work of the motor this energy can be released by two types of reaction, the more widespread of which is combustion. This reaction is used in the work cycle of the great majority of existing heat engines. Also used for the release of chemical energy is the reaction of decomposition of certain substances, if the process of decomposition is accompanied by the release of heat. In rocket motors, for instance, the reaction of the decomposition of hydrogen peroxide found wide application.

A rocket motor requires not only a source of energy but also a supply of mass which is ejected by the motor during its working cycle. The substance which is ejected by the rocket is usually called the working medium.

In existing motors the source of chemical energy is the fuel, the working medium being the products of combustion or decomposition reaction of fuel. In this way the fuel is first, the source of chemical energy and later, of kinetic energy.

A future source of energy for rocket motors will be nuclear energy. At the present time it is not applied to rocket motors, but without a doubt it will be used in the near future. It appears that nuclear fuel will be used only as a source of energy; the working medium would have to be contained aboard the rocket in addition to the fuel.

2. The Process of Combustion and Chemical Energy

The process of combustion is a chemical process involving two substances—the combustible and the oxidizer—which is accompanied by a release of a large quantity of heat.

Let us discuss the simplest combustion process—the reaction of combustion of combustible and oxidizing elements, taken in the pure atomic state.

The energy level of atoms is determined by the structure of the outer electron shell of the atom. The outer shell of the atom, when fully filled with electrons, can have only a very definite number of electrons. For elements

which are ordinarily used in fuels of liquid propelled rockets, this number is eight. Hydrogen is an exception; it may contain a maximum of two electrons in its outer shell.

The saturation of the outer shell by electrons is accompanied by the lowering of the energy level of the atom. An atom has a minimum energy level when its outer shell is occupied by a capacity number of electrons.

If we look at the atomic structure of combustible and oxidizing elements, we see that the outer shells are not occupied by electrons to capacity. So (see Fig. 4.1), the hydrogen atom lacks one electron (limit two), carbon lacks four, the oxygen atom lacks two, and the fluorine atom one electron (limit eight).

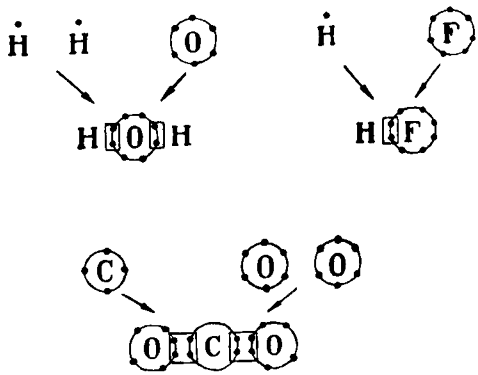


FIG 4.1. Schematic of chemical reactions. Electrons are represented by dots. Electrons enclosed in rectangles are contained in two atoms simultaneously.

In the chemical process of combustion, the structure of the atoms does not change, so that the total number of electrons in the combustible and oxidizer atoms remains constant. However, in formation of the combustion products, there is a mutual interlocking and rearrangement between electron shells of the reacting atoms, so that certain electrons find themselves simultaneously in the shells of both the combustible and oxidizer element atoms. For instance, as a result of the combustion reaction of hydrogen with oxygen and formation of water vapor, such a rearrangement in electron shells takes place. This results in two groups of two electrons each being simultaneously in the outer shells of oxygen and hydrogen atoms. Furthermore, the oxygen atom, contained in the molecule of water, has in its outer shell eight electrons; while the hydrogen atoms have two electrons each, i.e., the outer shells of all atoms are occupied by electrons to capacity.

In this same manner the simultaneous occupation of shells by electrons in the molecule of hydrogen fluoride (Fig. 4.1) leads to the saturation of the shells of all atoms which are contained in the product of combustion,

and the shells are occupied to capacity. For this reason the energy level of the molecules of the product of combustion is considerably lower than the energy level of the atoms prior to their union. The difference in these energy levels determines the amount of chemical energy released during burning.

Since the energy levels of the initial substances and the products of combustion depend on the structure of corresponding particles, the quantity of chemical energy released during combustion is constant, and is independent of any external conditions, such as pressure, temperature, volume, etc.

This is the process of combustion of substances in an atomic state. Actually, the combustible and oxidizing elements used in liquid propelled rockets are not found in an atomic state, but enter into the composition of molecules of simple or, most often, complex substances.

In this case the process of combustion, from the standpoint of energy release, can be conceived as taking place in the following manner. The substance, in a molecular state, decomposes into atoms of corresponding elements with expenditure and release of chemical energy. After that the atoms unite to form the products of combustion, always accompanied by a release of chemical energy. The over-all effect in relation to the liberation of chemical energy is determined by the algebraic sum of chemical energies corresponding to each of the substances participating in the reaction.

It should be noted that for liberation of chemical energy in the combustion process it is necessary to have initial energy of activation which is expended to bring the reacting atoms into such a state that the chemical reaction between them can take place. For instance, hydrogen and oxygen or carbon dioxide and oxygen can remain in the mixture as long as desired. The chemical reaction in such mixtures begins only after ignition. The process of ignition consists in applying the necessary energy of activation to a mixture which is ready to burn. If the combustion begins in some group of particles, then the energy of activation for other particles is derived from the neighboring particles which have completed the reaction. In this manner combustion progresses and becomes self-sustaining.

The magnitude of energy of activation for various mixtures differs. The greater the energy of activation the more difficult it is to ignite the mixture.

3. Chemical Energy and the Heat of Formation

Chemical energy liberated during combustion reaction can be determined by two methods.

The first method consists in calculating the energy level of particles (atoms and molecules) before and after reaction by means of spectronomy

and determining the magnitude of the released chemical energy by the difference of the found energies.

The second method consists in calorimetric determination of the heats of formation (also known as the heats of combustion) of substances. In this case the necessary reaction takes place in a special vessel (calorimeter) resulting in the transformation of the chemical energy into heat energy. This energy is expended to heat the products of reaction. At the end of the process of combustion the reaction products are cooled to the same temperature as the original substances before the start of the reaction. The amount of heat which is removed from the products of reaction is measured. The amount of heat determined in this manner is called the heat of formation of the substance, as differentiated from the chemical energy found by the first method.

The heat of formation has a definite relation to the chemical energy although it is not equal to it. The difference between the heat of formation and chemical energy is explained as follows:

During combustion and corresponding alteration of substances, a change in heat capacity of the original substance takes place. This change in the heat capacity

$$\Delta Q = \int_0^T c_2 dT - \int_0^T c_1 dT \tag{4.1}$$

where c_2 is the heat capacity of the reaction products, and c_1 is the heat capacity of the initial substances.

If the heat capacity is independent of the temperature, then

$$\Delta Q = (c_2 - c_1)T. \tag{4.2}$$

During the combustion reaction the heat capacity of the products of reaction is not equal to the heat capacity of the initial substances. This is because during combustion substances are formed whose properties are different from those of the initial substances.

Furthermore, the heat capacity of gaseous substances depends on the conditions (constant pressure or constant volume) under which the reaction takes place. Therefore, the change in the amount of energy contained in substances before and after the reaction depends on the conditions under which it takes place, i.e., pressure, volume, and initial temperature.

Temperature, T, has the greatest influence on the term ΔQ.

The heat of formation used in liquid fuel rocket calculations is usually related to the following conditions: constant pressure, equal to one absolute atmosphere and room temperature. In this case the heat of formation is considered as a change in magnitude of heat capacity H of the products of combustion and is designated ΔH_T^0. The upper index indicates pressure at which the heat of formation was determined and the subscript, absolute

temperature. If the heat of formation is determined when the absolute pressure $p = 1$ kg/cm² ($p = 14.7$ psi) then the upper index is equal to zero.

Knowing the heat of formation and the heat capacity of substances which participate in a reaction, it is always possible to calculate the chemical energy as an algebraic difference

$$\Delta H_T^0 \pm \Delta Q.$$

Since when $T = 0$, the term $Q = 0$, the chemical energy can be defined as the heat of formation at absolute zero and indicated as ΔH_0^0. This term serves as a measure of chemical energy and is widely used in many thermodynamic calculations.

Because the heat of formation of the products of combustion depends on surrounding conditions, it is determined at the so-called standard conditions. The heat of formation of substances is determined primarily at temperatures of 18°, 20°, and 25°C (291.16°, 293.16°, or 298.16°K). The difference between chemical energy and the heat of formation of the corresponding products is relatively small.

Since the heat of formation of a substance depends on the state of the elements forming a given substance (atomic, or in a form of simple or complex substances), the heat of formation is related not only to standard conditions but also to the standard state of the elements. These states of the elements are understood to be those in which the elements are most often found in nature. For instance, for standard states of the more frequently used fuel and oxidizing elements the following are used: for hydrogen, oxygen, and nitrogen—the molecular gases H_2, O_2, N_2; for carbon—β graphite; for metal—that crystalline form which is most often found in nature. The sign of the heat of formation is determined by the fact that the loss of energy by the system entering into the reaction is naturally considered as negative. For this reason, the heat of formation of combustion products always has a negative sign.

4. The Heating or Calorific Value of Fuel

The heating or calorific value of a fuel is the quantity of heat produced by the combustion of a unit weight of fuel. This quantity we shall represent by K_G, which is measured in kilocalories per kilogram.

The heating value of fuel is equal to the heat of formation of the products of combustion taken with an opposite sign, because the chemical energy of a substance, lost during the formation of the products of combustion, is wholly converted into heat according to the law of conservation of energy. In this manner,

$$K_G = -\Delta H_T^0.$$

The quantity of heat of formation is usually given in kilocalories per gram-mol, and can be easily converted into the quantity of heat per unit weight by making use of the relationship

$$\Delta H \text{ kcal/kg} = \frac{1000}{\mu} \Delta H \text{ kcal/gm mol}$$

where μ is the molecular weight of the products of combustion.

It should be noted that both the combustible and the oxidizer are contained aboard the rocket and consequently have equal value. Therefore, we will always relate the heating value or the heat of formation to the entire mass of the substances used in the process of reaction and not to the combustible alone, as is the practice in conventional thermodynamics where the expenditure of oxidizer—the atmospheric oxygen—is never taken into account since oxygen is obtained from the surrounding atmosphere.

Fuel components which contain the highest reserve of chemical energy should be used as combustible and oxidizer. In this case a greater supply of energy can be carried aboard a rocket of given dimensions. Since only a limited number of chemical elements exist in nature, it is natural to select those which possess the largest supply of chemical energy.

Even C. E. Tsiolkovski, in one of his first works, "Rocket into Interstellar Space," indicated the necessity for the existence of a definite relationship between the magnitude of chemical energy of the elements in connection with the periodic table of Mendelyeev. It is actually true that the heats of formation of products of combustion for different elements are systematically arranged, forming periods corresponding to the rows of elements in Mendelyeev's periodic law. More detailed investigations of this question led to the following results.

Of all the elements, only two—oxygen and fluorine—can be used as oxidizing elements yielding a large reserve of chemical energy, and insuring the required intensity of the combustion reaction. Other oxidizing elements, such as chlorine, bromine, and iodine, do not yield large chemical energies. As combustibles, the following are the most effective: beryllium, lithium, boron, aluminum, magnesium, silicon, hydrogen, and carbon.

Before we discuss in greater detail the properties and energy characteristics of various elements, let us more precisely define the basic indices according to which we can evaluate the fuel components (combustible and oxidizer) for liquid fuel rocket motors.

5. Basic Requirements for Rocket Fuel Motors

In evaluating various substances from the standpoint of utilization as rocket fuels, we must begin with the basic requirements demanded of fuels for the liquid rocket motors. These basic requirements are as follows:

1. High concentration by weight of chemical energy, determined by the magnitude of the calorific value of the fuel. The higher the magnitude of K_G, the higher the kinetic energy that can be imparted to the products of combustion, and the higher the exhaust velocity and specific impulse of the motor will be.

2. The greatest possible supply of energy per unit volume of fuel, i.e., the greatest possible volumetric fuel heat content. The higher this quantity, the smaller the volume necessary for the fuel supply, the smaller the relative weight of the rocket structure, and the smaller the value of μ_b for a given rocket.

The volumetric calorific value is

$$K_V = \gamma_t K_G \tag{4.3}$$

where γ_t is the specific weight of fuel in kg/liter.

The specific weight of the fuel composed of combustible and oxidizer is in its turn determined according to the formula

$$\gamma_t = \frac{1 + \nu}{1/\gamma_f + \nu/\gamma_0} \tag{4.4}$$

where γ_f is the combustible specific weight; γ_0, the oxidizer specific weight; and ν, the amount of oxidizer in kilograms necessary for one kilogram of the combustible.

Let us note that

$$\nu = \alpha\nu_0$$

where ν_0 is the theoretically required amount of oxidizer which is determined by the usual (weight) balance of the chemical reactions under the conditions of their total completion and minimum expenditure of oxidizer; and α is the excess coefficient of oxidizer; for liquid fuel rocket motors the coefficient α is ordinarily somewhat less than unity.

3. High specific heat of the products of combustion and correspondingly low burning temperature. Since the combustion process, in liquid propelled rocket motors, takes place at constant pressure, the combustion temperature, as a first approximation, can be determined by the relationship

$$T = K_G/c_p, \tag{4.5}$$

where c_p is the specific heat of the products of combustion at constant pressure. Therefore, with equal calorific value of the fuels, the higher the specific heat of the products of combustion, the lower the temperature in the combustion chamber will be.

The lowering of the combustion temperature of the fuel is very important in designing reliably operating motors. Aside from that, the lowering

of the combustion temperature lowers the degree of dissociation of the products of combustion (see page 154) and increases thermal efficiency of the motor.

Specific heat, c_{pm}, for gases with an equal number of atoms per molecule, expressed in kcal/gr mol deg, varies within narrow limits. Since one kilogram contains 1000μ gram mols of the products of combustion (μ is their molecular weight) the specific heat is:

$$c_p = c_{pm} \frac{1000}{\mu} \text{ kcal/kg deg.} \tag{4.6}$$

Consequently, the lower the molecular weight of the products of combustion, the higher their specific heat is and the easier it is to utilize a given fuel in the motor.

The molecular weight of the products of combustion can be defined by two other quantities, namely, the gas constant of the products of combustion, R, and the so-called specific volume, V_0.

As is well known, the universal gas constant for any gas is $R = 0.848$ kg m/gr mol deg or, in heat units

$$A\bar{R} = \frac{0.848}{427} = 1.986 \times 10^{-3} \text{ kcal/gm mol deg}$$

and the weight gas constant is

$$R = \frac{848}{\mu} \text{ kg m/kg deg.}$$

The lower the molecular weight of the products of combustion, the higher the gas constant is. Therefore, the magnitude of the gas constant for the products of combustion of rocket motors must be as high as possible. Let us utilize the equation of state for one kilogram of gas:

$$pV_0 = RT$$

or

$$V_0 = R(T/p).$$

We see that, under certain conditions, the volume of the formed products of combustion, V_0, is proportional to the value of the gas constant, R. The magnitude of the volume which the products of combustion occupy under normal conditions ($T = 291.16°K$, $p =$ kg/cm^2) is called the specific volume. It indicates how many liters of the products of combustion are formed from 1 kg of fuel under normal conditions. The higher this quantity, the higher the gas constant of the products of combustion is and the smaller their molecular weight is. Therefore, the specific volume during fuel combustion must be a maximum.

4. The physical condition of the combustion products exerts considerable influence on the process of expansion and transformation of heat energy into kinetic energy.

Transformation of heat energy into directed kinetic energy takes place, in its simplest form, in the case of the formation of gaseous products of combustion. For solid bodies the process of expansion is impossible, and they transmit the heat energy to the gas by means of heat conduction or radiation. Only that heat of the solid products which is transmitted to the gas by the above-mentioned methods can participate in the expansion process and can be utilized in creating the kinetic energy of the stream.

It is clear that the presence of solid particles in the nozzle is undesirable. Consequently, the boiling temperature of the products of combustion must be sufficiently low. Aside from that, the heat of evaporation of the products of combustion must be as low as possible, in order not to lose a considerable part of the heat released by combustion through evaporation.

6. Analysis of Combustible and Oxidizing Elements as Components of Rocket Fuels

The basic properties and energy characteristics of combustible and oxidizing elements of fuels for liquid propelled rocket motors are shown in Table IV.1.

In examining this table we must not be surprised at the apparently empty spaces and questionable data (numbers in parentheses). Even though the majority of substances listed in the table have been used in technology for some time, a more intense study of their properties and the properties of their compounds has begun only in recent years, due to the demands of rocket technology. The available data, listed in literature, pertaining to these substances is quite often contradictory and is frequently revised. However, the data included in the table is sufficient to give the basic evaluations of the listed elements as components of liquid rocket motor fuels.

Among the oxides listed, the greatest heat of formation per unit weight is possessed by beryllium oxide, BeO—5830 kcal/kg. This substance exceeds considerably the calorific values of the hydrocarbon fuels currently used. Boron oxide, B_2O_3; and lithium oxide, Li_2O; also have high heat of formation. A lower heat of formation is possessed by water, H_2O; magnesium oxide, MgO; aluminum oxide, Al_2O_3; and silicon oxide, SiO_2. Carbon dioxide, CO_2, has the lowest heat of formation.

Volumetric heat of formation is greatest for beryllium oxide and smallest for water. This last condition is a substantial deficiency of liquid hydrogen as rocket fuel.

The boiling temperature of oxides, with the exception of water and car-

bon monoxide gas, is quite high and the heat of evaporation has considerable magnitude, quite often greater than the heat of solidification. The application of fuels based on metallic combustibles and oxygenous oxidizers is not very profitable, inasmuch as conducting the process of combustion at high temperature is accompanied by a small release of energy as a result of dissociation, and conducting the process at low temperature requires the presence of a gaseous phase to absorb the heat of solid particles. The

TABLE IV.1. PROPERTIES OF BASIC COMBUSTIBLE AND OXIDIZING ELEMENTS AND THEIR PRODUCTS OF COMBUSTION

Element	Symbol or formula	State	Molecular weight	Specific weight, γkg/liter	Melting temperature, t_m°C	Boiling temperature, t_b°C
Hydrogen	H_2	liquid	2.016	0.07	−257.14	−252.79
Lithium	Li	solid	6.941	0.534	180	1400
Beryllium	Be	solid	9.01	1.85	1280	—
Boron	B	solid	10.82	1.73	2300	—
Carbon	C	solid	12.01	2.25	—	—
Magnesium	Mg	solid	24.32	1.74	650	1120
Aluminum	Al	solid	26.97	2.70	658	(2000)
Silicon	Si	solid	28.06	2.35	1414	(2400)
Oxygen	O_2	liquid	32.00	1.14	−223	−187
Fluorine	F_2	liquid	38.00	(1.14)	−218	−183

high order of molecular weights of the products of combustion determine the high temperature in the combustion chamber during the burning of metal.

Of the combustible elements of fluoride compounds the least heat of formation is possessed by hydrogen fluoride, HF (the heat of formation is the same as water); and carbon tetrafluoride, CF_4. The last has a lesser heat of formation than carbon dioxide, CO_2. Aside from that, the molecular weight of CF_4 is quite large. For this reason the utilization of fluorine as oxidizer for hydrocarbon fuels is not advantageous.

We must note, however, that the two atom molecule, HF, has considerably greater stability against dissociation than the three atom molecule, H_2O. Therefore, the application of fluorine oxides can become quite advantageous in conjunction with fuels very rich in hydrogen, because of the reduction of losses due to dissociation (see page 155).

The fluorine compounds of the remaining elements have a high heat of formation, both per unit weight and per unit volume. Among them lithium fluoride, LiF, stands out as having the maximum heat of formation per unit weight.

A situation worthy of note is that fluoride metallic compounds have

TABLE IV.1—*Continued*

Oxygen Combustion Products

Element	Chemical formula	State	Molecular weight	$\Delta H^\circ_{298.16}$ kcal/gr mol	K_G kcal/kg	ν_0 kg/kg	γ kg/liter	K_v kcal/liter	t_b °C	ΔH_{vapor} kcal/kg
Hydrogen	H_2O	gas	18.016	−57.798	3210	7.95	0.42	1350	100	—
Lithium	Li_2O	solid	29.88	−142.400	4760	1.15	0.75	3570	1300	(1160)
Beryllium	BeO	solid	25.01	−146.000	5830	1.78	1.52	8850	(3900)	quite high
Boron	B_2O_3	solid	69.64	−302.000	4350	2.21	1.28	5570	—	—
Carbon	CO_2	gas	44.01	−94.052	2140	2.66	1.32	2830	−78	—
Magnesium	MgO	solid	40.32	−143.840	3530	0.66	1.43	5050	(2250)	quite high
Aluminum	Al_2O_3	solid	101.94	−396.500	3900	0.855	1.66	6460	2700	(1137)
Silicon	SiO_2	solid	60.06	−203.300	3350	1.14	1.50	5030	1900	(1310)

Fluoride Combustion Products

Element	Chemical formula	State	Molecular weight	$\Delta H^\circ_{298.16}$ kcal/gr mol	K_G kcal/kg	ν_0 kg/kg	γ kg/liter	K_v kcal/liter	t_b °C	ΔH_{vapor} kcal/kg
Hydrogen	HF	gas	20.008	−64.20	3210	9.46	0.46	1475	19	—
Lithium	LiF	solid	25.94	−146.3	5650	2.74	0.87	4930	1680	(1960)
Beryllium	BeF_2	solid	47.01	−227.0	4830	4.22	1.23	5950	(1300)	(850)
Boron	BF_3	gas	67.82	−265.4	3910	5.26	1.21	4750	−101	—
Carbon	CF_4	gas	88.01	−162.5	1850	6.32	1.22	2250	−128	—
Magnesium	MgF_2	solid	62.32	−263.5	4210	1.56	1.31	5550	(2230)	(1110)
Aluminum	AlF_3	solid	83.97	−311.0	3710	2.12	1.41	5220	(1260)	(1100)
Silicon	SiF_4	gas	104.06	−361.3	3470	2.79	1.33	4600	−95	—

considerably lower boiling temperatures than oxide compounds and, no less important, comparatively small heat of evaporation. As a result of this, the application of fluoride oxidizers to metallic combustibles can have appreciable advantages as compared to oxygenous oxidizers. Lithium fluoride, LiF; and beryllium fluoride, BeF_2; are the two compounds of fluorine which have satisfactory molecular weights.

B. Contemporary Fuels for Liquid and Powder Rocket Motors

1. Required Properties of Liquid Fuels

Aside from the above-enumerated requirements of liquid fuels in general, the utilization of liquid fuels in the rocket motor places upon them many other, varied, and difficult to fulfill requirements. Let us look at some of the main ones.

The majority of rocket fuel components are chemical compounds. The formation of each chemical compound is accompanied either by absorption or release of heat.

Let us suppose that a certain chemical compound, used as a fuel component, has a negative heat of formation, i.e., it releases energy during its formation. In order to release heat from this substance in the combustion chamber it is obvious that it will be necessary to expend, at the expense of the heat of combustion, additional energy equal to the heat of formation of the component. Therefore, if the compound used as a fuel component has a negative heat of formation, the amount of heat release in the combustion chamber is lowered by a corresponding amount. If the heat of formation of a fuel component is positive, the calorific value of the fuel increases as compared to the calorific value of the elements referred to standard conditions.

From the above it follows that the fuel components must have a positive heat of formation. It is precisely on this principle that the proposals for using fuels in the atomic state in liquid propelled rockets are based.

Another requirement is that the fuel components be capable of absorbing large quantities of heat or have high heat absorption in order to cool the motor.

Quantitatively the heat absorption of a component is:

$$Q = c(T_b - T_0) \text{ kcal/kg}$$

where c is the specific heat of the component; T_b, the boiling temperature of the component at cooling jacket pressure (for a description of the cooling system, see below); and T_0, the component temperature equal to the temperature of the surrounding medium.

The highest heat absorption will be possessed by a component which has the greatest thermal capacity and the highest boiling temperature.

The heat absorption of a component is most correctly related to 1 kg of fuel burned in the motor, since both the amount of heat released in the motor and the amount of substance used for cooling are proportional to the total fuel consumption and not to the consumption of separate components.

The specific heat absorption of the combustible is

$$Q_f = c_f(T_b - T_0)\,\frac{1}{1+\nu}\ \text{kcal/kg of combustible}$$

and the specific heat absorption of the oxidizer is

$$Q_0 = c_0(T_b - T_0)\,\frac{\nu}{1+\nu}\ \text{kcal/kg of combustible.}$$

Having the same physical properties (specific heat and boiling temperature) the oxidizer will have greater specific heat absorption than the combustible, since $\nu > 1$. Therefore, if the liquid rocket motor uses an oxidizer with a high boiling temperature then, as a rule, it is the oxidizer that is used for cooling.

The over-all heat absorption of the cooling component, QG (where G is the weight of fuel consumed per unit time), must be greater than the total quantity of heat removed in unit time from the surface of the motor being cooled. If this condition cannot be satisfied for either of the fuel components separately, then the motor must be cooled by both components simultaneously.

The conditions imposed by the fuel feed system demand that the fuel components have low viscosity and that the viscosity be nearly independent of temperature. Otherwise the change in temperature of the surrounding medium can lead to the change in the ratio ν of the fuel components supplied to the combustion chamber to the detriment of efficient fuel utilization.

The conditions of ignition and combustion of fuel in the combustion chamber of the motor require low ignition temperatures for nonself-igniting fuels and the shortest possible ignition time delay for self-igniting fuels. The last requirement is of the greatest significance.

By ignition time delay is understood that increment of time which elapses from the moment of the initial contact between two liquid self-igniting fuel components and the instant of their ignition. Sometimes this quantity is called the induction period. It is clear that during chemical ignition the amount of fuel accumulated in the combustion chamber during the starting of the motor will be greater the longer the self-ignition time delay (all other conditions being equal). As has been pointed out earlier,

the accumulation of the mixture in a chamber can result in the explosion of the motor. During the starting of the motor the time delay of self-ignition must not exceed 0.03 sec for safety considerations.

It is necessary that the self-ignition time delay not be excessive at lowered fuel temperature and atmospheric pressures in order to insure reliable initiation of the operation of the motor under various meteorological conditions and altitudes.

The operating conditions of the motors and rockets require fuels having a physical and chemical stability which will allow storage of fuel components for prolonged periods of time without special precautions. Fuels must be explosion-proof and have high boiling and low freezing temperatures. The fuel must not be poisonous or corrosive in contact with structural materials. Besides, it is imperative that rocket fuel be cheap, easily produced in large quantities, and its production assured by availability of the raw material.

It must be stated that, at present, there are no fuels which satisfy all of the listed requirements simultaneously.

In spite of widespread research in the field of new substances a relatively limited number of chemical compounds is presently used as rocket fuel.

2. Classification of Rocket Fuels

Rocket fuels may be classified according to various criteria.

First of all, the various physical states must be differentiated. In accordance with this the fuels are divided into liquid and solid (powder) categories. Intermediate groups consist of mixed fuels, one component being liquid and the other solid. Attempts to use such fuels in liquid propelled motors may be noted. For instance, gelated benzine, with carbon rods arranged in the combustion chamber proper and a liquid oxidizer supplied by the tanks, was known to be used as rocket fuel.

The fuels may be classified according to the number of components of which they are comprised.

For simplicity in the fuel feed system and the construction of the motor chamber, the most convenient fuel is the single component fuel, or monopropellant, which contains both the combustible and the oxidizing elements. Such fuels have been in the process of development but, to this time, there is not a single liquid monopropellant fuel known which is safe, reliable, and has high calorific value. All presently employed liquid fuels are bipropellant.

The third and already known method of fuel classification is by its nonself-igniting (hypogolic) or self-igniting (hypergolic) characteristics.

A fuel classification chart is shown in Fig. 4.2.

The properties of fuel are determined primarily by the properties of the oxidizer. Therefore, the division of fuels into groups is also made according

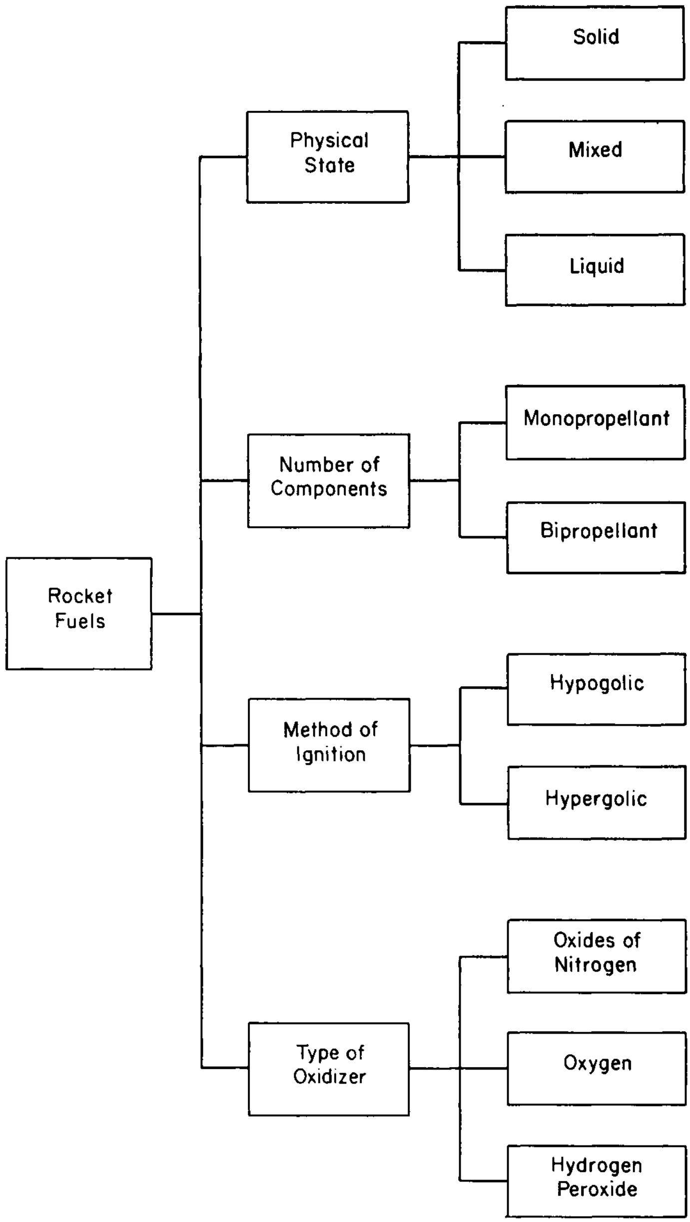


FIG. 4.2. Classification of rocket fuels.

TABLE IV.2. BASIC PROPERTIES OF PURE OXIDIZERS FOR CONTEMPORARY LIQUID ROCKET MOTORS

Name	Chemical formula	Molecular weight	Composition O	Composition C	Composition H	Composition N	Heat of formation kcal/gr mol	Specific weight kg/liter	Melting temperature‡ °C	Boiling temperature‡ °C
Nitric acid	HNO_3	63.02	0.762	—	0.016	0.222	−41.66	1.51	−41.6	+86
Nitrogen tetroxide	N_2O_4	92.02	0.696	—	—	0.304	+8.0	1.47	−9.3	+22
Tetranitromethane	$C(NO_2)_4$	196.04	0.653	0.061	—	0.286	+5.2	1.65	+13.8	+126
Liquid oxygen	O_2	32.00	1.000	—	—	—	0	1.14†	−227	−183
Hydrogen peroxide	H_2O_2	34.02	0.940	—	0.060	—	−45.20	1.46	0	+152
Water* (liquid state)	H_2O	18.02	0.889	—	0.111	—	−68.35	1.00	0	+100

* Water in the oxidizer serves as an inert stabilizer.
† At a temperature of −183°C.
‡ At atmospheric pressure.

TABLE IV.3. BASIC PROPERTIES OF COMBUSTIBLES FOR CONTEMPORARY LIQUID ROCKET MOTORS

Name	Chemical formula	Molecular weight	Composition C	Composition H	Composition O	Composition N	Heat of formation, kcal/gr mol	Specific weight, kg/liter	Boiling temperature, °C*	Melting temperature °C*
Kerosene	—	100	0.858	0.135	0.007	—	−46.0	0.76–0.84	170–150	−50
Ethyl alcohol	C_2H_5OH	46.07	0.522	0.131	0.347	—	−69.90	0.798	78.5	−117.3
Methyl alcohol	CH_3OH	32.04	0.375	0.125	0.500	—	−60.24	0.791	64.6	−97.9
Aniline	$C_6H_5NH_2$	93.08	0.774	0.176	—	0.150	+7.08	1.022	184.4	−6.2
Triethylamine	$(C_2H_5)_3N$	101.17	0.712	0.149	—	0.139	−42.2	0.728	89.5	−114.8
Dimethyl aniline	$(CH_3)_2C_6H_3NH_2$	121.12	0.792	0.092	—	0.116	−46.2	—	—	—
Furfuryl alcohol	$C_4H_3OCH_2OH$	98.06	0.613	0.062	0.325	—	−63.1	1.13	171	−32
Hydrazine hydrate	$N_2H_4 \cdot H_2O$	50.06	—	0.122	0.318	0.560	−63.13	1.03	118.3	−40
Vinyl ethyl ether	$C_2H_3OC_2H_5$	72.07	0.667	0.111	0.222	—	−46.0	0.754	36	−50

* At atmospheric pressure.

TABLE IV.4. BASIC PROPERTIES OF FUELS FOR CONTEMPORARY LIQUID ROCKET MOTORS

Oxidizer	Combustible	K_G kcal/kg	γ kg/liter	K_v kcal/liter	v liters/kg	T,† °K	I,† kg sec/kg
98% Nitric acid	Kerosene	1460	1.36	1980	800	3000	220
98% Nitric acid	Tonka 250*	1500	1.32	1800	784	2980	225
98% Nitric acid	Aniline (80%) + furfuryl alcohol (20%)	1520	1.39	1900	756	3050	223
Tetranitromethane	Kerosene	1620	1.47	2200	660	3300	238
Nitrogen tetroxide	Kerosene	1560	1.38	2000	680	3250	235
80% Hydrogen peroxide		680	1.35	920	1083	550	90–100
80% Hydrogen peroxide	Methyl alcohol (50%) + hydrazine hydrate (50%)	1020	1.30	1330	940	2600	190
Liquid oxygen	Kerosene	2200	1.00	2200	650	3550	260
Liquid oxygen	Ethyl alcohol 93.5%	2020	0.998	2000	789	3250	240

* German trade name.

† The values of combustion temperature, T, and specific impulse, I, are approximate, for average liquid rocket motors having a chamber to nozzle exit plane pressure ratio of 30:1.

to the type of oxidizer used. For this reason we will start the following description of fuels by describing the properties of oxidizers.

The properties of oxidizers, combustibles, and fuels as a whole are listed in Tables IV.2, IV.3, and IV.4. These tables show the composition by elements, i.e., content of carbon, C; hydrogen, H; oxygen, O; and nitrogen, N in 1 kg of combustible or oxidizer, the heat of formation, specific weight, melting temperature, and boiling temperature. For fuels, the specific calorific value per unit weight, unit volume, and specific volume are also listed, plus representative values of combustion chamber temperatures, and specific impulse for a rocket motor with average design parameters.

3. Nitric Acid and Nitric Oxides. Tetrantitromethane

Nitric acid, HNO_3, is a widely used household product. It was produced in large quantities prior to the appearance of liquid rocket motors. It is because of this that it was used as an oxidizer in the first liquid propelled rockets. At the present time, fuels based on nitric acid are widely used in rocket technology.

In its pure state, nitric acid is a colorless fluid. Commercial nitric acid always contains a certain amount of water and nitric oxides which give the acid a brownish-red color.

The presence of water in nitric acid is undesirable because it lowers the calorific value of the fuel. Therefore, liquid rocket motors use 98–96% concentrated acid, i.e., with water content not exceeding 2–4%.

Pure nitric acid (see Table IV.2) contains 76% O and has small negative heat of formation. This makes it a relatively powerful oxidizer. Of all the widely used oxidizers, nitric acid has the greatest specific gravity, which enables us to obtain fuel with high calorific value per unit volume.

Nitric acid has boiling and freezing temperatures (+86°C and −42°C, respectively) suitable for application in liquid rocket motors. Addition of water in amounts up to 10% to HNO_3 lowers the freezing temperature somewhat (to −68.5°C). Upon further dilution of nitric acid by water the freezing temperature becomes higher.

The boiling temperature of HNO_3 increases with increased pressure, so that at pressures prevailing in the cooling jackets of liquid rocket motors the boiling temperature exceeds 200°C.

The heat capacity of HNO_3 constitutes about 0.5 kcal/kg deg, which, coupled with high boiling temperature and relatively high content of nitric acid in fuel ($\nu_0 = 5.47$), makes HNO_3 a high quality coolant with a high degree of heat absorption.

Nitric acid also has a number of shortcomings. HNO_3 fumes are poisonous and, in contact with skin, nitric acid causes serious burns. Therefore, operation with nitric acid requires certain safety measures.

Nitric acid is quite corrosive with respect to metals and other structural materials. When diluted with water, HNO_3 has a particularly strong action on metal. Therefore, all parts which come in contact with nitric acid must be very carefully washed. Materials which can withstand the action of HNO_3 are stainless steel and certain plastics.

Nitric acid is quite volatile, which makes it inconvenient to store.

In order to improve the properties of nitric acid as an oxidizer it is mixed with various additives. They may be added in order to increase the calorific value of the fuel, to increase the specific weight of the oxidizer, to reduce corrosive action with respect to structural materials, to increase the action of oxidizer with respect to the combustible (especially with respect to self-ignition with nitric acid), and finally to lower the freezing temperature of fuel components. Many additives influence not one, but several properties of HNO_3, i.e., have a composite effect.

Two of the additives to nitric acid are nitric tetroxide and sulfuric acid.

Nitric tetroxide, N_2O_4, is an oxide of nitrogen, rich in oxygen content, with a positive heat of formation. It is a yellow volatile liquid. Its fumes decompose in the presence of air and have a distinctive yellow color. The use of pure N_2O_4 as an oxidizer is impossible because of the high freezing temperature ($-9.9°C$) and low boiling temperature ($+22°C$). Therefore, N_2O_4 is used only as an additive to HNO_3 in order to increase the calorific value of fuel. Besides, the addition of N_2O_4 to HNO_3 results in a solution with higher specific gravity than the specific gravities of N_2O_4 and HNO_3 taken separately. The maximum specific weight of such a solution having 40% N_2O_4 is 1.63 kg/liter. The addition of N_2O_4 also increases the reaction of the oxidizer and consequently facilitates the starting of liquid propelled rocket motors.

The addition of N_2O_4 to nitric acid, just as in the case of added water, changes the freezing temperature of the mixture. The lowest freezing temperature ($-73°C$) is obtained by addition of 18% N_2O_4. Further addition of N_2O_4 leads to the rise of freezing temperature.

A concentrated solution of sulfuric acid, H_2SO_4, is used as an additive, lowering the corrosive properties and improving the starting conditions of the motor, especially with self-igniting fuels. Addition of sulfuric acid to nitric acid in insufficient quantities leads to the lowering of the calorific value of the fuel.

Ferric chloride is added to nitric acid in order to lower the freezing temperature and increase the activity of HNO_3.

Among the nitrogen-oxygen compounds, tetranitromethane, $C(NO_2)_4$, can be used as an oxidizer, besides HNO_3 and N_2O_4. Tetranitromethane has a positive heat of formation. The big advantage of tetranitromethane as an oxidizer is its high specific weight (1.65 kg/liter), which is higher than

that of nitric acid. Tetranitromethane has no corrosive action on structural materials. The use of tetranitromethane is limited by its inclination to explosion. Besides, tetranitromethane is a very poisonous substance, affecting the mucus membrane.

The freezing temperature of tetranitromethane is quite high, $+13.8°C$. However, the freezing temperature is lowered to $-26°C$ when mixed with N_2O_4, which permits this mixture to be used as an oxidizer for liquid fuel rocket motors.

4. Combustibles used in Fuels Based on Nitric Acid and Oxides of Nitrogen

Of the fuel components used in conjunction with HNO_3 and oxides of nitrogen, the most widely used is kerosene. The basic physical-chemical properties of kerosene are shown in Table IV.3 and the properties of fuel combining nitric acid and kerosene in Table IV.4.

The successful application of kerosene to liquid fuel rocket motors is due to a number of positive qualities of this combustible. Fuels having kerosene in their composition have a high calorific value. Kerosene remains liquid within wide limits of temperature change. Kerosene can be used as a motor coolant; its heat capacity is 0.45 kcal/kg deg, and its boiling temperature, at elevated pressures, reaches 250°C. Transportation and storage of kerosene do not cause difficulties either, and its manufacture is insured by a well-developed oil refining industry.

Kerosene can be used as a combustible in conjunction with all oxidizers based on oxides of nitrogen. Quantitatively, the calorific value of fuels composed of nitric acid and kerosene equals 1,460 kcal/kg. For oxides of nitrogen this may be somewhat increased (up to 1,500 kcal/kg) due to superior properties of the oxidizer. The deficiency of kerosene as fuel is its comparatively small specific weight (0.80–0.85 kg/liter). A mixture of kerosene and HNO_3, and other fuels based on kerosene and oxides of nitrogen, are not self-igniting and require forced ignition.

Other hydrocarbons which, when paired with nitric acid, form nonself-igniting fuels, for instance, alcohols, did not find application in liquid fuel rocket motors.

Self-igniting fuels were also formed based on HNO_3 and oxides of nitrogen. The combustible in such fuels is usually a complex hydrocarbon: aniline, $C_6H_5NH_2$; furfuryl alcohol, $C_4H_3OCH_2OH$; xylidine, $(CH_3)_2C_6H_3$-NH_2; or triethylamine, $(C_2H_5)_3N$. Their composition and basic physico-chemical properties are shown in Table IV.3. Of the special properties of these hydrocarbons it is necessary only to point out the somewhat high specific gravity of aniline (1.03 kg/liter).

In order to get the least possible time delay period prior to self-ignition,

all other properties being satisfactory, the best mixture of combustibles is selected from various substances. In this manner a great number of self-igniting fuels of various compositions become available. Their common deficiency is high cost, and scarcity of available components. In practice, besides the application of self-igniting mixtures as the basic motor fuel, they are often used as initiators in systems of chemical ignition.

Of the more common self-igniting fuels are the following mixtures:

1. Nitric acid plus combustible, consisting of 50% xylidine and 50% triethylamine.
2. Nitric acid plus combustible, consisting of 80% aniline and 20% furfuryl alcohol.

5. Liquid Oxygen

Liquid oxygen is an even more powerful oxidizer than nitric acid, since it contains 100% of the oxidizing element. Liquid oxygen is a transparent liquid of slightly bluish coloration boiling at $-183°C$. Its specific weight is considerably less than that of HNO_3 and at boiling temperatures is equal to 1.14 kg/liter. The low boiling temperature does not permit liquid oxygen to be used as a coolant. For the same reason liquid oxygen cannot be used in rockets requiring storage in loaded condition. The filling of rocket tanks with liquid oxygen is done immediately prior to launching. Even under these conditions the loss of oxygen due to evaporation is considerable.

Liquid oxygen is relatively harmless to people. Upon contact with the skin, in small amounts, it boils and forms a layer of gaseous oxygen which protects the skin against freezing.

In recent years the use of liquid oxygen has been increased in many branches of technology, resulting in large scale manufacture. The problems of storage and transportation of liquid O_2 have also been solved satisfactorily. Therefore, regardless of the unaviodable losses due to evaporation (about 50% of the initial quantity of oxygen is lost when used in liquid propelled rocket motors), the cost of liquid oxygen used in rockets is not great.

6. Combustibles of Fuels Based on Liquid Oxygen

Any hydrocarbons may be used as combustibles with liquid oxygen. They all yield nonself-igniting fuels when combined with O_2. A mixture of liquid oxygen and kerosene has a high calorific value equal to 2,200 kcal/kg. This, in general, is the most powerful of contemporary liquid fuels. Attempts to utilize a mixture of oxygen with kerosene were made in the dawn of rocket technology development. The difficulties of using this mixture in liquid fuel rocket motors are in the high combustion temperature and also in the somewhat small amounts of kerosene in the fuel (approximately

20%), which complicates the cooling of the motor. These reasons limit the application of oxygen paired with kerosene to this day.

At the present time, fuels with liquid oxygen as an oxidizer and ethyl or methyl alcohol as combustible are widespread.

The basic properties of ethyl and methyl alcohols are shown in Table IV.3. Ethyl alcohol of 93.5% concentration is used in practice. Alcohols of higher concentration require too complex a technology. The calorific value of alcohols is lower than that of kerosene since they have greater negative heat of formation, but at the same time the temperature of combustion of alcohol in oxygen is lower. This facilitates the design of a reliably operating motor. The specific weight of alcohols is small (0.8 kg/liter). The boiling temperature is sufficiently high (taking the pressure in the cooling jacket of the liquid fuel rocket motor into consideration), which permits utilization of alcohol as a coolant. The heat capacity of alcohols is somewhat higher than that of kerosene and is euqal to approximately 0.6 kcal/kg deg. Due to the fact that the alcohol itself contains a considerable quantity of oxygen, the relative content of alcohol in the fuel increases to 40–45%. This fact also contributes to the satisfactory cooling of the motor by alcohol.

Ethyl and methyl alcohols mix with water in any proportion. This enables easy formation of fuels with various calorific values, thereby lowering temperatures in the combustion chamber and increasing the over-all heat absorption of the combustible to any degree. This was the road followed by the designers of the first ballistic rocket motors, who used as the combustible a 75% solution of alcohol and water, even though the specific impulse of the motor had been lowered considerably (to 204 kg sec/kg).

The low freezing temperature of alcohol permits its use through a wide range of ambient temperatures.

Alcohol is manufactured in very large quantities and is not a scarce combustible. Alcohol is not corrosive with respect to structural materials. This permits the use of relatively cheap metals for fabrication of alcohol tanks and fuel lines.

Ethyl alcohol can be replaced by methyl alcohol which, together with oxygen, forms a fuel of a somewhat lower quality. Methyl alcohol mixes with ethyl in any proportion, which permits its use during the shortage of ethyl alcohol; it can be added in some quantity to the combustible. Fuel based on liquid oxygen is used almost exclusively in long range rockets, which may mean, and due to their great weight even require, that they be loaded with fuel components on the launching pad.

7. Hydrogen Peroxide

In practice, hydrogen peroxide, H_2O_2, is not used in its pure form (i.e., 100% concentration) because of its highly unstable properties; decomposi-

tion easily resulting in an explosion under various, apparently insignificant external influences: a blow, light, the slightest contamination by organic substances and certain metallic admixtures.

More stable, highly concentrated solutions of hydrogen peroxide in water (most frequently of 80% concentration), are used in rocket technology. In order to increase the stability of hydrogen peroxide, small amounts of substances, preventing its decomposition, are added to it (for instance, phosphoric acid). The use of an 80% solution of hydrogen peroxide nowadays requires only the ordinary safety precautions necessary in handling strong oxidizers. Hydrogen peroxide in this concentration is a transparent, slightly bluish liquid with a freezing temperature of −25°C.

Hydrogen peroxide, decomposing into oxygen and water vapor, releases heat. This release of heat is explained by the fact that the heat of formation of peroxide is 45.20 kcal/gr mol, while the heat of formation of water is 68.35 kcal/gr mol. In this way, when peroxide decomposes according to the formula $H_2O_2 = H_2O + \frac{1}{2}O_2$, the released chemical energy is equal to the difference $68.35 - 45.20 = 23.15$ kcal/gr mol, or 680 kcal/kg.

An 80% solution of hydrogen peroxide can decompose in the presence of a catalyst, with a release of heat amounting to 540 kcal/kg and a release of free oxygen which may be used for oxidizing the combustible. Hydrogen peroxide has considerable specific weight (1.36 kg/liter for 80% concentrated solution). Hydrogen peroxide cannot be used as a coolant, since when heated it does not boil but immediately decomposes.

Stainless steel and very pure aluminum (with impurities not exceeding 0.5%) are used as fabricating materials for tanks and tubing of motors using hydrogen peroxide. The use of copper and other heavy metals is absolutely not permissible. Copper is a very strong catalyst aiding the decomposition of hydrogen peroxide. Certain types of plastics may be used for liners and reinforcements. Contact of concentrated hydrogen peroxide with skin causes severe burns. Hydrogen peroxide ignites organic substances with which it comes in contact.

8. Fuels Based on Hydrogen Peroxide

There are two types of fuels based on hydrogen peroxide. Fuels of the first type are represented by fuels of a divided feed system in which oxygen, released during the decomposition of hydrogen peroxide, is used for burning the combustible. Fuel used in the motor of an interceptor airplane, described earlier (see page 86), is an example of this type. It consisted of 80% concentrated hydrogen peroxide and a mixture of hydrazine hydrate ($N_2H_4 \cdot H_2O$) with methyl alcohol. By adding a special catalyst to the combustible, this fuel becomes self-igniting. Relatively low calorific value (1020 kcal/kg) and the low molecular weight of the products of combustion

determine the low combustion temperature and facilitate operation of the motor. However, due to the low calorific value, the motor develops low specific impulse (190 kg sec/kg).

Hydrogen peroxide with water and alcohol can form relatively explosion-proof triple mixtures which are examples of monopropellant fuel. The calorific value of these explosion-proof mixtures is relatively low: 800–900 kcal/kg. Therefore, they will hardly be used as the basic fuel for liquid fuel rocket motors. These mixtures may be used in steam generators.

The decomposition reaction of concentrated peroxide, as has been mentioned, is widely used in rocket technology for the generation of steam, which is the working medium of turbines in the turbopump fuel delivery system.

There are also motors in which the heat of the decomposition of peroxide serves to create thrust. The specific thrust of such motors is low (90–100 kg sec/kg). Two types of catalyst are used for decomposing peroxide: liquid (solution of potassium permanganate, $KMnO_4$), and solid. The use of the last is preferred since it dispenses with the delivery system of the liquid catalyst into the reactor.

9. Rocket Powders

Rocket powders are solid fuels used in powder rocket motors. In addition to the basic requirements of all rocket fuels further requirements for powder are:

1. Powder must have stable burning characteristics at low chamber pressures. This requirement is important because the combustion chamber of the powder rocket motor also serves as the container for the fuel. Thick walls at high pressures will result in high chamber weight and a large value of μ_b for the rocket.

2. It is necessary that the powder have high mechanical strength, since the powder is subjected to pressure in the combustion chamber and considerable inertial forces. The damage to powder grain during the combustion process leads to the increase of the burning surface, rise in chamber pressure, and explosion of the motor.

Some other requirements for powders will be indicated later, in the chapter devoted to fuel combustion.

Contemporary rocket powder consists of a solution of two or more nitro-organic compounds.

The basic substance entering into the composition of powder and insuring the necessary physical properties of the powder is nitrocellulose.

Nitrocellulose is obtained by treating cellulose, i.e., organic fibrous substances (wood cellulose and cotton cellulose), with concentrated nitric acid. In order to do this, bundles of cellulose are submerged in vats con-

taining nitric acid. During this treatment cellulose is transformed into organic esters containing nitrogen and oxygen.

The liquid substance which dissolves nitrocellulose and insures its gelatinization (i.e., homogeneous solid solution of uniform composition and physicochemical properties) is nitroglycerin. This is the second basic substance entering into the composition of powder.

Nitroglycerin is obtained by treating glycerin with a mixture of concentrated nitric and sulfuric acids. A molecule of nitroglycerin (chemical formula, $C_3H_5(ONO_2)_3$) contains a considerable quantity of oxygen necessary for burning the combustible elements—carbon and hydrogen. The calorific value of nitroglycerin is 1485 kcal/kg. Nitroglycerin is the basic constituent of powder having a supply of chemical energy. The calorific value of powder is determined by the percentage of nitroglycerin content.

At normal temperatures nitroglycerin is a heavy, oily liquid (specific weight, 1.6 kg/liter). Pure nitroglycerin is colorless; commercial nitroglycerin has a light yellow coloration. At elevated temperatures nitroglycerin exhudes a weak, sweetish odor. Its vapors are poisonous and cause headaches, and large doses entering the human organism can result in poisoning.

Glycerine, the basic substance for obtaining nitroglycerin, requires for its preparation the expenditure of esters. Therefore, in the manufacture of rocket powders the trend is to replace nitroglycerin, at least partially, by other gelatinizing substances. Some of these substances are diethyl phthalate, dinitrotoluene, diethylglycoldinitrate, and some others.

Aside from these basic substances small amounts of additives are also introduced into rocket powders.

Certain other additives, such as diphenylamine and ethyl centralite, are added to the powder to give it chemical stability. In preventing the decomposition of powder these substances also prevent a change in physical properties and powder structure. The change in powder structure during storage is most undesirable, since it can cause a variation in the normal process of combustion and explode the motor. Certain additives (vaseline and wax) serve to impart to the powder mass a degree of plasticity necessary in the manufacture of powder grains.

Inasmuch as the rocket powder is a mixture of various substances, the calorific value of the powder is determined by the composition and the calorific value of the composites.

Nitrogen, acting as a binder unifying into one molecule the combustible and oxidizing elements, is a ballast in powders just as it is in liquid fuels. Therefore, a large nitrogen content leads to the reduction of the calorific value of the powder. Nitrocellulose, usually used in powder, contains 12–14% N.

The calorific value of powder is also determined by the value of the so-called oxygen balance, i.e., the quantity ν for powder and its components. For the majority of powders $\nu < \nu_0$, or it can be said that the powder has a negative oxygen balance indicating lack of oxygen ($\alpha < 1$) and incomplete combustion of combustible elements.

The majority of rocket powder components, besides diphenylamine, have a negative heat of formation.

TABLE IV.5. Composition and Properties of Some Rocket Powders

Powder	Composition: Substance	Composition: Per cent by weight	Calorific value K_G, kcal/kg	Burning temperature at constant pressure T°K	Reference power of powder f_0, kg m/kg
J.P.	Nitrocellulose (13.25% N_2)	52.2	1230	3170	100,000
	Nitroglycerin	43.0			
	Diethyl phthalate	3.0			
	Diphenylamine	0.6			
	Potassium nitrate	1.1			
	Nigrosine	0.1			
J.P.N.	Nitrocellulose (13.25% N_2)	51.50	1230	3170	100,000
	Nitroglycerin	43.00			
	Diethyl phthalate	3.00			
	Ethyl centralite	1.00			
	Potassium sulfate	1.25			
	Carbon black	0.20			
	Candelilla wax	0.05			
Slow burning powder	Nitrocellulose (12.2% N_2)	56.5	880	2330	76,000
	Nitroglycerin	28.0			
	Dinitrotoluene	11.0			
	Ethyl centralite	4.4			
	Candelilla wax	0.1			

As a result of high nitrogen content, negative oxygen balance, and negative heat of formation of the components, powder has a relatively low calorific value. It varies within the limits of 820–1250 kcal/kg, depending on the content of nitroglycerin (or other solvent).

Quite often, aside from the magnitude of the calorific value, an evaluation of the energy characteristic of powder may be made by using the so-called reduced power of powder f_0, measured in kilogram meters per kilogram.

The power of powder is the product of the gas constant, R, and the temperature of the combustion products, T. The combustion temperature used corresponds to powder burning at constant pressure in a rocket motor. The reduced power of powder enters directly into the expressions which determine the exhaust velocity and, therefore, in certain cases it is more convenient for calculation than calorific value.

The composition and properties of certain rocket powders are shown in Table IV.5.

10. Means of Increasing the Content of Chemical Energy in Fuels

The desire to increase the specific impulse of liquid fuel rocket motors leads to attempts to find new, more productive fuels. In the literature there are many references to attempts to utilize ozone, fluorine, and its compounds as new oxidizers; and metallic suspensions in kerosene, boron compounds with hydrogen, and a number of metal-organic compounds as combustibles. However, there are no known motors using these fuels.

As can be seen from the data presented in Table IV.1, there are strong possibilities of increasing the calorific value of rocket fuels at present. At the same time these possibilities are limited by considerable heat losses

TABLE IV.6.
Properties of Certain Organometallic and Hydrometallic Compounds

Substance	Chemical formula	Melting temperature °C	Boiling temperature °C	Heat of formation, kcal/gr mol	Specific weight, kg/liter
Pentaborane	B_5H_9	(50)	(60)	(0)	(0.64)
Diboranimine	B_2H_7N	−66	76	(−10)	(0.70)
Diethylberyllium	$Be(C_2H_5)_2$	12	(200)	(−35)	(0.60)
Trisilane	Si_3H_8	−117	53	(−20)	(0.88)
Trisililamine	$(SiH_3)_3N$	−106	52	(+10)	0.895

caused by dissociation of combustion products at high chamber temperatures (see Chapter V), and also by the difficulty involved in delivering solid substances to the motor. Great difficulties must be expected in attempting combustion of solid substances and also in the introduction into rocket technology of the very toxic and corrosive fluorine oxidizers.

The application of solid (metallic) combustibles may be conceived in the form of suspensions of finely divided metal in an ordinary combustible; in metal-organic, metallic, and hydrogen compounds, which under normal conditions are liquid. The physical properties of certain organometallic and hydrogen-metallic compounds are shown in Table IV.6 (in parentheses are shown the insufficiently confirmed physico-chemical constants).

Other methods of increasing the volumetric and gravitometric calorific value of fuel have been discussed in literature concerned with rocket technology.

The calorific value of fuels utilizing chemical energy can be increased by means of selecting as components not the pure elements in their standard form, but in such forms or compounds which (aside from the chemical energy of the elements which is released during combustion) possess positive heat of formation which was required in their formation.

This heat is released during combustion reaction and is added to the chemical energy of the products of combustion.

Examples of substances with a positive heat of formation are the combustible and oxidizing elements which are found not in the standard, molecular state, but in an atomic state. It must be noted that during formation of atomic elements considerable energy is expended (see Table IV.7). For

TABLE IV.7. Table of Substances Having a Positive Heat of Formation

Substance	Chemical formula	Heat of formation		Specific weight, kg/liter
		kcal/gr mol	kcal/kg	
Hydrogen, atomic	H	51.6	51,500	—
Oxygen, atomic	O	58.6	3,640	—
Ozone	O_3	35.0	730	1.71 at −183°C
Acetylene	C_2H_3	54.85	2,120	0.618 at −81.5°C
Tetranitromethane	$C(NO_2)_4$	16.0	81.5	1.65 at −20°C
Nitrogen tetroxide	N_2O_4	8.0	87.0	1.49 at 0°C

instance, 1 kg of atomic hydrogen contains 51,500 kcal of heat. However, until now, there was no known way of obtaining atomic hydrogen or oxygen in pure form, and storing them for any appreciable length of time.

Other substances which may serve as examples are: ozone, O_3, requiring for its formation from molecular oxygen 730 kcal/kg; and acetylene, C_2H_2, which is formed from carbon and hydrogen with a heat expenditure of 2,120 kcal/kg.

The defect in these fuels is their instability and tendency toward explosion during storage and burning, which creates difficulties in their exploitation in motors. Only substances with a very small positive heat of formation —tetroxide, N_2O_4, and tetranitromethane, $C(NO_2)_4$—found, and still find, application in rocket technology. It is worthy of note that many complex fuels and oxidizers have a specific weight greater than the elements of which they are composed. This is an advantageous property which leads to the increase of the volumetric calorific value of the fuel.

C. The Energy of Nuclear Reactions and Its Exploitation in Rocket Motors

1. The Nucleus of an Atom and Mass Defect

Quite an appreciable increase in the specific impulse of a rocket motor can be obtained by the exploitation of the energy of nuclear reaction, or the so-called atomic energy.

Nuclear reactions, as opposed to chemical reactions, take place in a way that changes the atomic structure.

According to contemporary theories, the nucleus of an atom consists of two kinds of heavy particles having (relatively) large mass: protons and neutrons. A proton consists of a positively charged a particle whose mass almost equals the mass of a hydrogen atom. A neutron, as is implied by its name, has no charge, but its mass also approximates the mass of a hydrogen atom. The total number of protons and neutrons contained in the nucleus is called the mass number.

In forming the nucleus of an atom from free protons and neutrons, just as in forming molecules from atoms, energy is released. The release of energy is explained by the fact that the nuclei of elements represent a stable system held together by a binding energy, whose rise during the formation of the nucleus must be accompanied by the lowering of the potential energy of the system. The calculation of the energy of nuclear formation is most easily performed by using the principle of the equivalence of mass and energy, according to which these quantities are interrelated by a relationship

$$E = mc^2 \tag{4.7}$$

where E = energy, m = mass, and c = velocity of light equal to 3×10^8 m/sec (186×10^3 mi/sec).

Energy equivalent to the mass of 1 kg sec^2/m consists of 9×10^{16} kg m. In converting into units of heat for 1 kg of weight we have

$$\frac{9 \times 10^{16}}{9.81 \times 427} = 2.15 \times 10^{13} \text{ kcal/kg.}$$

It follows from the principle of the equivalence of mass and energy that the binding energy of the elementary particles of the nucleus, which form a stable (with a negative-energy potential) atom, is accompanied by a reduction in their mass as compared to the mass of these same particles when they are so separated that interaction between them is precluded. This reduction in mass, Δm, during nuclear reactions is called mass defect and can be determined experimentally. According to the relationship in

Eq. (4.7), the energy lost in the system, and consequently the magnitude of concurrently released energy is:

$$E = \Delta mc^2.$$

2. Nuclear Reactions

The energy of formation and, consequently, the mass defect of different nuclei, varies. Therefore, in principle, there are possible nuclear reactions which will lead to the formation of nuclei with greater mass defect than the mass of the initial nuclei. As a result, the newly formed nuclei will be more stable. The energy released during the formation of a new nucleus is equivalent to the difference between the mass defects in the newly formed and in the original nuclei. Nuclear reactions, involving various elements, proceed differently. The most easily realized nuclear reactions are those using heavy elements having high mass numbers. It so happens that the greater the mass number of the nucleus, the less stable it is, and the easier it is to disintegrate it. It is precisely the heavy nuclei of the radioactive elements which are characterized by strong radiation during the splitting of the nuclei. Artificial nuclear reactions involving transformation of light elements have also been known for a long time.

However, in order to obtain a large and constant release of energy during nuclear reactions it is necessary to excite the nuclei artificially by bombardment. The magnitude of the energy of bombardment itself must be considerable. Also, the bombarding particles, conveying the necessary energy, must penetrate into the nucleus. Charged particles are not suitable for intensive bombardment of nuclei, since the greater part of their energy is lost in overcoming the force of the electric field surrounding the nucleus. The most suitable particles for bombarding the nucleus are neutrons which have no charge.

In order to maintain a continuous nuclear reaction it is necessary to have a constant source of neutrons, having the necessary supply of energy, or the nuclear reaction itself must be a source of neutrons for the continual bombardment of nuclei. It is after the attainment of these exact conditions that the practical application of the heavy nuclei energy became possible.

3. The Problem of Utilizing the Energy of Nuclear Reaction in Rocket Motors

In examining the question of utilizing the energy of nuclear reaction in rocket motors we come up against two of the basic properties of nuclear energy; its especially high concentration, and the necessity for having a critical mass for the realization of certain nuclear reactions.

First let us examine the question of atomic energy concentration.

Each nuclear reaction is characterized by its energy effect. Its measure is the mass defect expressed in increments, δ, of the initial mass of the active substance. The magnitude of δ is very small, considerably smaller then the relative defect during the formation of the two nuclei of the elements, since it is determined by the difference of the two nuclei. For instance, for the pressure reaction* of uranium, U_{92}^{235}, the quantity $\delta =$ 0.000731; during the reaction with light atoms the quantity δ is considerably higher. For the reaction of helium formation from lithium and hydrogen, $Li_3^7 + H_1^1 \rightarrow 2He_2^4$, the quantity $\delta = 0.00232$, and for the transformation of hydrogen into helium, $4H_1^1 \rightarrow He_2^4$, $\delta = 0.00715$.

However, even with these insignificant mass defects, due to the enormous magnitude of energy corresponding to the unit of mass, the energy yield K_G, will be very large. In accordance with Eq. (4.7) the quantity $K_G = 2.15 \times 10^{13}$ kcal/kg. For the reactions indicated above we have:

Reaction	K_G, kcal/kg
Uranium decomposition	1.57×10^{10}
$Li_3^7 + H_1^1 \rightarrow 2\ He_2^4$	2.67×10^{10}
$4\ H_1^1 \rightarrow He_2^4$	1.54×10^{11}

When nuclear reactions are utilized in rocket motors, the released energy must be absorbed in the form of heat energy by some working medium. Afterward, just as in the case of a conventional engine, this energy must be transformed into kinetic energy. A substance selected to serve as the working medium must have sufficient gravimetric heat capacity, i.e., as has been shown above, it must be a substance with a small number of atoms per molecule and the lowest possible molecular weight. Such substances (of the ones available) are, first of all, hydrogen, H_2, followed by ammonium hydroxide, NH_3, and water, H_2O, whose gravimetric heat capacities are also comparatively high.

It must be noted that dissociation, undesirable during utilization of chemical energy, can become advantageous in utilizing nuclear energy since, during dissociation of the working medium, the nuclear energy can accumulate in the form of chemical energy of the dissociated gas.

Table IV.8 shows the quantity of heat, K_G, accumulated in 1 kg of the working medium at temperatures of $T = 4000°$, and 6000°K (not taking dissociation into account). In the same table the amounts of active substance (uranium, U_{92}^{235}, or plutonium, Pu_{94}^{230}) necessary for heating 1 kg of the working medium to the required temperatures are shown.

* In nuclear reactions the conventional notation describes nuclear structure. The upper index following the element symbol shows the mass number and the lower shows the number of protons in the molecule.

TABLE IV.8. AMOUNT OF ENERGY, K_G, STORED IN 1 KG OF A WORKING MEDIUM AND CORRESPONDING EXPENDITURE OF ACTIVE MASS G_a

Working substance	T = 4000°K				T = 6000°K			
	K_G, kcal/kg	$G_a \times 10^{10}$, kg/kg	I_{sp}, kg sec/kg	I'_{sp}, kg sec/liter	K_G, kcal/kg	$G_a \times 10^{10}$, kg/kg	I_{sp}, kg sec/kg	I'_{sp}, kg sec/liter
Hydrogen, H_2, undissociated	15500	7.20	822	57.6	2500	12.0	1045	78.2
Water, H_2O, undissociated	2450	1.14	330	330	4700	2.18	453	453
Ammonia upon decomposition into H_2 and N_2	1940	0.90	293	200	3000	1.39	366	250
Atomic hydrogen, H	71500	33.4	1105	—	81400	37.8	1350	—
Atomic oxygen, O	4900	2.28	275	—	5500	2.56	337	—
Atomic nitrogen, N	9500	4.42	298	—	10200	4.75	360	—

NOTE: The specific impulse is calculated with a chamber to nozzle exit pressure ratio of 100. In calculating the specific impulse developed by substances in an atomic state, the recombination of atoms into molecules was not taken into account.

From Table IV.8 it follows that the amount of active mass relative to the working medium is quite small. This may lead, however, to substantial difficulties in designing heat exchangers for the transfer of energy to the working medium.

The thrust characteristics shown in Table IV.8 tell us that the specific impulse, considerably greater than that in conventional engines, can be obtained either by utilizing hydrogen as a working medium or by considerably increasing the temperature of the working medium in the chamber.

The application of hydrogen is made more difficult by its small specific weight in the liquid state, and the necessity for elevating the temperature in the chamber presents a whole array of problems connected with the difficulties of cooling.

The process of heat exchange, during which the energy of the active mass is transferred to the working medium, also poses a serious problem.

The second problem relating to the utilization of atomic energy consists in the fact that for the progression of nuclear reactions it is necessary to have a certain minimum—critical—mass. This requirement is explained by the fact that the dimensions of the atomic nucleus are very small (the cross section of the nucleus equals approximately 10^{-24} cm^2) and, to insure a sufficiently high probability of neutron collisions with the nucleus, it is necessary to have a considerable path which the neutron must traverse in the active substance.

The magnitude of the critical mass, and the correspondingly critical volume, depends on the form of the active substance and the conditions of nuclear reaction. For pure substances, U^{238}, U^{235}, and Pu^{239}, the critical volume is equivalent to the volume of a sphere having a radius of a few centimeters. The weight of such a sphere is 20–30 kg. However, in rocket motors it is difficult to expect the use of such small quantities of active substances since, under these conditions, the surface available for heat transfer is very small.

In order to increase the heat transfer surface, decelerate the nuclear reaction, and control it, it is necessary to add the so-called retarder to the fission substance. The retarder must have the ability to slow down the neutrons without absorbing them. Graphite and heavy water are used as retarders.

Such a scheme for utilizing the active substance serves as the basis of all existing proposals for using nuclear energy for commercial purposes. The magnitude of the critical mass, with the application of the retarder, rises rapidly, and reaches hundreds of kilograms.

The first of several possible schematic arrangements of a motor operating on nuclear energy is shown in Fig. 4.3.

The liquid working medium (the passive mass) is located in the tank, *1*.

It is transferred through the cooling jacket into the chamber by the pump, *2*. The amount of expenditure is controlled by the regulator, *3*. Passing through the head of the chamber, *6*, the working medium comes into contact with the pile of the reacting active substance, is heated to a high temperature, and then expands in the nozzle.

The basic difficulty arising from this scheme consists of the fact that the temperature of the active substance must be very high, higher than the temperature of the working medium, in order to insure the transfer of heat to the working medium. The solution to this problem is so difficult because

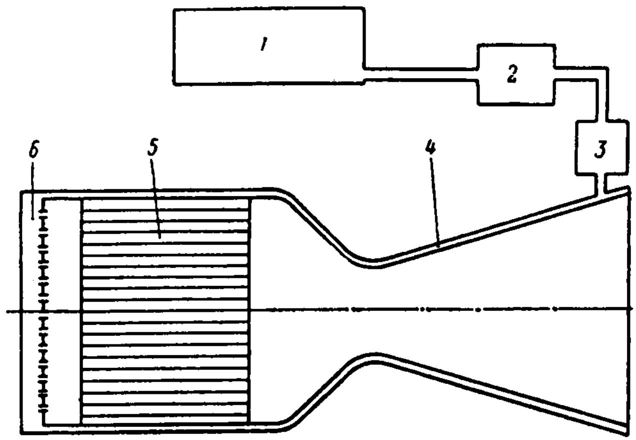


FIG. 4.3. Schematic of an atomic rocket motor containing a pile of solid reactive substance: *1*, tank with a liquid working medium; *2*, pump; *3*, working medium supply regulator; *4*, cooling jacket; *5*, reactive pile (atomic fuel); and *6*, head with jets.

there are no known materials which remain solid at temperatures of 4000°–6000°K. Uranium melts at $T = 1150°K$. A substance with a higher melting point, uranium oxide, melts at $T = 2100°K$. Even graphite vaporizes at a temperature of about 4000°K.

For these reasons we must find a design solution in which not all parts of the pile containing the active substance are subjected to this intensive heating; the cooler areas would insure the necessary mechanical strength of the pile as a whole. Partial melting or vaporization of the active substance may be allowed.

It must be noted, in general, that the supply of the active substance is not determined by the minute quantities which are shown in Table IV.8 and which were calculated on the basis of energy balance. This supply is determined by the size of the critical mass. We can also expect that the quantity of the active substance will be increased even more in order to obtain the necessary heat transfer surface.

The second possible scheme for utilizing atomic energy in a rocket motor is shown in Fig. 4.4.

The working medium is contained in the tank, *1*. It is fed to the motor by a pump, *2*. Having passed through the regulator, *3*, cooling jacket, *4*, and chamber head, *5*, the working medium enters the chamber of the motor in an atomized state.

The active substance, in the form of a solution or suspension, is contained in a tank, *9*. It is directed to the annular manifold by a pump, *8*,

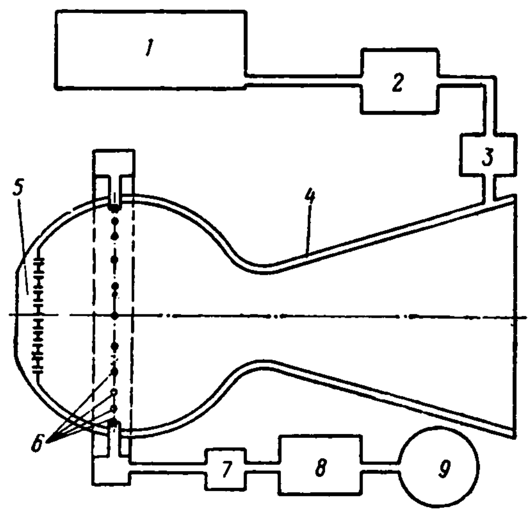


FIG. 4.4. Schematic of an atomic rocket motor with the injection of reactive substance into the chamber: *1*, tank with liquid working medium; *2*, working medium pump; *3*, working medium feed regulator; *4*, cooling jacket; *5*, head with working medium jets; *6*, circle of reactive substance (atomic fuel) substance; *7*, feed regulator; *8*, reactive substance pump; and *9*, tank with the solution of a liquid reactive substance.

through a regulator, *7*. The active substance passes from the manifold into the motor chamber and is atomized by a system of jets, *6*. Due to nuclear reactions which take place in the chamber, the working medium is heated, and subsequently expands in the nozzle. Such a system, utilizing atomic energy, is very attractive, since it insures the best conditions for heat transfer in a turbulent flow.

The basic difficulty in realizing the second scheme is the excessively large chamber dimensions. The magnitude of the critical mass, in this case, is replaced by the critical product of chamber pressure, p_k, and its radius, R_k. According to certain calculations, using hydrogen as a working medium and a chamber temperature of 5000°K, the necessary magnitude of p_kR_k is 12,000 kg m/cm². Therefore, with a chamber pressure of 100 kg/cm², the

chamber must have a minimum diameter of 240 meters. It is useless even to think about making a chamber of such dimensions. The same calculations for other working mediums yield even greater values of the critical chamber dimensions.

Thermonuclear reactions, independent of the magnitude of the critical mass, have just recently become known. However, in order to initiate such reactions a constant source of high temperature is necessary. For the time being the only such source is the atomic explosion, which does not permit utilization of thermonuclear reactions for a slow release of energy.

The brief considerations concerning the possibility of utilizing nuclear reaction energy in rocket motors presented above indicate that this problem is very complex and will require a considerable amount of work before the application of nuclear energy to the rocket motor can be realized.

V. The Processes in the Combustion Chamber of a Rocket Motor

A. Combustion in a Liquid Fueled Rocket Motor

1. Preliminary Processes and Combustion in the Chamber of the Liquid Fueled Rocket Motor

Chemical reactions of combustion take place in the combustion chamber, resulting in the transformation of the chemical energy of the fuel into the heat energy of the combustion products.

There is a great deal of evidence that the fuel components must vaporize before entering into the reaction and that combustion takes place in the gaseous phase. At the same time there are examples of the reaction taking place in the liquid phase as, for instance, in the case of the combustion of self-igniting fuels. It can be supposed, however, that in this case a large part of the fuel enters the reaction only after vaporization.

In either case, the chemical reaction can take place only when the molecules of the combustible and the oxidizer come in contact in the necessary proportions for combustion. Therefore, in order to bring about the combustion of liquid fuels, it is necessary, at first, to create as homogeneous a mixture of combustible and oxidizer vapors as is possible in order that the relationship of oxidizer and combustible, at any point in the chamber, be the closest to that which has been chosen for the motor as a whole. The process of forming such a mixture is called atomization. The head of the motor is the atomizing member.

Jets injecting the combustible and oxidizer in a finely atomized form into the chamber are located in the motor head. The mixing of the fuel components can begin even in the liquid phase by union of droplets and the mutual dissolving of the combustible and oxidizer, but the main part of the mixture is formed after evaporation of droplets and the mixing of the vapors of the components.

The vaporizing and mixing of the components is related to the transfer of particles from one point of the chamber to another, i.e., with diffusion and convection currents in the combustion chamber. Apart from that, there must be a heat flow from the hotter regions of the chamber, in order to vaporize and then to raise the vapor to the temperature at which combustion can take place. The processes of particle transfer and heat transfer take place simultaneously and are closely related.

The laws of mechanical and thermal exchange to which the process of atomization is subjected are extremely complex. Therefore, in the following discussion of atomization, we will restrict ourselves only to the qualitative conclusions.

It is very clear that in designing a motor it is always desirable that the head and the jets located in it insure the most uniform distribution of fuel within the chamber volume. Naturally, it is easier to achieve a homogeneous mixture as the droplets of the atomized fuel become smaller, the distribution of drops in the jet cone becomes more uniform, and a greater number of jets of small capacity are located in the head. The size of the drops and the uniformity of their distribution is determined, primarily, by the type of the jet.

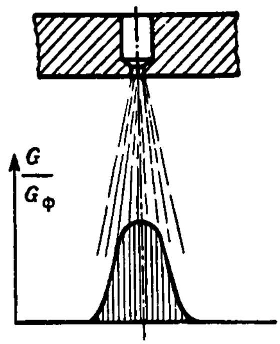


FIG. 5.1. Straight flow jet and cross sectional droplet distribution.

The liquid fueled rocket motors use two types of jets: straight flow and centrifugal.

The straight flow jets (Fig. 5.1) consist of a simple cylindrical opening of a small diameter, insuring the passage of a fine liquid stream. The disintegration of the stream into drops takes place as a result of friction between the liquid and the gas contained in the chamber.

The characteristic of the straight flow jet is a long cone of fuel; the atomization begins comparatively far from the jet. The distribution of drops along the section of the cone is not uniform. The greatest number of drops is along the axis of the cone (see Fig. 5.1). In order to eliminate this defect the straight flow jets are arranged in such a manner that two or more streams, emerging from different jets, intersect at one point in the chamber (Fig. 5.2). Also, it is often the practice to have the streams of the combustible and oxidizer meet at the point of intersection. As a result of the impact of the streams they are more easily atomized and the mixture of the fuel components is improved.

The best atomization is obtained by centrifugal jets (Fig. 5.3). In these

jets, the fuel passing through the duct of the jet receives a rotary motion which is maintained during the exit from the duct. Under the influence of centrifugal forces the emerging stream stretches into a film which quickly disintegrates into droplets.

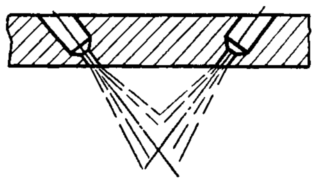


FIG. 5.2. Straight flow jets with intersecting axes.

The cone of centrifugal jets is wide and short, and the droplets are distributed more uniformly, even though the greatest concentration of fuel occurs around the circumference at a certain radius about the axis of the jet.

The rotary motion in the duct of the centrifugal jet is imparted to the liquid by either a tangential entrance into the duct (Fig. 5.3), or the motion

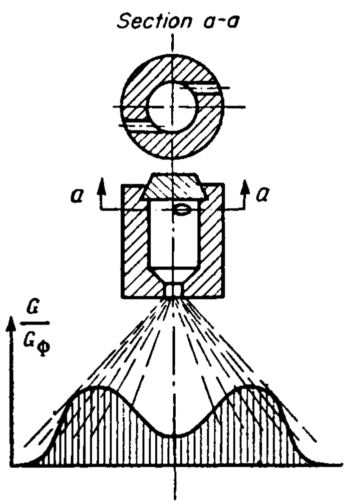


FIG. 5.3. Centrifugal jet and cross sectional fuel distribution.

along a spiral passage formed between a grooved insert (screw) and the wall of the jet.

Occasionally, both the centrifugal and straight flow jets are employed simultaneously in motor heads.

The discharge of components through jets is determined by the cross section of the passage and the pressure drop across the nozzle; the discharge is proportional to the square root of the pressure drop. The larger the pres-

sure drop, the better is the quality of fuel atomization. On the other hand, increase in pressure drop leads to the increase of the necessary delivery pressure which, in turn, leads to an increase in weight of the motor, especially in the displacement system of fuel feed.

As has been mentioned earlier, in order to obtain a homogeneous mixture, a large number of jets, with small discharge per jet, is required.

In reality, existing motors have a large number of small jets. For instance, in the motor of the V-2 rocket the atomization of fuel in the amount of approximately 6.6 lb/sec per antechamber is accomplished by means of 68 jets, (of which 44 are centrifugal and 24 straight flow), so that, on the average, each jet discharges only 1.5 oz/sec.

In order to obtain the most uniform distribution of components along the cross section, the combustible and oxidizer jets are distributed around

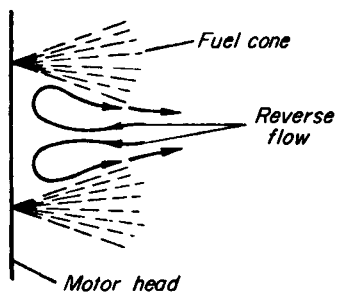


FIG. 5.4. Schematic showing the development of vortex flow at the head.

the head in a specific order. The requirement of uniform distribution of the combustible and oxidizer may be waived around the periphery of the head. Often, only the combustible jets are arranged around the periphery of the head in order to form a vapor boundary which protects the wall of the chamber from burning through.

The heat necessary for the vaporization and preheating of fuel vapor is transferred to the drops in three ways: by turbulent motion of gas at the head, by radiation from the volume of gas at high temperature and the hot walls of the motor and, finally, after the start of the reaction of combustion, directly in the form of heat released during the reaction. Of the greatest significance in heat transfer to the drops of fuel during the vaporization process is the turbulent flow of gas near the head. The same turbulent flow aids in mixing the vaporized fuel.

The turbulent flow near the head is accompanied by a reverse flow in the spaces between fuel cones (Fig. 5.4). These reverse currents carry with them heat which is necessary for vaporization of fuel, and aid in mixing the combustible and the oxidizer. As the preheating progresses a chemical

reaction starts within the mixture of combustible and oxidizer vapors which results in the release of sufficient heat for sustaining further reaction in the entire volume of the chamber.

In accordance with this concept of the combustion process in the chamber of the liquid fuel rocket motor, the chamber can be conventionally divided into several characteristic zones (Fig. 5.5).

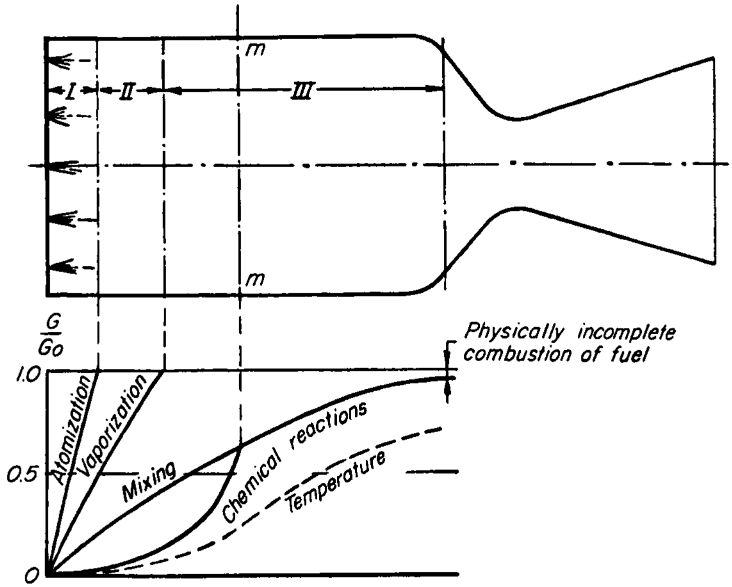


FIG. 5.5. Head of the combustion chamber and division of the chamber into zones: *I*, atomization zone; *II*, zone of vaporization; *III*, zone of mixing and kinetic combustion, *m–m*, transition section from the kinetic zone into the diffusion zone.

The disintegration of fuel streams into droplets takes place in the first zone, directly adjacent to the surface of the head. Therefore, this zone may be called the atomization zone. Other processes—vaporization and mixing—take place in this zone only to a very small degree.

As the fuel moves along the chamber, vaporization becomes more intensive and mixing begins. The chemical reactions also start in this zone but their speed is small due to low temperature. Therefore, the second zone may be conventionally called the zone of vaporization and mixing.

Finally, as the quantity of fuel mixture is increased (in a gaseous state) and its temperature raised, the chemical reactions begin to take place with intensity in the next or third zone. The speed of chemical reaction is still small in the first part of this zone. Therefore, the burning of the fuel is determined by the velocity or, as it is called, the kinetics of the chemical reaction. This region of the chamber is called the region of kinetic combustion.

The rise in temperature leads to a very sharp increase in the velocity of chemical reaction so that, starting at a certain temperature, all of the fuel which is mixed burns almost instantly. Now the speed of combustion will depend almost entirely on the speed with which the components are mixed. Since the rate of mixing is determined by the rate of diffusion, this region is called the region of diffusion combustion.*

The combustion process in the liquid fuel rocket motors takes place primarily in the diffusion region, so that the time necessary for combustion is determined by the rate of mixing. Therefore, the third zone of the combustion chamber is the zone of mixing and chemical reactions.

2. Residual Time of Fuel in the Combustion Chamber

The combustion chamber dimensions should be such that the mixing and chemical reactions shall be completed prior to the entrance into the motor nozzle. This will insure the more complete conversion of chemical energy into heat energy and will reduce incomplete combustion.

The necessary dimensions of the chamber are determined by a conventional quantity τ—time of the fuel in the chamber.

If the fuel consumption, the combustion product temperature at the chamber exit, and the pressure are equal to G, T_c, and p_c, respectively, then the total volume of gas passing through the chamber in unit time is

$$V = G(RT_c/p_c).$$

This volume of gas will remain in the chamber during the period of time

$$\tau = V_c/V = V_c p_c/GRT_c \tag{5.1}$$

where V_c = volume of combustion chamber.

The quantity τ is called the residual time. It only indirectly indicates the actual time that the fuel and the products of its combustion remain in the chamber. The volume of an amount of the fuel, as it burns in the chamber, increases from an insignificantly small volume of liquid fuel to the value V_c and the residual time of the fuel in the chamber is based on this greater volume. Therefore, the actual time that the fuel is in the chamber is greater than the magnitude of τ but has a definite relation to it.

The residual time τ necessary for sufficiently complete combustion of fuel is determined experimentally and on the basis of studying the designs of developed motors. In existing motors it constitutes from 0.003 to 0.008 sec. At increased pressures the residual time increases. Consequently, with

* Here we mean the so-called turbulent diffusion which consists of random motion of small volumes of gas, and not the motion of separate molecules.

the same fuel consumption at high pressure the required dimensions of the chamber decrease.

For a given design of the head the residual time in the chamber is the basic factor determining the degree of the completeness of fuel combustion, i.e., that amount of the fuel which is able to enter into the chemical reaction in the chamber.

3. The Design of Motor Combustion Chambers and Injection Heads

As can be seen from expression (5.1) the residual time τ is independent of the combustion chamber configuration, so that for a given volume the chamber can be of any shape. However, the choice of the combustion chamber configuration cannot be made at will. In a long chamber with a small cross section, the head cannot accomodate the necessary number of jets. With a short chamber, the atomization zone occupies a considerable volume of the chamber, and the length of the mixing and combustion zones becomes very small. The usual relation of the area of the transverse section of the chamber to the area of the critical section is within the limits 3 to 10.

It must be noted that the necessary combustion chamber volume depends on the performance of the injector head. The more uniform the fuel mixture supplied by the injector head, the smaller may be the combustion chamber volume. Aside from that, in order to decrease the necessary chamber volume, the head must distribute the fuel mixture along the transverse section of the chamber in such a way as to enable the fuel to fill all parts of the chamber, so that the process of combustion along the section of the chamber may be uniform. In other words, the shape of the combustion chamber and the shape of the head must be closely interrelated.

The two geometrical shapes most frequently used in contemporary combustion chambers are cylindrical and spherical (or approaching spherical).

The advantage of the spherical chamber is that this chamber has the least surface as compared to chambers of any other shape having equal volume. The small surface of the chamber determines its small weight and the small amount of heat which will be transferred to the cooling system.

The spherical chamber is desirable from the standpoint of structural strength as well. For equal strength, the walls of the spherical chamber are half the thickness of the walls of a cylindrical chamber. Therefore, when the thickness of the chamber walls is not determined by technological or functional considerations, but rather by strength (and this is always the case with large motors having high pressure in the chamber), the chamber with the spherical form should have preference. The disadvantage of the spherical chamber is the complexity of its manufacture and the difficulties connected with obtaining good performance from the injector head and cham-

ber at the same time. The cylindrical chamber has a considerable advantage in this respect.

In fact, in examining Fig. 5.6, which shows the chamber of a liquid propelled booster, we see that the flat head installed in this motor permits

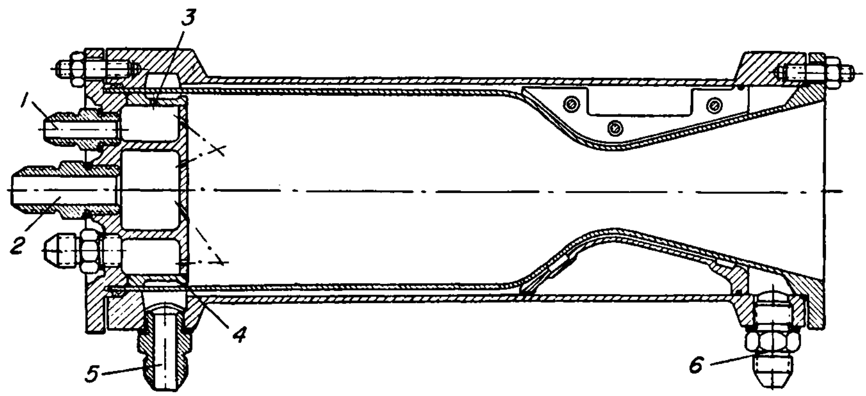


FIG. 5.6. Cylindrical chamber of a liquid fuel jet assist rocket: *1*, combustible supply; *2*, oxidizer supply; *3*, ports for internal cooling by the combustible; *4*, injection clearance for internal cooling by the combustible; *5*, oxidizer supply for cooling; and *6*, oxidizer outlet from the cooling jacket.

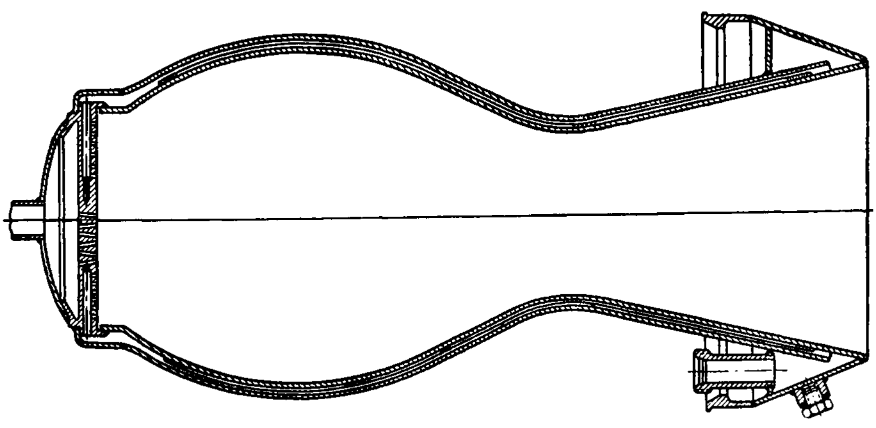


FIG. 5.7. Spherical chamber with a flat head.

uniform distribution of fuel along the transverse section of the chamber, leaving no area lacking in fuel, where combustion would not take place, that is, no so-called "dead spots."

A similar flat head installed in a pear shaped motor (Fig. 5.7) does not allow the full utilization of the volume of the chamber for combustion. The part of the chamber which is outside the cylindrical volume defined

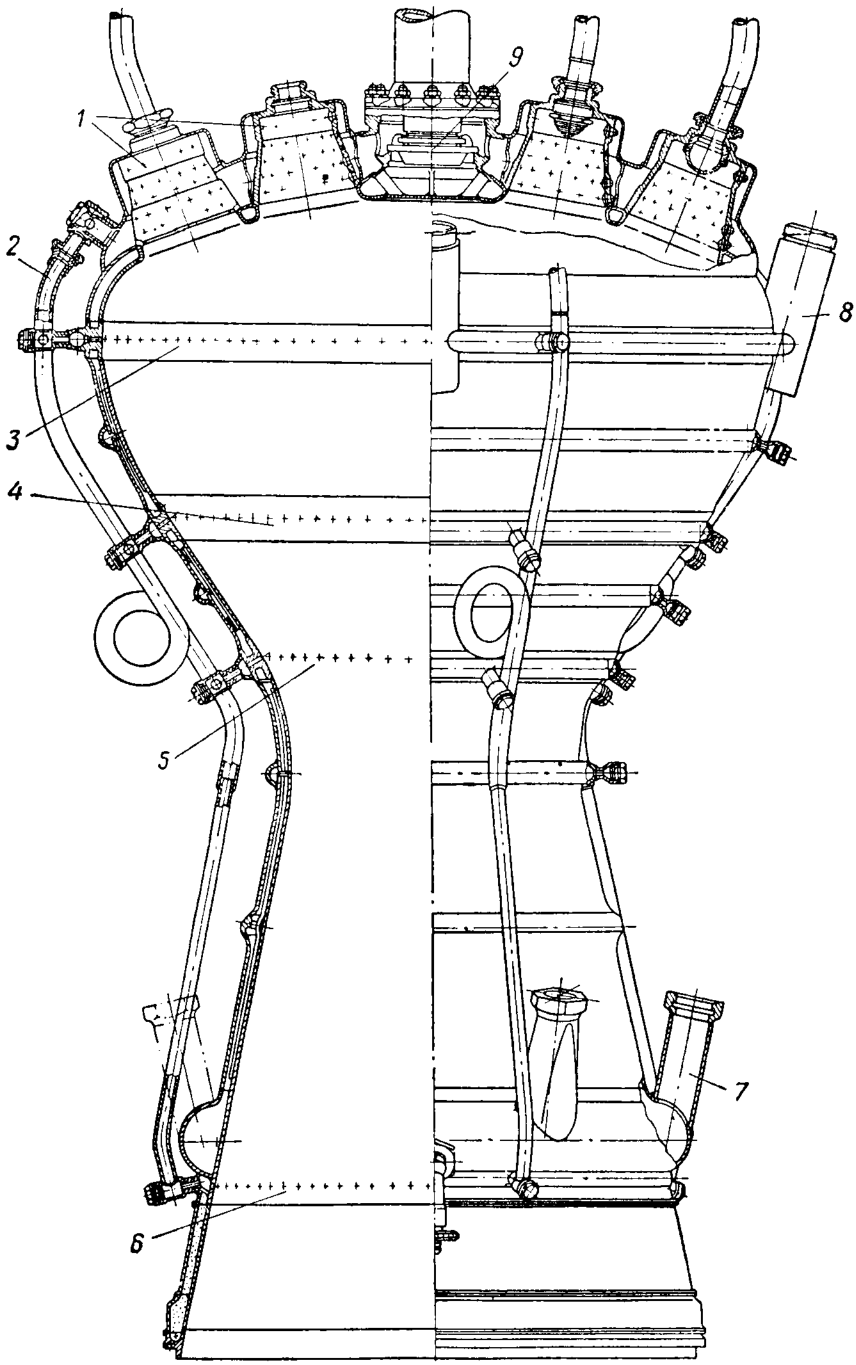


FIG. 5.8. Motor chamber of the V-2 long range rocket: *1*, antechambers; *2*, internal cooling tubing; *3*, *4*, *5*, and *6* circumferential ports for injecting combustible against the inner surface of the chamber walls; *7*, combustible lines to the cooling jacket; *8*, motor bulkhead attachment fittings; and *9*, main alcohol valve.

by the diameter of the head is practically useless for combustion of fuel. At the same time, the dimensions of the flat head make it impossible to locate a large number of centrifugal jets on it. In this motor we had to restrict outselves to the use of less practical through-flow jets.

The combustion chamber volume of the V-2 motor (Fig. 5.8), whose head contains 18 antechambers, is also used inefficiently. The fuel mixture prepared in the antechamber flows forcefully into the chamber in the shape of a cone, and even though these cones collide with each other and mix

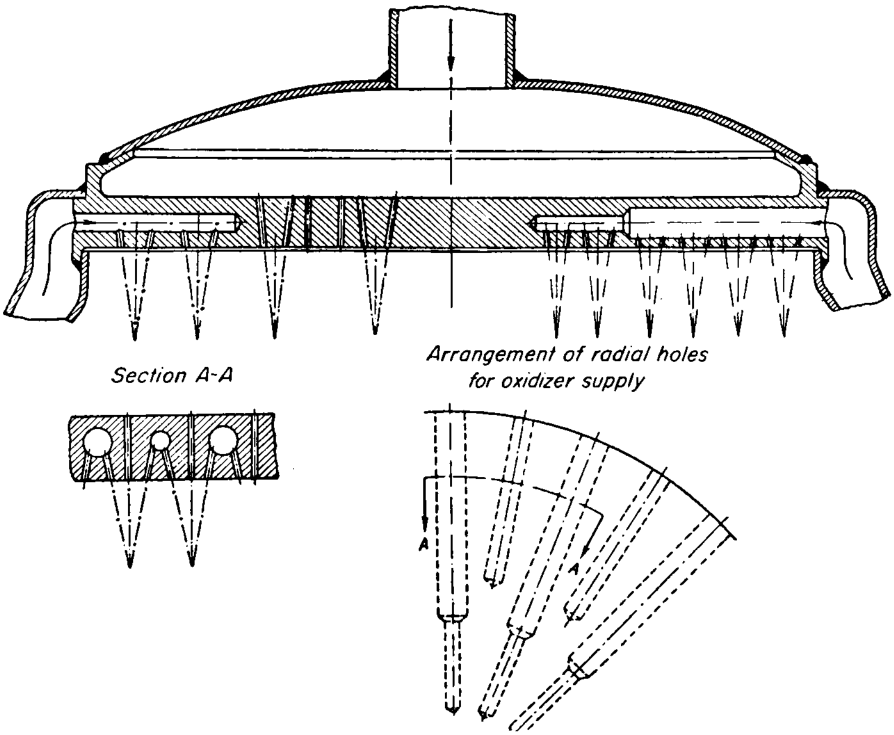


FIG. 5.9. Flat chamber head.

thoroughly, a considerable portion of the chamber volume, included between the cones emerging from the antechamber, is not used for the process of combustion.

An idea of the construction of a flat head with straight flow jets can be obtained from Fig. 5.9.

This head has 432 jets of 0.06 dia. for the combustible and 324 jets of 0.10 dia. for the oxidizer. The oxidizer and combustible ports are drilled at an angle to each other to insure pulverization of streams during collision and beginning of atomization in the liquid phase. The combustible is

delivered from the upper cavity of the head. Long radial holes must be drilled for the oxygen supply, and great care must be taken that they do not intersect the holes for the combustible. The fabrication of such heads becomes extremely complicated.

Centrifugal jets can also be located in the flat head. We can suppose that they will give a more uniform mixture along the transverse section of the chamber. Besides, centrifugal jets insure the intersection of atomizing cones of fuel and oxidizer, with a normal distribution of jets, i.e., when their axes are parallel to the chamber axis. In this manner the installation of centrifugal jets does not require the drilling of inclined holes. However,

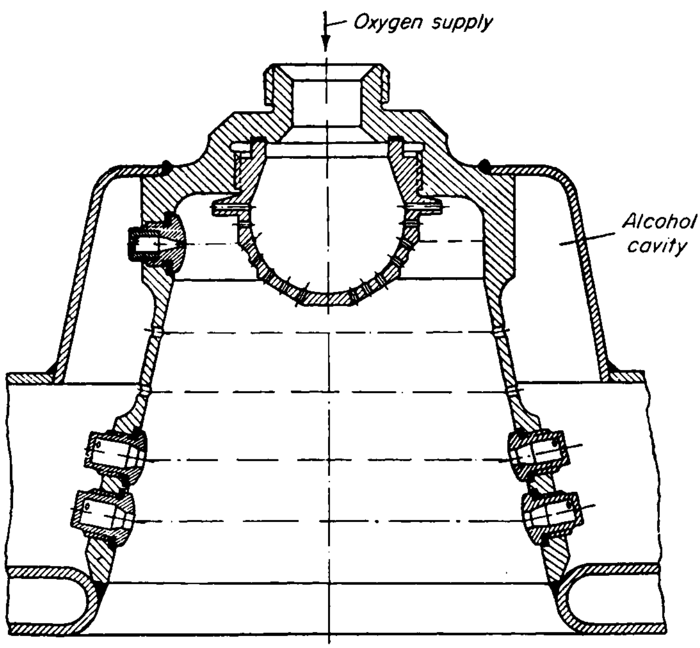


FIG. 5.10. Antechamber of a V-2 long range rocket.

centrifugal jets occupy more space and the head must be larger. This can easily be accomplished in the cylindrical chamber without increasing the volume of the chamber by increasing its diameter and decreasing its length.

Because of assembly considerations, it is very difficult to arrange jets directly at the base of a spherical combustion chamber. Furthermore, due to the small surface of the base, it is difficult to accommodate the necessary number of small jets. Therefore, motors with spherical combustion chambers have antechambers (Fig. 5.10). In each antechamber the oxidizer is atomized by one jet having a large number of ports. The combustible is

supplied by jets distributed along the conical side surface of the antechamber. It is clear that this scheme of mixing cannot insure (at least by ordinary means) a uniform distribution of components along the transverse section of the antechamber. In order to improve the formation of mixture, a complicated system of coordinated jets is employed in the described antechamber. The oxygen is injected through openings arranged on concentric circles and inclined to the axis of the antechamber at different angles in order for the streams of oxidizer to fill the volume of the antechamber more uniformly.

Rows of combustible jets are radially arranged to correspond to the rows of oxidizer openings. The upper row contains centrifugal jets. The short reach of these jet cones protect the walls of the antechamber from direct impingement of the oxidizer upon them.

The next two rows consist of straight flow jets which, due to their greater penetration, carry the combustible to the center of the antechamber volume. The lower rows again have centrifugal jets. All of the enumerated measures do permit better mixture formation but, on the whole, it is not as good as in a motor with a flat head. The latter results in the increase of the relative volume and weight of the combustion chamber.

4. Fuel Ignition in Liquid Fuel Rockets

The progress of the established process of combustion in the motor has been described above. The initiation of burning of the liquid fuel in the combustion chamber—ignition—poses, in certain cases, certain requirements with respect to the motor.

As we already know, fuels may be self-igniting or nonself-igniting. The conditions of their ignition in the combustion chamber are different. The nonself-igniting fuels are ignited by injecting them into the igniting pilot flame which fills the combustion chamber. The igniting pilot flame is made sufficiently powerful to ignite the basic components in the quantities required, as they are supplied at the start. The ignition of nonself-igniting fuels does not present special requirements in the design of the head. The self-igniting fuels begin to react and release heat upon contact even in the liquid form. Therefore, for their reliable ignition it is necessary to insure good contact of components in the liquid phase.

According to some data, it is more desirable for self-igniting fuels to use jets with intersecting axes.

Self-igniting fuels have a certain delay period of self-ignition. During the start of the motor, liquid unburned fuel accumulates in the chamber. The following combustion of concentrated fuel leads to a sharp increase in pressure which may be detrimental to the motor.

In order to decrease the concentration of fuel in the chamber during the

initial stages of starting it is necessary to reduce the fuel consumption artificially.

It is for this reason that, in motors using self-igniting components, or in motors with pyrotechnic ignition, means are employed to insure a slow increase in fuel supply during start. Either the throttling valves are slowly opened during start (see description of anti-aircraft rocket motors) or a rotary sleeve valve at the head of the motor gradually increases the amount of fuel reaching the jets.

5. Pulsating Burning (Chuging)

During static tests and use of liquid fuel rocket motors, it has been noted that a pulsating combustion can start in the combustion chamber. It consists of a change in gas pressure in the chamber with the frequency up to 200 ∼ (Fig. 5.11). The amplitude of pressure oscillations can reach

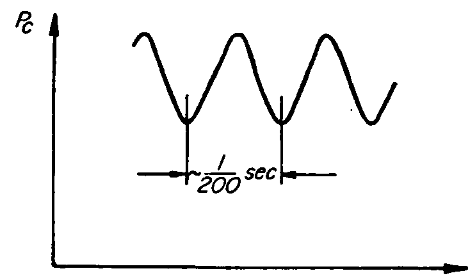


Fig. 5.11. Variation in chamber pressure during pulsating burning.

considerable magnitudes, sufficiently large to create danger to the motor.

The physical picture of the initiation of pulsating combustion is not completely clear. However, it may be explained qualitatively to some degree.

Let us suppose, for instance, that the pressure in the delivery system in the tanks of a motor with displacement delivery remains constant. Further, let us suppose that for some accidental reason the pressure in the combustion chamber dropped with respect to the nominal. Then the fuel delivery by jets will increase, since the pressure drop across the jets increases. The increased fuel delivery by jets will continue until the first quantity of fuel with increased concentration, in the time of the order τ, is converted into gas and begins to flow out of the motor. From that moment, the pressure in the chamber of the liquid fuel rocket motor increases in proportion to the increasing discharge of the combustion products through the throat (see page 179 below) and will be greater than the nominal. Due to this, the pressure drop across the jets and the fuel consumption will be reduced. After

an interval of time of the order of τ, the exhaust of the gaseous products will also be reduced, which will cause a pressure reduction in the chamber and, consequently, reestablishment of conditions for the repetition of the preceding cycle of oscillation.

The magnitude of consumption variation at constant initial pressure disturbance in the chamber depends on the nominal pressure drop across the jets. The greater this pressure drop, the lower the relative change in the pressure drop across the jets and the smaller the change in consumption will be. In this way, the increase in pressure drop across the jets counteracts the initiation of pressure fluctuations and pulsating burning.

In the same manner, the volume of the combustion chamber affects the initiation of pulsating. The greater the volume of the chamber, the more excess fuel is used to vary the supply of gas in the chamber. By this means the pressure fluctuations are damped out in a chamber of large dimensions.

Pressure fluctuations in the chamber can also create fluctuations of the fuel column in the delivery piping, which under certain frequency conditions can lead to an increase in the amplitude of the combustion pulsations. If the pulsating combustion gives rise to pressure fluctuations of such magnitude that the fuel supply to the chamber momentarily ceases, then in a subsequent renewal of fuel delivery an explosion of the motor is quite possible.

Aside from the increased pressure drop across the jets, other means of preventing the initiation of pulsating burning are increased speed of combustion, which requires decrease in time necessary to convert fuel into gaseous products, and the selection of geometrical dimensions for the chamber and delivery system, which will prevent development of fluctuations.

B. Combustion of Rocket Powders

1. Rate of Powder Combustion

Rocket powder is a homogeneous mass, impervious to gas penetration, whose smallest volume contains the necessary mixture of combustible and oxidizing elements. The powder burns at the surface and the flame front penetrates into the mass only as the previous layers burn away.

It has been determined, both theoretically and experimentally, that the burning of the powder is preceded by the thermal decomposition of substances on its solid surfaces. The intensity of the decomposition reaction is determined basically by the velocity of heat transfer from the zone of continuing combustion of gaseous products of decomposition, which were formed on the surface of the burning powder. The heat is supplied to the surface by means of conduction and radiation, since the flow of gaseous

products is always directed away from the burning surface, making heat transfer to the grain by convection impossible.

The basic characteristic of powder burning is the magnitude of the burning rate, u_p, i.e., the thickness of the powder layer which is consumed in a unit of time. Usually this quantity is expressed in in./sec.

Since a powder is a homogeneous mass it is reasonable to expect uniform burning of the powder along its entire surface. Experiments with sudden interruption of powder grain burning have confirmed this supposition.

The powder weight burned in a unit of time, and consequently the amount of formed combustion products is

$$G_p = S_p u_p \gamma_p \tag{5.2}$$

where S_p is the burning surface of the powder grain; u_p is the powder burning rate; and γ_p is the specific weight of the powder.

Since the specific weight of powder is a constant quantity ($\gamma_p \approx 100$–106 lb/ft^3), the quantity of formed gas depends on the size of the burning surface and burning rate.

The burning rate of the powder is, first of all, determined by the pressure at which combustion takes place. Increase in pressure facilitates heat transfer to the powder grain and accelerates reactions taking place on its surface.

At pressures up to 2,800 lb/in.^{2}, characteristic of the combustion chambers of powder motors and powder pressure accumulators, the dependence of powder burning rate on pressure p can be approximated by the empirical formulas

$$u_p = \alpha + \beta p \tag{5.3}$$

and

$$u_p = b p^n \tag{5.4}$$

where α, β, b, and n are experimental constants.

The magnitude of the exponent n in formula (5.4) varies within the limits of 0.6–0.8. Coefficients α, β, and b depend not only on the composition of the powder but, to a considerable extent, on the initial temperature of the powder (before start of burning).

The influence of the initial temperature of the powder on the burning rate is quite natural; the increase in this temperature facilitates the reaction on the surface of the grain and increases the rate of burning. At the same time, because of relatively high burning rate and low heat conductivity, there is no heating of the entire powder grain during the process of burning, so that the powder temperature remains constant (there is preheating of a very thin layer which for all practical purposes has already entered the reaction).

The dependence of the burning rate of the powder charge on its initial temperature is quite perceptible and results in a considerable change in the gravimetric consumption of combustion products in the temperature range from −60° to +120°F, which is representative of the different seasons of the year and climatic conditions.

The typical dependence of the burning rate of powders on pressure and temperature is shown in Fig. 5.12 in logarithmic coordinates.

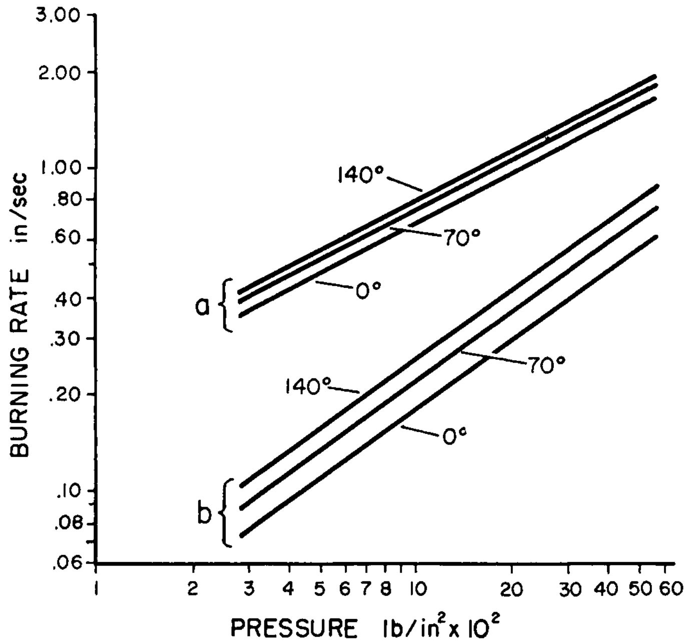


FIG. 5.12. Dependence of powder burning velocity on pressure and temperature: *a*, rapidly burning powder, and *b*, slowly burning powder.

The first group of curves is for a powder with a high burning rate (1 in./sec at the pressure of 1130 lb/in.^{2} and initial grain temperature of t = 70°F). However, the burning rate of this powder is not appreciably affected by pressure (n = 0.52). The initial temperature also has little effect.

The lower group of curves is for a slow burning powder (the burning rate is 0.25 in./sec at the same pressures and temperatures). However, the dependence of the burning rate on temperature and especially on pressure is much greater for this powder (n = 0.71).

2. THE SHAPE OF ROCKET POWDER GRAINS

The amount of gas formed in a unit of time at constant pressure is determined, as follows from expression (5.2), by the size of the burning surface

of the grain. During the process of powder grain burning the size of the surface, S, generally speaking, does not remain constant, and may either increase or decrease. If during the burning process S decreases, then the amount of gas formed in a unit time also decreases. This is called regressive burning. If the burning surface increases with time, the amount of gas formed in a unit time also increases. In this case the burning of the grain is called progressive.

By giving different shapes to the powder grain we can, within certain limits, regulate the rate of gas generation.

In designing powder rockets the object is usually to insure constant thrust on the motor during the trajectory. Obviously, it is necessary to obtain a constant amount of gas in a unit time, i.e., have a constant burn-

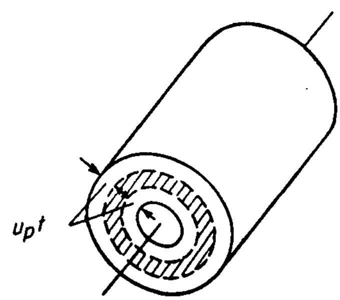


FIG. 5.13. Burning of a tubular grain. Dashed lines indicate the burning surface after t sec. The cross hatched section is the end burning area.

ing surface of powder grains. In order to fulfill this condition, the grains are made in special shapes. An example of this is a hollow, cylindrical, so-called tubular grain, shown in Fig. 5.13. In this grain, the burning of the external cylindrical surface leads to a reduction, and the burning of the internal surface to a corresponding increase of the burning surface. In this way, the change in the size of the burning surface is due only to the reduction in the burning areas of the ends. If the grain has appreciable length, then the influence of the end areas on the total burning surface is insignificant, and we can consider that the burning surface remains practically constant. It may be said that the burning is slightly regressive.

Let us note that in certain cases, for instance, to obtain high rocket velocity during launching from "zero length" rails, it is necessary to obtain rapid burning of the powder (in 0.1–0.3 sec). In such cases the grain surface is increased and the thickness (web) is decreased. The charge may be made multigrained (Fig. 5.14,a). Other, more complicated, grain shapes are employed in order to insure the required change in the burning surface (Fig. 5.14,b and c).

In special cases it is necessary to have a charge with a small burning surface which burns, however, for a considerable length of time (tens of seconds). This is necessary, for instance, in powder pressure accumulators. The so-called inhibited grains are used for reducing the burning rate.

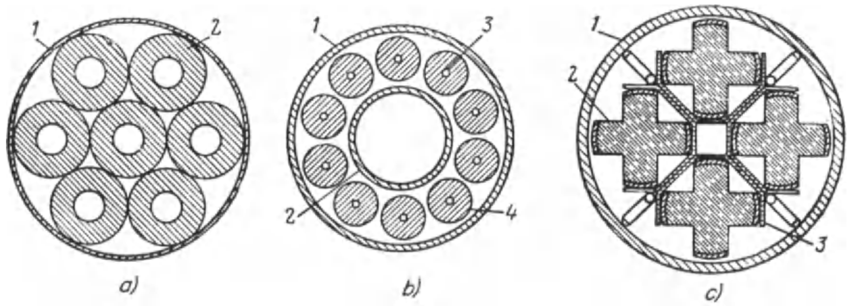


FIG. 5.14. Grain shapes of rocket charges: *a*, hepta-grain tubular charge—*1*, chamber walls; *2*, powder grain; and *b*, tubular grains retained by rods—*1*, rocket chamber wall; *2*, central tube for explosive charge; *3*, grain retaining rod; *4*, powder grain; and *c*, charge made up of cruciform grains—*1*, rocket chamber wall; *2*, cruciform powder grain; *3*, steel partitions.

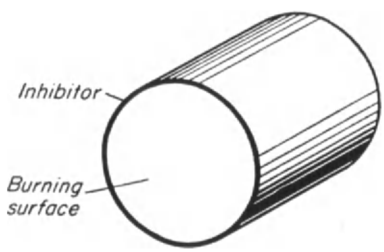


FIG. 5.15. Inhibited grain.

These grains have a portion of the powder surface covered by a plastic (for instance, cellulose acetate) which does not burn and which prevents ignition of the grain surface it covers. Fig. 5.15 shows a grain inhibited on all sides with the exception of one end.

3. BURNING OF THE POWDER CHARGE IN THE CHAMBER OF A POWDER MOTOR

The above-discussed relations are characteristic for a powder charge burning in the absence of gas motion along the surface of the grain. When the grain burns in the chamber of a powder motor, however, the generated gases, in moving toward the nozzle, flow across the surface of the grain. Experience shows that the burning rate of the powder depends on the veloc-

ity of gases across the grain; the higher the gas velocity the higher the burning rate is. This is explained by increased heat transfer from the hot gases to the powder.

In powder motors, the combustion chamber volume occupied by the fuel must be as large as possible. The space between burning grains is not large and the velocity of gas flowing across grain surfaces becomes quite appreciable. The gas velocity increases with approach to the exit. For this reason, the powder grains burn faster at the nozzle end.

Low pressuies in the combustion chamber of the motor can give rise to fluctuating burning (chuffing). Chuffing consists of a periodic extinguishing and reignition of the powder charge. The pressure at which chuffing occurs depends on the powder composition and on the temperature of the charge. The lowering of the initial charge temperature facilitates chuffing. For rocket powders chuffing starts at pressures of 300–550 lb/in.^{2}.

4. The Ignition of the Rocket Powder Charge

The ignition of a rocket powder charge is performed by the igniter. The construction of the igniter is shown schematically in Fig. 5.1.6 An electric impulse traveling along the wires of the electrical firing device, *4*, heats the

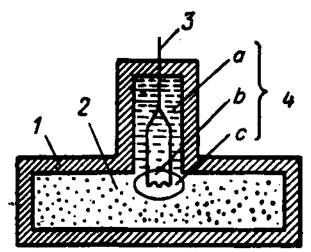


Fig. 5.16. Construction of an igniter: *1*, igniter body; *2*, igniter charge; *3*, electric wires supplying current to the squib; *4*, squib—*a*, potting compound; *b*, hot wire; *c*, easily ignited mixture.

glow wire, *b*, which ignites the sensitive primary explosive (squib), *c*, which in turn ignites the booster charge, *2*. The electrical firing device is usually imbedded in a potting compound for hermetic sealing.

The igniter chamber, *1*, can be fabricated from plastic or metal.

The hot gases formed in the igniter flow over the surface of the main powder charge and ignite it. In the process of ignition there is, first of all, a rise in temperature along the surface of the charge to ignition temperature and, secondly, an increase in chamber pressure to a value assuring the normal burning of the charge.

Ignition time delay of the powder charge must be as short as possible.

This requires an intensive heat transfer of the products of combustion from the igniter to the powder charge.

Under ignition conditions, when the velocity of gases along the chamber is small, radiation plays an important part in heat transfer. However, the ability of gases to radiate is low. In order to increase radiation, the igniter charge is compounded in such a way that its products of combustion contain considerable quantities of solid particles, rapidly radiating heat. Therefore, the igniter charge is made either from black powder, which yields a certain amount of solid particles, or from a mixture of magnesium (or aluminum) powder and potassium perchlorate ($KClO_4$).

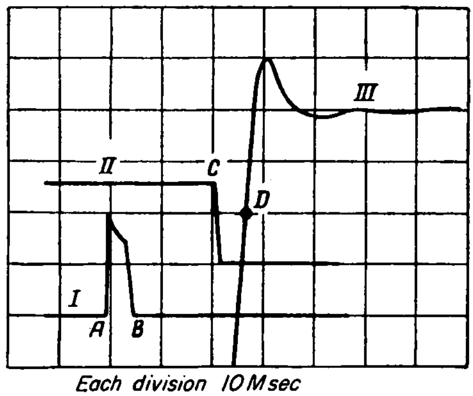


FIG. 5.17. Ignition with respect to time: *I*, curve of the current intensity in the hot wire circuit—*A*, instant of current supply to the hot wire; *B*, burning of the hot wire; and *II*, curve of internal stresses in the housing—*C*, start of housing rupture; and *III*, curve of pressure increase in the combustion chamber—*D*, pressure sufficient for initiation of grain burning.

Igniters composed of $KClO_4$ and Mg (or Al) insure faster ignition (5–10 milliseconds. However, they are more dangerous to handle. Besides, the metallic powder which is contained in their composition may oxidize during long storage, resulting in misfire. The ignition time of a rocket charge by black powder is longer (25–30 milliseconds), but such an igniter is more reliable during storage. The time history of the ignition process is shown in Fig. 5.17.

The igniter is located in the combustion chamber of the powder motor near the ends of the charge. A more reliable ignition of the powder charge is assured by an igniter located at the head of the chamber. In this case, the igniting gases flow along the entire length of the charge and heat it before emerging through the nozzle.

C. Products of Combustion and Their Properties

1. Parameters of State of a Gaseous Mixture

The process of combustion, taking place in the combustion chamber of a rocket motor, consists of a combination of complex chemical reactions, and of preliminary processes necessary for their accomplishment.

The main result of the combustion process is the conversion of liquid or solid fuel into gaseous products of combustion heated to a high temperature. Since the fuel is always composed of several elements, the products of combustion consist of a mixture of various, for the most part gaseous, chemical compounds. Let us examine the basic properties of the gaseous products of combustion and introduce quantities which determine their state.

The state of a gas is characterized by the following parameters: absolute pressure, p; absolute temperature, T; density, ρ (or specific weight, γ); and the gas constant, R.

As is known, parameters p, ρ, and T are related by the equation of state for ideal gases (Clapeyron Equation).

$$p/\rho = gRT. \tag{5.5}$$

The density of the gas is related to the specific volume by the relationship

$$g\rho = 1/v. \tag{5.6}$$

In this connection the equation of state may be written

$$pv = RT. \tag{5.7}$$

The value of the gas constant for a mixture of gases is determined by the constituents of the mixture. In calculating R, we may make use of the relationship

$$R = \bar{R}/\mu_\Sigma \tag{5.8}$$

where $\bar{R}$ is a universal gas constant referred to 1 kg mol of any gas or gas mixture ($\bar{R}$ = 848 kg m/kg mol deg or, in heat units, $A\bar{R}$ = 1,986 kcal/kg mol deg), and μ_Σ is the apparent molecular weight of the mixture.

The apparent molecular weight of the mixture is

$$\mu_\Sigma = \sum \mu_i r_i \tag{5.9}$$

where μ_i is the molecular weight of a gas making up the mixture and denoted by subscript i, and r_i is the volumetric portion of the gas with the molecular weight μ_i.

The volumetric portion of gases in the mixture can be most simply expressed by partial pressures, p_i.

As is known, the term partial pressure means that pressure of the gas which it would have had if it alone occupied the entire volume which contains the gas mixture. The total pressure of the gas mixture, p_Σ, is equal to the sum of the partial pressures.

$$p_\Sigma = \sum p_i.$$

The volumetric portion of gas which interests us is

$$r_i = p_i/p_\Sigma. \tag{5.10}$$

Taking this relationship into account, we will get, instead of Eq. (5.9)

$$\mu_\Sigma = \frac{1}{p_\Sigma}\sum \mu_i p_i. \tag{5.11}$$

For a gas mixture of constant composition the values of μ_Σ and R do not change. Conversely, if the composition of the mixture changes, the apparent molecular weight and the gas constant for the mixture will also change.

2. Internal Energy and Specific Heat of a Gas

One of the energy characteristics of a gas is its internal energy. The internal energy of a gas is the energy of motion of gas molecules and is usually thought of as the amount of heat contained in the gas. The energy of the motion of the molecule is determined by the temperature of the gas and the structure of the molecule itself.

The monatomic molecule has the simplest structure.

In studying the properties of gas, and in determining its energy level, interatomic motion (for instance, the motion of the electrons) is not taken into consideration and the atom is considered as a physical point. Therefore, the motion of a monatomic molecule (or, more precisely, its position in space) is determined by three coordinates. As a reminder, the number of independent coordinates determining the position of a system in space is called the number of degrees of freedom of this system. Therefore, the monatomic molecule has three degrees of freedom. These three degrees of freedom correspond to the displacement of a molecule along three mutually perpendicular paths, i.e., its translational motion.

A more complex diatomic molecule (Fig. 5.18) has a greater number of degrees of freedom. Actually, the position of a molecule composed of two atoms having a bond between them is determined by six coordinates; specifically, by three coordinates determining the position of the center of gravity of the molecule in space, by two angles determining the orientation of the molecular axis in space and, finally, by a coordinate defining

the deviation of the atoms from their certain mean position within the molecule.

Each degree of freedom of a molecule is customarily related to one form of motion or another. The three degrees of freedom corresponding to the coordinates of the center of gravity are related to the translational motion of the molecule. The two degrees of freedom which determine the position of the molecular axis in space are related to the rotation of the molecule about two axes which do not coincide with the axis of the molecule. At the same time, to speak of rotation of the molecule about its own axis is senseless, inasmuch as we consider atoms as physical points with infinitely small dimensions. The coordinate determining the change in distance between atoms corresponds to the vibratory motion of the atoms within the molecule.

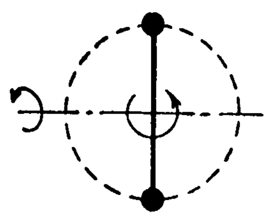


FIG. 5.18. Rotary motion of diatomic molecules.

A triatomic molecule has an even greater number of degrees of freedom, to be exact, nine.

The internal energy of the gas, U, is the sum of the energies of molecular motion along each one of the molecule's degrees of freedom. The magnitude of the internal energy depends, therefore, upon the number of degrees of freedom and the intensity of the molecular motion "along a given degree of freedom."

It has been determined that the quantity which determines the motion of the molecules along their inherent degrees of freedom is temperature, exclusively. Therefore, the energy of motion in a given degree of freedom, U_i, can be expressed in the form of some function

$$U_i = \varphi(T). \tag{5.12}$$

The derivative,

$$\partial U_i/\partial T = c_i \tag{5.13}$$

is the rate of energy increase in a given degree of freedom, with increase in temperature. If this is referred to one degree of temperature and a unit mass, for instance, gram mol, then the quantity c_i will represent the heat capacity of a given degree of freedom with the dimensions kcal/gr mol deg. At the same time it is characteristic of any degree of freedom that the heat capacity, c_i, increases with temperature, but only up to a definite limit.

At certain temperatures the heat capacity for a given degree of freedom reaches a maximum value $c_{i\max}$, after which it does not change. Such temperature may be called the saturation temperature, Ts. The characteristic dependence of heat capacity on temperature for a given degree of freedom is shown in Fig. 5.19.

The value of the maximum heat capacity $c_{i\max}$, referred to 1 gr mol of gas, depends only on the type of the degree of freedom to which it is related, and is completely independent of the individual properties of the gas. It has been shown theoretically, and confirmed experimentally, that the value

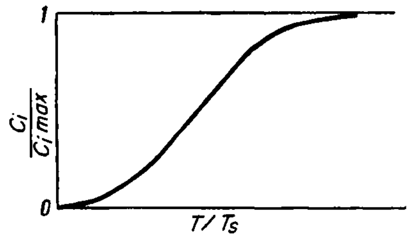


FIG. 5.19. Dependence of heat capacity ratio $c_i/c_{i\max}$ for the ith degree of freedom on temperature ratio T/T_s.

of the maximum heat capacity for translational and rotational degrees of freedom, referred to 1 gr mol of gas,* is $R/2$, and for vibratory degrees of freedom it is R, where R is the gas constant.

The magnitude of the temperature at which a given degree of freedom is saturated depends upon the type of motion and upon the properties of a given molecule. The saturation of translational degrees of freedom takes place at very low temperatures for all molecules, without exception. Therefore, the heat capacity of the three translational degrees of freedom, i.e., the heat capacity of the translational motion of the molecule, is always constant and is equal to $3/2R$.

At relatively higher temperatures (but practically still very low), about 10°–30°K, the rotational degrees of freedom are saturated, and their heat capacity from then on is $R/2$ for each degree of freedom. As for the vibratory degrees of freedom, the saturation of the majority of the diatomic and triatomic gases, comprising the products of combustion of the rocket motors, takes place only at very high temperatures, exceeding the temperature of combustion. Therefore, the heat capacity of the vibratory degrees of freedom is variable and changes in a way which depends only on the temperature of the gas, and increases with the increase in temperature.

The total heat capacity of any gas is the sum of the heat capacities of all degrees of freedom which a molecule of a given gas possesses.

* The quantity R here is expressed in the same units as the heat capacity.

As can now be easily understood, the heat capacity of the monatomic gases remains constant through a wide range of temperatures and is equal to $3/2R$; their internal energy is directly proportional to the temperature and is equal to $3/2RT$. Diatomic gases at low (for liquid fuel rocket application) temperatures have the heat capacity of three translational, and two rotational degrees of freedom, i.e., $5/2R$; and at very high temperatures their heat capacity increases and tends to reach a limiting value of $7/2R$. Qualitatively, the same dependence of the heat capacity of a gas on temperature is true for the triatomic gases.

Above, we discussed the so-called molar heat capacity of gases, c, expressed in cal/gr mol deg. An important quantity, of which we were already convinced in evaluating the fuels of rocket motors, is the gravimetric heat capacity

$$c_G = c/\mu \quad \text{kcal/kg deg.}$$

As has been shown above, the heat in each degree of freedom and the energy of the given degree of freedom depend only on temperature. For this reason the internal energy also depends upon, and is completely determined by, temperature alone, and is therefore a function of the gas state. At first glance, it may seem that this statement is contradicted by the fact that the sum of heat capacities of degrees of freedom of a gas is usually written as $\sum c_i$, and is called the specific heat at constant volume c_v; and the internal energy or, more precisely, the change in internal energy, is ordinarily written in the form

$$U = \int_0^T c_v dT. \tag{5.14}$$

Therefore, at first glance, it may seem that the internal energy of a gas depends upon the conditions (for instance, a constant volume or constant pressure) under which the heat was supplied to the gas. Actually, the term specific heat at constant volume means that, in heating the gas under the condition of constant volume, the heat was used only to increase the internal energy of the gas and was not expended for any other purpose.

3. Heat Content of a Gas. Adiabatic Exponent

The second function which characterizes the energy state of a gas is the so-called heat content or enthalpy, H. The heat content differs from the internal energy in that the product pv or, in heat units, Apv, is added to the value of the internal energy. The product is a measure of the potential energy contained in 1 kg of gas which occupies volume v at pressure p. Therefore, the heat content is a measure of the sum of the internal energy and the potential energy due to gas pressure. For instance, the total energy

of a compressed spring is composed of the internal energy of the spring material heated to a given temperature and of the energy which was expended for its compression. This total energy of the compressed spring is analogous to the heat content of the compressed gas. It is obvious that the name of the function under discussion does not correspond to its physical significance, since the quantity of heat contained in the gas is determined by its internal energy.

The heat content is the most important energy characteristic of a gas. This is explained by the fact that, in virtually all cases, in changing the state of a gas during various technical processes, potential as well as internal energies change. In this way, the total change in gas energy which takes place during various gas processes is always determined by the magnitude of change in heat content ΔH.

According to definition, the heat content is

$$H = U + Apv \tag{5.15}$$

or, substituting from (5.7) and (5.14)

$$H = \int_0^T c_v dT + ART. \tag{5.16}$$

It is easy to see that the change in the heat content of gas ΔH corresponds to the expenditure of heat to heat the gas at constant pressure. During an increase in temperature from T_1 to T_2, and expansion of a gas from the value of the specific volume v_1 to v_2, the change in heat content is

$$\Delta H = \Delta U + p(v_2 - v_1)A.$$

Here $p(v_2 - v_1)$ represents the work of gas expansion at pressure p. The derivative of heat content with respect to temperature is

$$dH/dT = c_v + AR = c_p \tag{5.17}$$

and is called the specific heat at constant pressure.

The values of the specific heat and of the internal energy are functions of gas state parameters. The change in heat content during any process does not depend on the type of process, but is determined only by the initial and final gas states.

The heat content (or specific heat) of liquid and solid substances (for instance, fuel components) is almost exactly equal to their internal energy because, due to the small specific volumes, the potential energy of compression is inconsequentially small. Let us note that the specific heat c_p, for all gases and at all temperatures, is greater than the specific heat c_v, by a quantity AR. In thermodynamics, the ratio of specific heat at constant pressure

to specific heat at constant volume is very significant. This ratio is called the adiabatic exponent and is denoted by k:

$$k = c_p/c_v = 1 + AR/c_v. \tag{5.18}$$

It follows from formula (5.18) that the value of k depends on the specific heat of the gas at constant volume, i.e., on the structure and temperature of the gas. For gases comprising the products of combustion, the value of k varies within wide limits, depending upon the temperature. The value of k decreases with an increase in temperature. For instance, for diatomic gases, the value of k changes from 1.4 at low temperatures to 1.28 at very high temperatures.

Monatomic gases have a maximum value of $k = 1.67$, and the triatomic gases at high temperatures, the minimum value of $k = 1.15$.

The specific heat at constant pressure may be expressed in terms of adiabatic exponent k, in the following manner:

$$c_p = [k/(k - 1)]AR. \tag{5.19}$$

The calculation of internal energy, the heat content (enthalpy) of the gas, or corresponding specific heats c_p and c_v, is done on the basis of experimentally determined molecular constants by the methods of statistical thermodynamics. Within narrow temperature intervals the change of specific heat with temperature can be presented as a linear function. However, such relationships cannot be used for the entire range of temperature changes of the products of combustion in the liquid fuel rocket motor.

4. Chemical Energy and Total Heat Content. The Basic Equation of Combustion

In the process of combustion chemical energy is transformed into heat. Therefore, the energy characteristic of fuel and the products of combustion must contain a chemical energy term.

The sum of heat content and chemical energy is called total heat content (or energy content). For a liquid this will be the sum of heat and chemical energies; for gases, the sum of heat, potential, and chemical energies.

As has been noted above, the magnitude of chemical energy is independent of external conditions, and is determined only by the structure of the chemical substances which participate in the reaction. Quantitatively, the value of chemical energy depends on the reference system used in calculation and, specifically, on the substances whose chemical energy is taken as being equal to zero.

If it is assumed, as is usual, that in the standard state the molecular gases and carbon, in the form of β–graphite, have zero chemical energy, then the chemical energy of carbon dioxide gas, for example, will be equiv-

alent to -94.05 kcal/gr mol, or $-2{,}410$ kcal/kg. Water vapor has also a negative chemical energy equal to -57.80 kcal/gr mol, or $-3{,}210$ kcal/kg. The formation of these gases results in conversion of chemical energy into heat energy.

Certain gases composing products of combustion, for instance, gases in the atomic state, have a positive chemical energy. This means that during their formation the chemical energy was not released but, on the contrary, was absorbed.

Using the concept of the total heat content, it is easily possible to write the basic equation of combustion. In order to do that we must apply to the process of combustion the law of conservation of energy. First, let us suppose that the combustion is not accompanied by any energy losses. Then the total heat content of the products of combustion E_g at temperature T, which they will have as a result of combustion, must be equal to the total heat content of the fuel E_f which entered the chamber:

$$E_g = E_f. \tag{5.20}$$

During the process of combustion various energy losses may take place, due, for instance, to heat flow to the chamber walls or physically incomplete combustion caused by poor mixing. They may be taken into account by introducing the coefficient of combustion efficiency η_c. The equation of combustion in this case can be written in the following form:

$$E_g = \eta_c E_f. \tag{5.21}$$

In order to determine the temperature of combustion according to Eqs. (5.20) and (5.21) it is necessary to know the composition of the products of combustion, since not only chemical energy, but the value of the heat content, depends on the composition of the gas mixture (inasmuch as heat capacities of various gases are different).

Dissociation has an appreciable effect on the composition of the products of combustion and the completeness of chemical energy conversion in the combustion chambers of liquid fuel rockets.

D. Thermal Dissociation and Composition of Combustion Products

1. Thermal Dissociation and Equilibrium Constants

In describing processes which take place in the combustion chambers of rocket power plants, and especially liquid rocket motors, we have paid particular attention to the problem of insuring complete combustion of the fuel and, consequently, of the total conversion of its chemical energy into heat energy. However, the degree of full conversion of chemical energy into

heat is determined not only by the design of the head and the chamber but also by the nature of the physical and chemical processes at high temperatures.

Combustion processes at high temperatures are characterized by quite intensive dissociation; processes of chemical compound formation at these temperatures are partially accompanied by their decomposition.

During the reverse reaction a reconversion of energy takes place. Due to dissociation there is a loss of heat energy and the degree of recovery of chemical energy is lowered. For instance, the reaction of oxidation of carbon monoxide is necessarily accompanied by the reaction of decomposition of carbon dioxide:

$$CO + \tfrac{1}{2}O_2 \rightleftarrows CO_2.$$

From the kinetic point of view, the possibility of reversible reaction is explained by the fact that in a gaseous mixture collisions between molecules of CO_2 with each other and with molecules of CO and O_2 take place, and that a sufficient force of collision is accompanied by the decomposition of a molecule of CO_2 into its constituents. The source of energy for splitting CO_2 is the energy of heat motion.

As the combustion reaction progresses the amount of the constituent molecules, i.e., CO and O_2, gradually diminishes, and consequently the velocity of the reaction decreases. The velocity of the dissociation reaction, on the contrary, increases as the content of the products of combustion in a gas (in our case CO_2) increases, since the number of collisions in which molecules of CO_2 take part will be on the increase. As a result, a time is reached when the velocities of combustion and dissociation reactions become equal to each other—a state of chemical equilibrium. At this time, the average chemical composition of the gas will not be changed.

The quantitative relationship established between dissociated and nondissociated gases, under conditions of chemical equilibrium, is determined by the so-called constant of chemical equilibrium or equilibrium constant.

In liquid rocket motors the equilibrium constant, K_e, expressed in terms of partial pressures, is used for calculations. Each partial pressure enters into the expression of the constant in a degree proportional to its effect on the velocity of the reaction. For instance, the equilibrium constant for the reaction

$$CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2.$$

is expressed in the following manner:

$$K_e = p_{CO}p_{O_2}^{1/2}/p_{CO_2} \tag{5.22}$$

where p_{CO}, p_{O_2}, and p_{CO_2} are the partial pressures of gases composed of a given mixture, i.e., those pressures which the enumerated constituents would have individually if they occupied the total volume of the mixture. In the above expression for the equilibrium constant, it can be seen that the stronger the dissociation, the greater the magnitude of K_e (and the greater the pressures p_{CO} and p_{O_2}).

The value of the equilibrium constant for a given reaction depends only on temperature. This dependence is extremely complex and it is impossible to have an analytical expression relating the equilibrium constant and the temperature for reactions between the products of combustion of rocket motors.

Equilibrium constants are presently determined by the methods of statistical thermodynamics. In order to be able to calculate an equilibrium constant it is necessary to know the molecular constants and, most important, the magnitude of chemical energies of corresponding substances. Tables of equilibrium constants for the necessary range of temperature changes are used for calculations.

2. The Effect of Temperature and Pressure on the Composition of Combustion Products

The values of the equilibrium constants of the dissociation reactions rise sharply with an increase in temperature, and the content of the products of dissociation increases correspondingly in the products of combustion. This is also clear from the kinetic point of view. If the temperature, T, of the gas mixture is increased, then the number of molecules having a large supply of energy is increased, leading to the increase in the rapidity of dissociation reaction of the combustion products and the disturbance of the equilibrium which had been established at a previous temperature. At a new, higher gas temperature, a new equilibrium condition will be established, characterized by equal velocities of the forward (burning) and reverse (dissociation) reaction, but at a higher content of the dissociation products in a gas mixture. In this way, the temperature of the gas mixture influences the composition of this mixture in such a manner that, with the increase in temperature, the amount of gases whose formation required an expenditure of heat increases.

The equilibrium constant for ideal gases does not depend on pressure. This does not mean, however, that the composition of a gas mixture always remains constant during a change of pressure.

Many dissociation reactions are accompanied by a change in the volume of the gas mixture. For instance, during the dissociation of carbon dioxide gas, there is an increase in the number of moles, and consequently an increase in the volume of gas mixture, by $\frac{1}{2}$ mole for each mole of fully dissociated carbon dioxide gas.

For dissociation reactions which take place with an increase in the number of moles, the composition of the gas mixture will depend upon the pressure to which it is subjected. The increase in pressure leads to the suppression of dissociation reactions and increases the content of the products of complete combustion in the gas mixture. In other words, increased pressure reduces the degree of gas dissociation if it is accompanied by an increase in the number of moles. For dissociation reactions which take place without increase in the number of moles the composition of the products is independent of pressure. Since the majority of dissociation reactions of rocket fuel combustion products take place with an increase in the number of moles, the increase in the combustion pressure decreases the degree of dissociation somewhat (but not significantly).

3. Composition and Temperature of Combustion Products in Rocket Motors. Completeness of Chemical Energy Release

Ordinary rocket motor fuels are composed of four elements: carbon, hydrogen, oxygen, and nitrogen.

If the combustion was not accompanied by dissociation, the products of combustion would consist of carbon dioxide, CO_2; water vapor, H_2O;

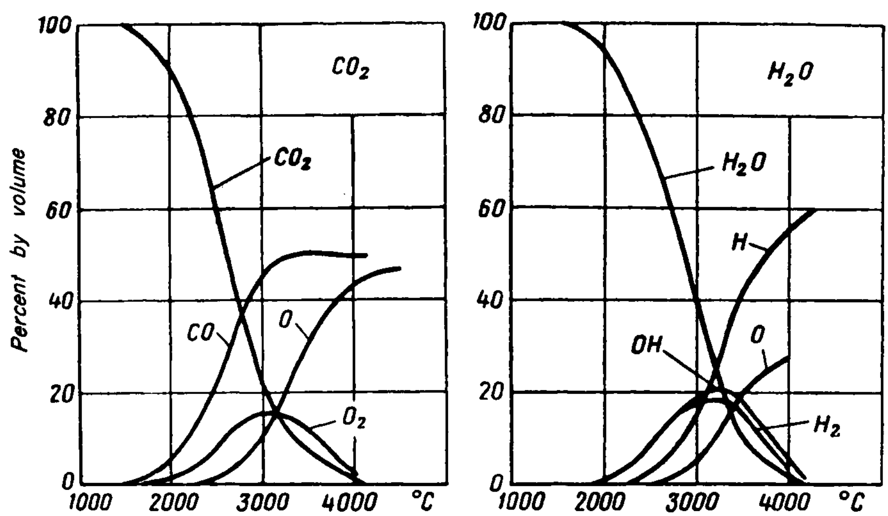


FIG. 5.20. Dissociation of water vapor, H_2O, and carbon dioxide, CO_2, with temperature.

and molecular nitrogen, N_2.* However, even at moderate temperatures—about 2800°K—considerable dissociation of carbon dioxide and water vapors may be observed (Fig. 5.20). In addition, carbon monoxide, CO;

* For the stoichiometric relations of components.

hydroxyl radical, OH; molecular oxygen, O_2; and hydrogen, H_2; are formed. At a still higher temperature, the presence in combustion products of nitrogen oxide, NO; and gases in an atomic state: hydrogen, H; oxygen, O; and nitrogen, N; also becomes significant.

The composition of the combustion products in the chamber is determined by the equilibrium constants corresponding to the dissociation reactions, taking into account the pressure in the chamber, p_c. Aside from that, the composition of gases depends, obviously, on the relative content of various elements in the fuel.

Since the composition of combustion products depends on temperatures, the solution of Eq. (5.20) or (5.21) requires a considerable amount of calculation. Ordinarily, it is necessary to assume a temperature, then to find the composition of the products of combustion, i.e., partial gas pressures, according to the composition of fuel and equilibrium constants, and, finally, to check the basic equation of combustion

$$E_g = \eta_c E_f.$$

The actual temperature and its corresponding composition of combustion products is determined, therefore, by a method of selection.

Calculation results show that because of the sharp increase in the degree of dissociation of the combustion products with increase in temperature above 3000°K, increase in the availability of chemical energy in the fuel does not yield a proportional increase of temperature in the chamber. For instance, combustion temperatures of two fuels may be compared: kerosene plus nitric acid, and kerosene plus oxygen. The calorific value of the second fuel is 2400 kcal/kg, which is approximately 70% higher than the calorific value of the first (1400 kcal/kg). However, the combustion temperature of kerosene plus oxygen fuel (3550°K) is only 15% higher than the combustion temperature of kerosene with nitric acid (3050°K). This is a direct result of the intensive dissociation of the combustion products and reduction in the completeness of the release of chemical energy.

Let us further examine the dependence of combustion temperature on pressure.

This dependence is well illustrated by a graph in Fig. 5.21, drawn for the combustion products of the fuel: kerosene plus liquid oxygen with $\alpha = 0.7$. From this dependence it may be seen that, initially, at low values, the increase in pressure leads to a rapid increase in temperature, and that subsequent increase in pressure leads to a less rapid increase in temperature.

The temperature and the composition of combustion products, besides being affected by pressure, are also affected by the relationship of components within the fuel.

Not taking dissociation into account, the maximum combustion tem-

perature and the maximum quantity of heat released during combustion would occur when the theoretical relationship of combustible and oxidizer is ν_0, i.e., when $\alpha = 1$. However, the dissociation phenomenon reduces the combustion temperature, and the unequal stability of combustion products with respect to dissociation results in the fact that the maximum temperature and maximum heat release correspond to values of $\alpha < 1$ in contemporary liquid rocket fuels, i.e., fuels which have a deficiency of oxidizer and an excess of combustible.

Figs. 5.22 and 5.23 show the dependence of the combustion temperature and the quantity of released heat, Q, on the oxidizer excess coefficient α and pressure for kerosene plus oxygen fuel. The shift of maximum temperature and heat release in the direction of $\alpha < 1$ is explained by the fact that

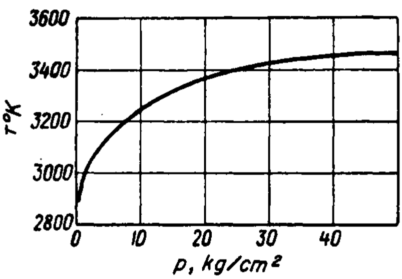


FIG. 5.21. Dependence of oxygen plus kerosene fuel combustion products temperature on pressure when $\alpha = 0.7$.

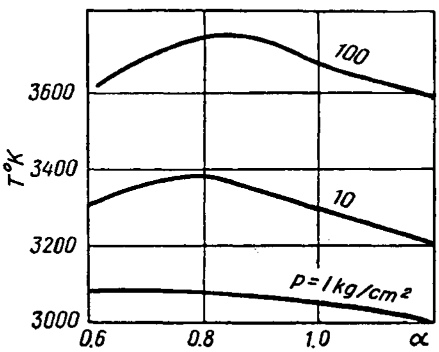


FIG. 5.22. Dependence of oxygen plus kerosene fuel combustion temperature on the oxidizer excess coefficient α and pressure p.

in the presence of insufficient oxidizer the relative content of carbon monoxide, stable with respect to dissociation, is increased in the combustion products. As the curves of Fig. 5.23 show, the loss of heat due to dissociation in the combustion chamber is quite high—at a pressure of 14.7 lb/in.² it constitutes more than 30% of the calorific value of fuel whose heat content is 2400 kcal/kg.

We have examined the effect of dissociation using as an example a fuel which has the maximum value of heat content. Inasmuch as the intensity of dissociation is reduced at lowered temperatures, the influence of dissociation is reduced for fuels with lower calorific value (oxygen plus alcohol or nitric acid plus kerosene), even though it still remains considerable. For instance, the loss of heat in the chamber due to dissociation for the oxygen plus alcohol fuel constitutes 18–25% of the calorific value, and for nitric oxide fuels it is 12–18%.

The composition of combustion products determines the value of their gas constant. The simultaneous influence of the composition and temperature on the heat capacity of gases leads to the corresponding change in the adiabatic exponent k. The influence of temperature and pressure of the

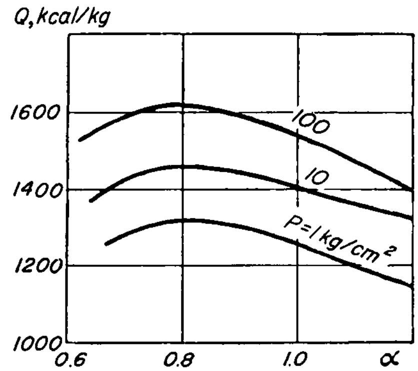


Fig. 5.23. Dependence of the quantity of released heat during the combustion of oxygen and kerosene fuel on oxidizer excess coefficient α and pressure p.

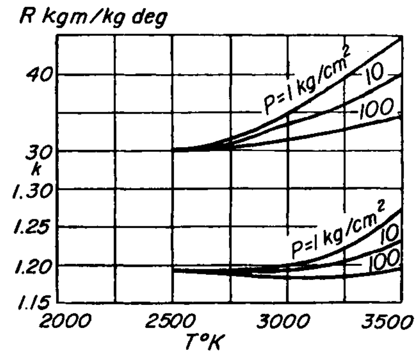


Fig. 5.24. Dependence of combustion products gas constant R and adiabatic exponent k on temperature T and pressure p.

combustion products on the value of gas constant R and adiabatic exponent k is illustrated by the graph in Fig. 5.24. As is shown in this graph, the gas constant of the combustion products increases with increasing temperature. This happens because, as a result of dissociation, there is a reduction in the content of polyatomic gases in the products of combustion. For the same reason, the adiabatic exponent of the products of combustion increases with an increase in temperature and a decrease in pressure.

VI. Flow of the Combustion Products through the Nozzle of a Rocket Motor

A. Basic Laws of Gas Flow

1. Gas Flow Parameters

The products of combustion, formed in the chamber of the motor, flow into the nozzle, where they are expanded and accelerated. In the process of motion along the nozzle the flow parameters change. Parameters of gas flow in the nozzle, aside from the parameters of state p, v (or ρ), and T, mentioned earlier, also include velocity w, and the area of the stream cross section S.

During the flow along the nozzle, reduction in gas temperature and pressure take place. Reduction in temperature leads to the reduction in the degree of dissociation of a gas which was strongly dissociated in the chamber—recombination of atoms and radicals into molecules.

The reduction in pressure does not hinder this process significantly. It is clear that the recombination results in an additional (with respect to the combustion chamber) release of heat and facilitates more complete conversion of the fuel chemical energy into kinetic energy of the gas flow emerging from the nozzle.

For the time being, let us not take into account the recombination phenomenon, and let us examine the flow of a gas of constant composition along a nozzle. As a basic equation for determining the parameters of flow, let us use the equation of the gas state

$$p/\rho = gRT \tag{6.1}$$

or, in another form,

$$pv = RT.$$

The second equation, which contains the parameters of the gas flow, is the equation of the thermodynamic process. We shall dwell on this equation in some detail.

The change in the gas state during thermodynamic processes can take place in various ways; for instance, at constant volume, at constant pressure, or at constant temperature. Depending on this the interrelation between gas parameters varies.

The more general form of the thermodynamic process equation is

$$p/\rho^n = \text{constant}$$

or

$$pv^n = \text{constant}. \tag{6.2}$$

In the future we will agree to discuss only those processes whose exponent, n, remains constant for the entire process.

Taking various values of the exponent n, it is possible to describe basic thermodynamic processes which take place in a gas. For instance, taking $n = 0$, we will get $p = \text{constant}$. Consequently, Eq. (6.2) will express the equation of an isobaric process. Such a process takes place, for instance, in the combustion chamber of a rocket motor. This process in pv coordinates is shown in Fig. 6.1 by a line I (isobar).

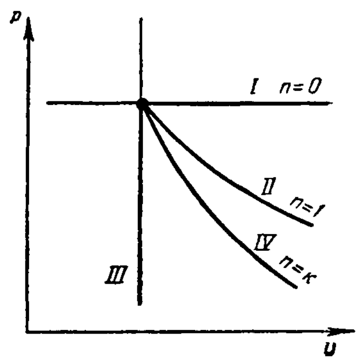


FIG. 6.1. Curves of the basic gas thermodynamic processes.

When $n = 1$, we will get $pv = \text{constant}$ or, taking into account the equation of state, $T = \text{constant}$, i.e., the equation of an isothermal process (line II in Fig. 6.1).

When $n \to \infty$, Eq. (6.2) represents the equation of an isochoric process, $v = \text{constant}$.

Line III in Fig. 6.1 (isochor) divides two characteristic areas: the area of expansion processes (increase in the specific volume v), and the area of compression processes (reduction in specific volumes v).

Of all the possible gas processes, we will be most interested in the process of the expansion of a gas of constant composition taking place without heat exchange between the gas and the surrounding medium. It would seem that such conditions for gas flow, especially in the nozzle of a rocket motor, cannot exist, since the gas is at a high temperature and comes in contact with rapidly cooled walls of the nozzle. However, in actuality this is not the case. The time during which the gas is in contact with nozzle

walls is very short. Due to the high flow velocity, this time, even in motors having considerable dimensions, represents a magnitude in the order of 0.001 sec. Besides, a considerable portion of the gas passes at some distance from the walls and releases heat to them only to a small degree.

For the above reasons the amount of heat given up by the gas to the walls is insignificant by comparison with its total supply, and the process of the gas expansion in the nozzle of a rocket motor may be considered as taking place without heat exchange to the walls. Such an expansion process is called adiabatic.

An adiabatic process taking place in the gas is characterized by the manifestation of the law of the conservation of energy in its simplest form.

Reduction in gas temperature and pressure takes place during adiabatic expansion. Concurrent with the reduction in temperature there is a reduction in the internal energy of the gas. The reduction in pressure also causes a reduction in the potential energy due to gas pressure. According to the law of conservation of energy, the difference in energies which the gas possessed at the beginning and at the end of the process is entirely converted into the work of expansion of the gas.

The work of gas expansion is variously exploited in different machines. In the piston engine, for instance, it is converted into the work of displacing a piston; in reaction motors, into kinetic energy of a gas flow. The amount of work obtained during adiabatic gas expansion is easily calculated by determining the change in the internal and potential energies of the gas during the expansion process. This we shall do below during the derivation of the energy equation.

The exponent n for an adiabatic process is equal to the adiabatic exponent

$$n = k = c_p/c_v.$$

As can be seen from Fig. 6.1, the expansion curve with an exponent, $n = k > 1$ (line IV), passes between the isotherm and isochor but with a steeper slope than the isotherm.

2. Stable and Unstable Gas Flow

It is not sufficient, in determining the relationship between the parameters of a moving gas, to use only the thermodynamic relationships.

Of all the possible flows, let us first emphasize the stable or steady state flow. Steady state flow is that flow which is characterized by unchanging gas parameters (velocity, pressure, temperature, density) at any point in the gas stream. If these parameters are not constant with respect to time, then the flow is considered unstable.

In many technical problems involving motion of gases, the gas flow may

be considered as stable, even when it is not entirely stable, at least as an average. This considerably simplifies the solution of many practical problems.

It is this type of flow which we will consider as taking place in rocket motor nozzles. At the same time, the period of starting and stopping the motor or the time of transition from one working regime to another, when the gas rate of discharge varies, will be excluded from this examination, since such changes vary the parameters of gas flow.

An example of an obviously unstable flow is the gas flow in the pulsating ducted motor. In this case, gas parameters vary not only along the length of the motor but also with respect to time.

3. Velocity Distribution in the Stream Cross Section. One-Dimensional Flow

Gases, like liquids, have viscosity. A characteristic of viscosity is the coefficient of viscosity μ, determined by the relationship

$$\tau = \mu(\Delta w/\Delta y),$$

where τ is the sheer force related to a unit area, arising between two layers of gas or liquid moving parallel with respect to each other and located at a distance Δy from each other; and Δw is the difference in velocities between these layers.

The gas viscosity is considerably less than the viscosity of liquids and becomes apparent only when there is a large difference in velocities; for instance, immediately at the surface of a body placed in the gas stream. The particles of gas come in intimate contact with the surface and would seem almost to attach themselves to it and remain immobile.

As the distance from the surface of the body increases, the velocity of gas particles increases rapidly, and then remains constant and equal to the velocity of flow.

The thin layer of gas in which the velocity change from zero to the velocity of the free stream takes place is called the boundary layer. The gradual increase in velocity within the boundary layer is explained by the action of internal friction forces within the gas.

The thickness of the boundary layer at the surface of the body in the gas stream increases in the direction of flow, and with great linear dimensions of the body can reach large magnitudes.

The motion of gases and liquids, as is known from the course in physics, may be laminar and turbulent. In laminar—stream line—flow, streams of gas do not mix, whereas during turbulent flow there is an intensive mixing of fluid volumes.

Laminar flow may be either stable or unstable. Turbulent flow is always unstable. However, since the mixing of fluid layers takes place within volumes considerably smaller than the over-all flow dimensions, the turbulent flow may be considered, on the average, as being stable.

During gas flow along a wall in the boundary layer, the flow is laminar for some distance, and then becomes turbulent. The change from laminar to turbulent flow depends on the conditions of gas flow.

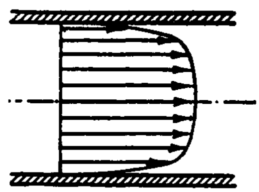


FIG. 6.2. Traverse of velocity distribution in a turbulent flow.

The law of velocity change across the thickness of a boundary layer is different for laminar and turbulent flows. In the turbulent boundary layer, due to the intensive mixing of gas layers, the velocity increases considerably faster than in the laminar layer.

The conditions of gas flow along the rocket motor nozzle are such that the boundary layer is always turbulent, with a rapid increase in velocity to the full velocity of the stream (Fig. 6.2). Therefore, in examining the

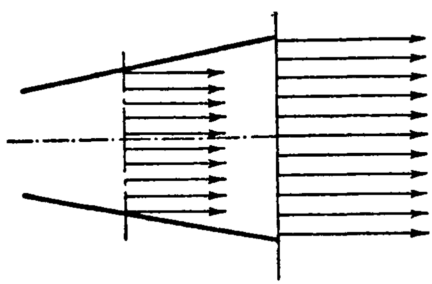


FIG. 6.3. Schematic of a one-dimensional flow in the nozzle of a rocket motor.

flow of gas along the nozzle of a liquid rocket motor, the boundary layer is usually not taken into account, considering that, at any point of a given section, the velocity is the same and is equal to the velocity in the center of the flow where the influence of the walls is not felt.

The nozzle of a rocket motor is a duct with changing cross section, in which radial gas flow must take place toward the axis of the duct in the convergent part and away from the axis in the divergent part. However, the radial velocities of gas in the nozzle may be neglected as a first approximation.

Therefore, at each section of the motor, the velocity is considered to be constant and equal to the velocity at the axis of the duct (Fig. 6.3). Such fluid flow is called one-dimensional.

If the radial velocity is not considered, then the calculated (theoretical) velocity of flow will be greater than actual. Consequently, we can consider that the radial flow results in some loss in exit velocity and specific impulse of the motor. This loss is included in the total losses which accompany the flow of gas along the nozzle.

4. Equation of Discharge

Let us examine two sections, perpendicular to the direction of velocity of one-dimensional flow (Fig. 6.4), and calculate the mass of gas passing through both sections during the time Δt. The mass discharge is determined, in this case, by the volumetric discharge $Sw\Delta t$, multiplied by the density ρ. Therefore, the gas mass passing through the first section of one-dimensional flow will be $\rho_1 w_1 S_1 \Delta t$, and through the second, $\rho_2 w_2 S_2 \Delta t$.

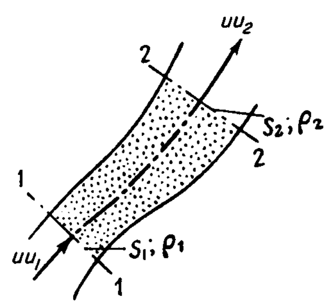


Fig. 6.4. Derivation of consumption equation.

However, in a steady state flow the gas parameters cannot change at any point between the first and the second section during the time Δt. Therefore, there will be no concentration or diminution in the gas mass in the volume bounded by the sections. The passage of gas through section 1–1 must therefore equal the discharge through section 2–2, from which it follows that for one-dimensional gas flow in the steady state

$$\rho w S = \text{constant}. \tag{6.3}$$

The derived equation is called the equation of discharge. It represents the expression of the law of conservation of mass for the case of gas flow.

For an incompressible fluid ρ = constant. The equation then takes the form

$$wS = \text{constant}. \tag{6.4}$$

At low velocities, gas may be considered an incompressible fluid. From Eq. (6.4) it follows that, for this case, the velocity will be inversely proportional to the area of the cross section of the stream. For a compressible gas, due to change in density ρ, the picture changes not only quantitatively but also qualitatively. As we shall see later on, at supersonic velocities in a divergent duct the gas velocity does not diminish, but increases.

5. Equation of Energy

Let us examine the energy relationships characteristic of gas flow. These relationships result from the law of conservation of energy. Let us consider that a heat exchange between the flow and the walls (surrounding medium) is absent, i.e., we shall examine an adiabatic gas flow. In this case, the energy, E, of some gas mass will remain constant for any section of the stream.

Let us calculate the value of E for 1 kg of gas at some section of the stream.

Gas energy is composed of kinetic energy, potential energy due to pressure and weight, and the internal (heat) energy of the gas. First of all, at velocity w, the gas has a kinetic energy equal to

$$mw^2/2$$

or, for 1 kg of gas, $w^2/2g$, and, in heat units, $Aw^2/2g$.

The potential energy due to pressure of 1 kg of gas in heat units is equal to Apv. The potential energy of weight, mgz, for 1 kg of substance is equal to the height, z, of the center of gravity of the gas mass with respect to some datum line. For low density gas flows, the potential energy due to weight is negligible compared to the potential energy due to pressure.

Finally, the internal gas energy

$$U = c_v T.$$

Therefore, if the potential energy due to weight is neglected, the total energy of 1 kg of gas

$$E = Aw^2/2g + U + Apv.$$

The sum of the internal energy, U, and the potential energy due to pressure, Apv, is called the heat content, H (see Chapter V). Therefore

$$E = Aw^2/2g + H. \tag{6.5}$$

For adiabatic flow, E = constant. Consequently

$$H + Aw^2/2g = \text{constant}. \tag{6.6}$$

or, for two arbitrary sections, 0–0 and 1–1 of the gas stream,

$$H_0 + Aw_0^2/2g = H_1 + Aw_1^2/2g. \tag{6.7}$$

The derived equation represents an expression of the law of conservation of energy, which states that energy does not disappear or appear, but is transformed from one form into another. Equation (6.7) will be referred to from now on simply as the energy equation.

Equation (6.7) is often used in determining the velocity of gas flow.

$$w_1 = \sqrt{w_0^2 + 2g(H_0 - H_1)/A}. \tag{6.8}$$

If the flow of gas from a vessel of large dimensions is considered, where velocity w_0 is low, then

$$w_1 = \sqrt{2g(H_0 - H_1)/A}. \tag{6.9}$$

Let us present the energy equation in a different form, which will be useful to us in the future. In order to do this, we will take advantage of relationships presented in Chapter V, and will express the heat content of the gas in terms of the parameters of state

$$H = kRT/(k - 1) = kp/g\rho(k - 1).$$

The energy equation can now be rewritten in the form

$$kp/\rho(k - 1) + w^2/2 = \text{constant}. \tag{6.10}$$

In deriving the energy equation for incompressible fluid the heat energy should not be taken into account, since it does not change during the motion of such a fluid. On the other hand, due to the large specific weight of liquids, it is imperative to consider the change of the potential energy due to weight. The equation of conservation of energy for incompressible fluid is written in the following form:

$$p/\gamma + w^2/2g + z = \text{constant}. \tag{6.11}$$

The equation of energy for an incompressible liquid Eq. (6.11), is called Bernoulli's equation.

B. Gas Flow in a Supersonic Nozzle

1. Velocity of Sound in Gases

A very important gas characteristic is the speed of sound propagation.

By the term speed of sound, we mean the velocity of propagation of longitudinal vibrations in the medium. This includes not only vibrations which are audible to the human ear as sound, but also gas vibrations whose frequencies are beyond the threshold of hearing.

Let a motionless gas mass at pressure p, density ρ, and temperature T, be confined in a cylindrical tube (Fig. 6.5). Further, let an impulse—for instance, a short shock—be imparted to the gas with the aid of a movable piston at the left end of the tube. The gas in the vicinity of the piston will compress and then, in expanding, will set in motion particles of gas located to the right. A wave will start to travel along the tube from left to right.

At a certain time t_1, the wave will reach section 1–1. After Δt sec (at time t_2), it will be displaced to section 2–2.

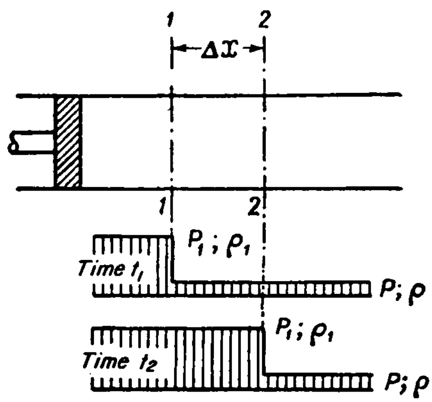


FIG. 6.5. Derivation of the speed of sound.

Beyond section 2–2, the gas pressure will be the same as before the imparting of the impulse to the gas. To the left of section 2–2, the pressure will be greater than in the undisturbed gas. Let us designate it by p_1, ($p_1 > p$).

The velocity of wave propagation obviously will be $c = \Delta x/\Delta t$, where Δx is the distance between sections 1–1 and 2–2.

The increase in the gas mass in the tube volume corresponding to the region Δx is due to the inflow into this volume, from the left, of a certain gas mass with some velocity w. The magnitude of this mass is

$$\Delta M = \rho_1 S w \Delta t,$$

where S is the area of the tube cross section, and ρ_1 is the gas density to the left of the wave front.

On the other hand, the increase in the gas mass can be expressed in terms of the change in density within the volume $S\Delta x$

$$\Delta M = (\rho_1 - \rho) S \Delta x.$$

Equating the right members of the last two equations we find

$$w = (\rho_1 - \rho)c/\rho_1. \tag{6.12}$$

In order to eliminate the unknown velocity w, we will take advantage of the momentum theorem.

The gas mass in the volume $S\Delta x$ will be $\rho S\Delta x$. This mass during the time Δt begins to move with the velocity w. The change in the momentum must be equal to the impulse

$$\rho S(w - O)\Delta x = (p_1 - p)S\Delta t,$$

from which

$$p_1 - p = \rho wc.$$

Substituting into this equation expression (6.12) for w we get

$$c = \sqrt{\rho_1(p_1 - p)/\rho(\rho_1 - \rho)}. \tag{6.13}$$

For weak gas disturbances such as sound vibrations, ρ_1 does not vary appreciably from ρ, and p_1 from p

$$\rho_1 = \rho + \Delta\rho; \qquad p_1 = p + \Delta p \tag{6.14}$$

and

$$c = \sqrt{\Delta p/\Delta\rho}. \tag{6.15}$$

The quantity $\Delta p/\Delta\rho$ depends on the process of the gas compression. Newton, who first derived this expression in 1687, supposed that the gas temperature remained unchanged during the passage of a wave. In this case

$$p/\rho = \text{constant}$$

from which we get

$$\Delta p/\Delta\rho = p/\rho$$

and

$$c = \sqrt{p/\rho}.$$

This formula gives a value for the speed of sound in air almost 15% less than that obtained experimentally. In his time, Newton explained this discrepancy by the presence in the atmosphere of suspended solid particles and water vapor.

Considerably later, in 1810, Laplace showed that the process of gas compression during the passage of a wave should be regarded not as isothermal but as adiabatic, inasmuch as during rapid compression and expansion of the gas the heat transfer within a gas does not have time to take place.

In the case of an adiabatic process in a wave

$$p = \rho^k \text{ constant}$$
$$\Delta p = k\rho^{k-1}\Delta\rho \text{ constant}$$

from which

$$\Delta p/\Delta\rho = k(p/\rho)$$

and

$$c = \sqrt{k(p/\rho)}, \tag{6.16}$$

or, in accordance with the equation of state, $p = \rho gRT$,

$$c = \sqrt{kgRT}. \tag{6.17}$$

The last two formulas yield a higher value for the speed of sound and are in good agreement with experiments.

Therefore, the speed of sound in gas depends not on absolute values of pressure and density but on their relationship, i.e., on the temperature.

For air, $k = 1.4$, $R = 53.3$, and the expression for the velocity of sound takes the form

$$c = 49.0 \sqrt{T}.$$

The velocity of sound in air at 32°F equals 1080 ft/sec. For combustion products in the chamber of a rocket motor when $T = 5410°\text{R}$, $k = 1.2$, $R = 62.6$, and speed of sound $c = 3610$ ft/sec.

The speed of sound has a definite physical significance, representing the velocity of propagation of weak disturbances in a gas. The velocity of sound also has a definite energetic significance. In order to clarify it, let us rewrite the expressions for the square of the sound velocity and for the heat content of a gas.

$$c^2 = kgRT$$
$$H = kART/(k - 1).$$

Eliminating from these two equations the term T, we will find

$$c^2 = HA/g(k - 1).$$

The obtained expression means that the square of the speed of sound is a measure of the heat content of a gas.

The concept of the speed of sound has a tremendous significance in aerodynamics and gas dynamics. The flow of gas past bodies, through tubes and fittings, and, in general, the behavior of any gas motion, depends upon the ratio of gas velocity to the velocity of sound in a gas. Depending upon the value of this ratio, it is customary to speak of subsonic and supersonic regimes of flow and flight velocities. The ratio of flow velocity to the velocity of sound is cutomarily represented in all aerodynamic and gas dynamic calculations by the letter M, and is called the Mach number

$$M = w/c. \tag{6.18}$$

Initially (in 1868), the relationship of gas velocity (or a body moving through the gas) to the velocity of sound was introduced into scientific circles by a Russian scientist—ballistician N. V. Mayevsky. Later, it was also used by an Austrian physicist, Mach, and is widely known in technology as the Mach number.

Let us return to expression (6.13).

If the pressure p_1 and density ρ_1 differ considerably from the pressure p and density ρ in the undisturbed gas, then the disturbing wave is called a strong, or shock wave, as differentiated from the weak accoustical (sound) wave. The velocity of propagation of the shock wave is

$$c_s = \sqrt{[(\rho + \Delta\rho)/\rho](\Delta p/\Delta\rho)}. \tag{6.19}$$

By comparing expressions (6.15) and (6.19) it follows that the velocity of propagation of the shock wave is always greater than the velocity of sound.

If a strong disturbance is imparted to the gas, i.e., if a large difference in pressures, $p_1 - p$, is created, then the formed wave, during its propagation, will partially dissipate energy imparted to the gas. The strength of the wave, measured by the difference of pressures $p_1 - p$, will be decreasing. Its velocity will decrease correspondingly and the shock wave, after some time, will turn into a weak wave propagating with the velocity of sound.

2. The Maximum Exhaust Velocity

Let us imagine a vessel, for instance, the combustion chamber of a rocket motor, which contains an immobile gas ($w_0 = 0$), with unchanging parameters, p_0, ρ_0, T_0. Let the gas flow out of this vessel into a region where the parameters of this gas will be p, ρ, and T.

It follows from the energy equation that the velocity of gas will be greatest in those sections of the stream in which the heat content is least. A maximum velocity will be obtained if all of the heat content is converted into the kinetic energy of the stream of the exhaust gas. At the same time, absolute temperature of the gas must become equal to zero. From the equation of the adiabatic process which is assumed to take place during gas flow from the vessel,

$$T = T_0(p/p_0)^{(k-1)/k} \tag{6.20}$$

it follows that in order to get gas temperature equal to zero it is necessary for the pressure, p, in a gas stream to equal zero also. Therefore, the maximum velocity of a gas can be obtained by exhausting into a vacuum.

According to expression (6.9) the value of the maximum velocity is

$$w_{\max} = \sqrt{(2g/A)H_0},$$

where H_0 is the heat content of the immobile gas in the vessel.

The term $w_{\max}$ may be expressed in terms of parameters of state and velocity of sound in a motionless gas

$$w_{\max} = \sqrt{2kp_0/(k-1)\rho_0} = \sqrt{2kgRT/(k-1)} = c_0\sqrt{2/(k-1)}. \tag{6.21}$$

For air at room temperature, $w_{max} \approx 2{,}460$ ft/sec; for combustion products of rocket motor fuel ($T_0 = 5410°K$, $R = 62.6$, $k = 1.2$), $w_{max} \approx 11{,}500$ ft/sec.

The maximum exhaust velocity, as follows from formula (6.21), depends only on temperature T_0, and is independent of pressure. From the energetic point of view this is completely clear. The maximum velocity will result when the initial heat content of the gas is completely converted into kinetic energy, and the amount of heat content is determined only by the initial temperature of the gas. At maximum velocity there is a complete conversion of the thermal random motion of molecules into the directed motion of the flow.

It would seem, at first glance, that with increased pressure the flow velocity w_{max} must increase, inasmuch as, conventionally speaking, the force pushing the gas out of the vessel increases. However, with the increase in pressure, the density ρ_0 increases in the same proportion (when T_0 = constant) and, consequently, the mass contained in a unit volume also increases. It is understandable that the increased pressure imparts to the mass, which has been increased in the same proportion, the same velocity w_{max}.

3. Dependence of Gas Parameters on Local Flow Velocity

Let us examine the manner in which the parameters of a moving gas will change with respect to flow velocity. Let us write the energy equation for two states of flow

$$kgRT_0/(k-1) = \tfrac{1}{2}w^2 + kgRT/(k-1).$$

Here, w and T are related to some arbitrary section, and T_0 represents the temperature of gas when $w = 0$. For temperature T we get the following expression:

$$T = T_0\left(1 - \frac{w^2}{2kgRT_0/(k-1)}\right),$$

or

$$T = T_0\left(1 - \frac{w^2}{w_{max}^2}\right). \tag{6.22}$$

Consequently, the higher the flow velocity of a gas the lower its temperature.

In an adiabatic gas flow

$$\rho/\rho_0 = (T/T_0)^{1/(k-1)}; \qquad p/p_0 = (T/T_0)^{k/(k-1)}.$$

Consequently,

$$p = p_0\left(1 - \frac{w^2}{w_{\max}^2}\right)^{k/(k-1)} \tag{6.23}$$

$$\rho = \rho_0\left(1 - \frac{w^2}{w_{\max}^2}\right)^{1/(k-1)}. \tag{6.24}$$

Therefore, both the pressure and the density of the gas are reduced with the increase in flow velocity. In reaching maximum velocity, the pressure and density of the gas are reduced to zero.

4. The Relationship between the Local Speed of Sound and Flow Velocity. Critical Velocity

As we already know, the velocity of sound in a gas is determined only by the temperature. However, the gas temperature may be different at various points within the stream. Consequently, the velocity of sound will

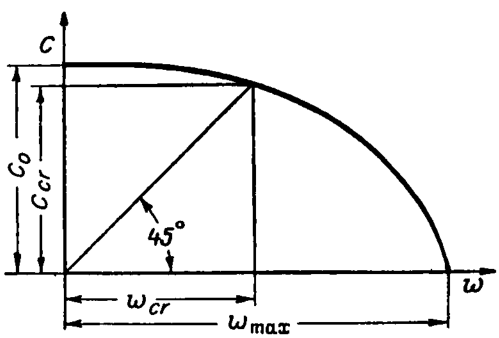


Fig. 6.6. Dependence of the speed of sound in a gas on flow velocity.

also be different. Therefore, in considering a moving stream, we must consider not only the local velocity, temperature, pressure, and density, but also the local velocity of sound.

The energy equation, after introducing the concept of maximum velocity, may be written in the following manner:

$$\tfrac{1}{2}w^2 + kgRT/(k-1) = \tfrac{1}{2}w_{\max}^2. \tag{6.25}$$

From this, taking into consideration that $kgRT = c^2$, we get

$$c = \sqrt{(w_{\max}^2 - w^2)(k-1)/2}. \tag{6.26}$$

The relationship between the speed of sound and gas velocity is shown in Fig. 6.6. When $w = w_{\max}$, the speed of sound is reduced to zero, inasmuch as, under these conditions, $T = 0$.

The flow velocity may be either greater or less then the local speed of sound. The flow velocity equal so the local speed of sound is called the critical velocity. It may be found by means of expression (6.26) by letting $c = w = w_{cr}$.

$$w_{cr}^2 = w_{max}^2(k - 1)/(k + 1).$$

But since

$$w_{max}^2 = 2kgRT_0/(k - 1),$$

then

$$w_{cr}^2 = 2kgRT_0/(k + 1) = 2c_0^2/(k + 1). \tag{6.27}$$

Therefore, the value of w_{cr} depends only on the temperature of gas in the vessel from which flow takes place.

Let us establish the local gas parameters at critical velocity. According to expression (6.22)

$$T_{cr} = T_0\left(1 - \frac{w_{cr}^2}{w_{max}^2}\right)$$

or, since

$$w_{cr}^2 = w_{max}^2(k - 1)/(k + 1)$$

$$T_{cr} = 2T_0/(k + 1). \tag{6.28}$$

Considering expressions (6.23) and (6.24) we also get

$$p_{cr}/p_0 = [2/(k + 1)]^{k/(k-1)}; \tag{6.29}$$

$$\rho_{cr}/\rho_0 = [2/(k + 1)]^{1/(k-1)}. \tag{6.30}$$

In general, for air ($k = 1.4$)

$$p_{cr} = 0.528\ p_0;$$
$$T_{cr} = 0.833\ T_0;$$
$$\rho_{cr} = 0.634\ \rho_0;$$

for combustion products of a rocket motor ($k = 1.2$)

$$p_{cr} = 0.565\ p_0;$$
$$T_{cr} = 0.909\ T_0;$$
$$\rho_{cr} = 0.621\ \rho_0.$$

Therefore, in order to get supersonic airflow exhausting to the atmosphere, it is necessary to have a chamber pressure approximately twice that of the atmospheric pressure.

5. Configuration of a Supersonic Nozzle

Up to now we have discussed dependence of the gas flow parameters on velocity, but we have not examined the problem of reaching this velocity.

It is well known that the increase in velocity of an incompressible fluid or gas may be obtained by the constriction of the duct. However, experience shows that in a constricted duct, velocities greater than critical may not be obtained. Supersonic flow velocities of a gas can be obtained by means of the Laval nozzle, which consists of a duct whose cross section first decreases and then increases (Fig. 6.7).

It has been found that, if gas flows through a Laval nozzle in such a way that it reaches critical speed at the narrowest section, then beyond the throat the flow velocity, w, begins to increase. In this manner, it has been

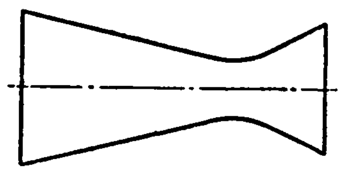


FIG. 6.7. Laval nozzle configuration.

determined by experiment that for supersonic flow, the flow laws are directly opposite to the flow laws of a subsonic stream. In other words, a stream flowing with subsonic velocity is accelerated in a convergent duct and decelerated in a divergent section. Supersonic flow, on the contrary, is decelerated in a convergent duct and is accelerated in a divergent duct.

Let us examine this question in more detail. Let us write the discharge equation, Eq. (6.3), in the form

$$S = \text{constant}/\rho w. \tag{6.31}$$

For an incompressible fluid (ρ = constant), the velocity is inversely proportional to the area of the duct section. For a compressible fluid at normal velocities, the density, ρ, as follows from formula (6.25), decreases with increase in velocity, but decreases so unnoticeably that the character of the gas flow remains qualitatively the same as for the incompressible fluid, and the area of the flow section decreases with increase in velocity. However, at high, supersonic velocities the value of ρ decreases with increase in velocity faster than the increase of w. Consequently, the flow cross section S, in this case, must increase.

Let us examine the dependence of ρw on w. According to expression (6.24)

$$\rho = \rho_0(1 - w^2/w_{\text{max}}^2)^{1/(k-1)}.$$

Consequently

$$\rho w = \rho_0 w(1 - w^2/w_{\text{max}}^2)^{1/(k-1)}.$$

The term ρw may be called the mass specific discharge, which means the discharge of a gas mass in a unit of time through a unit area of cross

section. When $w = 0$, the specific discharge is reduced to zero. When $w = w_{\max}$, it also reduces to zero since, at this velocity, the density ρ becomes zero. In the interval between these two limiting values of velocity there obviously exists a maximum for the function.

Fig. 6.8 shows the dependence of the specific discharge on velocity.

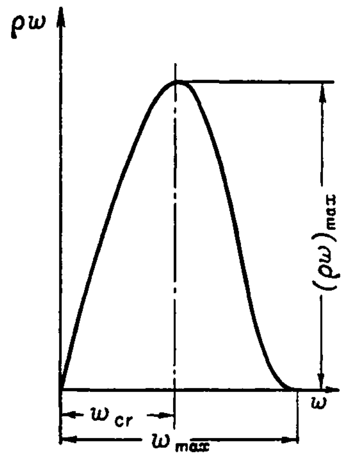


FIG. 6.8. Relationship between the quantity ρw and the flow velocity w.

Let us see at what value of velocity the maximum specific discharge takes place. Differentiating the expression for ρw with respect to w, and equating the derivative to zero, we get

$$\rho_0 \left(1 - \frac{w^2}{w_{\max}^2}\right)^{\frac{1}{k-1}} - \rho_0 w^2 \left(\frac{1}{k-1}\right)\left(1 - \frac{w^2}{w_{\max}^2}\right)^{\frac{1}{k-1}-1} \frac{2}{w_{\max}^2} = 0$$

and, from that

$$\left(1 - \frac{w^2}{w_{\max}^2}\right)^{\frac{1}{k-1}-1} \left[1 - \frac{w^2}{w_{\max}^2} - \left(\frac{2}{k-1}\right)\frac{w^2}{w_{\max}^2}\right] = 0. \tag{6.32}$$

Consequently, either $w = w_{\max}$, or the expression within brackets in Eq. (6.32) equals zero.

In the first case, $\rho w = 0$, and we obtain a minimum shown on the curve of Fig. 6.8. In the second case, when

$$w^2 = w_{\max}^2(k-1)/(k+1) \tag{6.33}$$

the specific discharge ρw has its maximum value.

However, the velocity determined by the relationship (6.33) is none other than the critical velocity equal to the local velocity of sound. The

maximum point on the curve of Fig. 6.8 separates subsonic flow velocities from supersonic flow velocities.

In examining the curve ρw, it is not difficult to determine how the transverse section of the duct must vary in order to obtain supersonic velocities. The cross sectional area, according to Eq. (6.31), must first decrease and then increase. At the point where the term ρw has its maximum value, the area of section S must be minimum. The local velocity of sound and the critical velocity are reached at this point. Therefore, the minimum section of the nozzle is called the critical section. In this manner we see that attain-

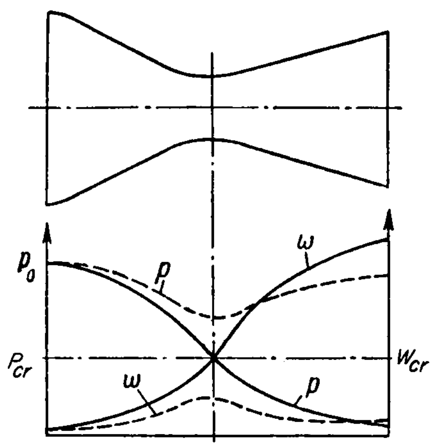


FIG. 6.9. Variation in flow parameters along a supersonic nozzle for different values of various gas parameters at the entrance.

ment of supersonic velocities is possible in the Laval nozzle. However, this is not always possible. If the difference in pressures at the entrance and exit of the nozzle is insufficient for creation of critical velocity at the narrow section, it will be impossible to obtain supersonic velocity.

Fig. 6.9 shows possible cases in the operation of the nozzle. The solid curves correspond to the basic operational case. The relationship of pressure at the entrance to the exit pressure is less than a critical ratio

$$p/p_0 < [2/(k + 1)]^{k/(k-1)}.$$

In the meantime, the gas pressure drops continuously and the velocity increases.

In cases in which the pressure ratio

$$p/p_0 > [2/(k + 1)]^{k/(k-1)},$$

i.e., when the exit nozzle pressure becomes too great or too small at the entrance, the critical velocity at the narrow section of the nozzle will not

be attained, even though the velocity there will be maximum. In the diverging portion of the nozzle the velocity will be decreasing and the pressure increasing (dashed curves in Fig. 6.9).

C. The Performance of a Rocket Motor Nozzle

1. The Area of the Critical Nozzle Section

The geometrical dimensions of a supersonic nozzle of a rocket motor are determined by the areas of the critical and exit sections.

First, let us find the required area of the critical section. For this, let us express the mass rate of fuel (gas) discharge G, in terms of the gas parameters at the nozzle critical section,

$$G = g\rho_{cr}w_{cr}S_{cr}.$$

But, according to Eqs. (6.30) and (6.27)

$$\rho_{cr} = \rho_0[2/(k+1)]^{1/(k-1)} \qquad \text{and} \qquad w_{cr}{}^1 = \sqrt{2kgRT_0/(k+1)}.$$

Therefore

$$G = g\rho_0 S_{cr}[2/(k+1)]^{1/(k-1)}\sqrt{2kgRT_0/(k+1)}. \tag{6.34}$$

Since

$$\rho_0 = p_0/gRT_0$$

then

$$G = (p_0S_{cr}/\sqrt{RT_0})[2/(k+1)]^{1/(k-1)}\sqrt{2kg/(k+1)}. \tag{6.35}$$

Equation (6.35) relates the gas discharge, G, to the area of the critical section of the nozzle, S_{cr}, and to the gas parameters p_0 and T_0 in the combustion chamber. The last, as we already know, are determined from the combustion calculations in the liquid propelled rocket.

It may be seen, incidentally, from expression (6.35) that the increase in pressure, p_0, leads to the proportional increase in discharge G, while the flow velocity w_{cr} remains unchanged. In this manner, the volumetric gas discharge $w_{cr}S_{cr}$, through the critical section, and consequently through the entire nozzle, at constant chamber temperature remains constant. This means that at an increased pressure, the discharge G increases only because of the increased density of the gas ρ in the stream. At constant temperature, increase in density leads to the proportional increase in pressure at all points within the stream. It should be noted that the increase in temperature T_0 at constant gas discharge rate also leads to the increase in pressure within the chamber and the nozzle.

It follows from expression (6.35) that at increased chamber pressure the value of the required discharge G may be obtained with smaller dimen-

sions of the critical section S_{cr}, resulting in the reduction of all other nozzle dimensions.

The composition of the combustion products influences the required area of the nozzle section by the values of the adiabatic exponent k and the gas constant R. The greater the amount of substances with a small number of atoms in the molecule and with low molecular weight, which are characterized by large values of k, in the products of combustion, the lower the gas discharge through a nozzle will be with a given area of section S_{cr}.

In many types of rocket motors (specifically, aircraft), it is necessary to regulate the thrust. This is most simply accomplished by varying fuel

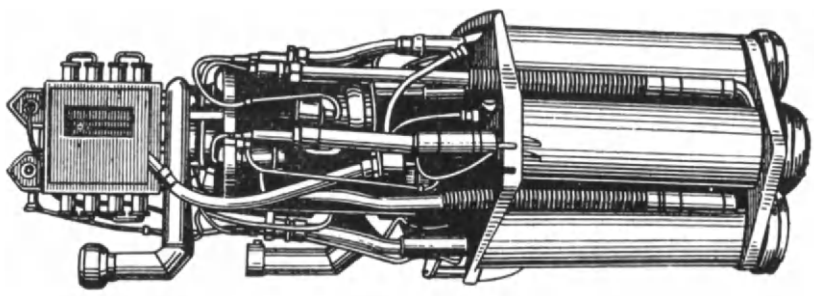


FIG. 6.10. External view of a four chamber motor.

discharge G. However, such regulation of thrust, especially if it is performed within wide limits of discharge variation, leads to a considerable change in chamber pressure. Reduction in the chamber pressure affects the combustion process adversely and, specifically, can give rise to pulsating burning. Reduction in chamber pressure also leads to the impairment of nozzle performance (see page 187). Therefore, if it is necessary to regulate thrust within wide limits, the motor nozzle must be designed with a variable critical section in order to maintain constant chamber pressure.

Mechanization of the nozzle (variation of the critical section during operation of the motor) represents an extremely difficult problem. Therefore, in practice, this problem is solved by designing motors with several combustion chambers. When it becomes necessary to reduce the thrust, one or several chambers are cut off and the remaining chambers continue to operate with normal or near normal combustion chamber pressures. Motors with two, and even four, chambers are known to be in existence (Fig. 6.10).

2. AREA OF THE NOZZLE EXIT SECTION

It has been shown above that the attainment of the maximum flow velocity, w_{max}, is related to the complete expansion of the gas to $p = 0$, and, consequently, requires an infinitely large area of the exit section of the

nozzle S_e. This is a very important condition, since, as we now see, for the attainment of maximum velocity the mere presence of a vacuum in the environment is insufficient. This is understandable, since for the attainment of maximum velocity it is necessary to cool the gas to $T = 0$ within the stream, i.e., to obtain a vacuum, not in the environment, but at the exit plane of the motor nozzle.

The effort to increase flow velocity may lead to the unreasonable increase in the dimensions and weight of the nozzle. For this reason, the maximum velocity remains unattainable in practice and the value of flow velocity is limited by the consideration of the reasonable dimensions of motor nozzles.

Let us find the connection between the dimensions of the exit section of the nozzle, S_e; flow velocity, w_e; and the gas exit pressure, p_e. In order to do this, we will express the flow velocity, w_e, making use of Eq. (6.9), in terms of heat content of gas in the combustion chamber and upon exit from the motor.

$$w_e = \sqrt{(2g/A)(H_0 - H_e)}.$$

Since

$$H_e = c_p T_e;$$
$$T_e = T_0(p_e/p_0)^{(k-1)/k};$$

and

$$c_p = kAR/(k - 1);$$

then the exit velocity

$$w_e = \sqrt{[2kgRT_0/(k - 1)][1 - (p_e/p_0)^{(k-1)/k}]}. \tag{6.36}$$

Further, let us express the time rate of discharge of the fuel, G, in terms of gas parameters at the exit plane of the nozzle

$$G = g\rho_e w_e S_e.$$

Taking into account that

$$\rho_e = \rho_0(p_e/p_0)^{1/k}$$

and also using expression (6.36), we find

$$G = g\rho_0 S_e(p_e/p_0)^{1/k}\sqrt{[2kgRT_0/(k - 1)][1 - (p_e/p_0)^{(k-1)/k}]}. \tag{6.37}$$

Equating the right side of Eq. (6.37) with the right side of Eq. (6.34), we get

$$\frac{S_e}{S_{cr}} = -\frac{\sqrt{[(k - 1)/2][2/(k + 1)]^{(k+1)/(k-1)}}}{\sqrt{(p_e/p_0)^{2/k} - (p_e/p_0)^{(k+1)/k}}}. \tag{6.38}$$

As is seen from formula (6.38), the pressure ratio p_0/p_e for a given gas with a constant adiabatic exponent k depends only on the ratio S_e/S_{cr}. The last we shall call the nozzle expansion ratio.

Relationship (6.38) enables determination of gas parameters at any section of the nozzle S_x. For this S_x must be substituted for S_e, and p_x for p_e in Eq. (6.38). Knowing the values of S_x and S_{cr}, p_x/p_0 can be calculated for a given section and then, with the aid of formulas for the adiabatic process, all the other flow parameters may be found for a section with an area S_x.

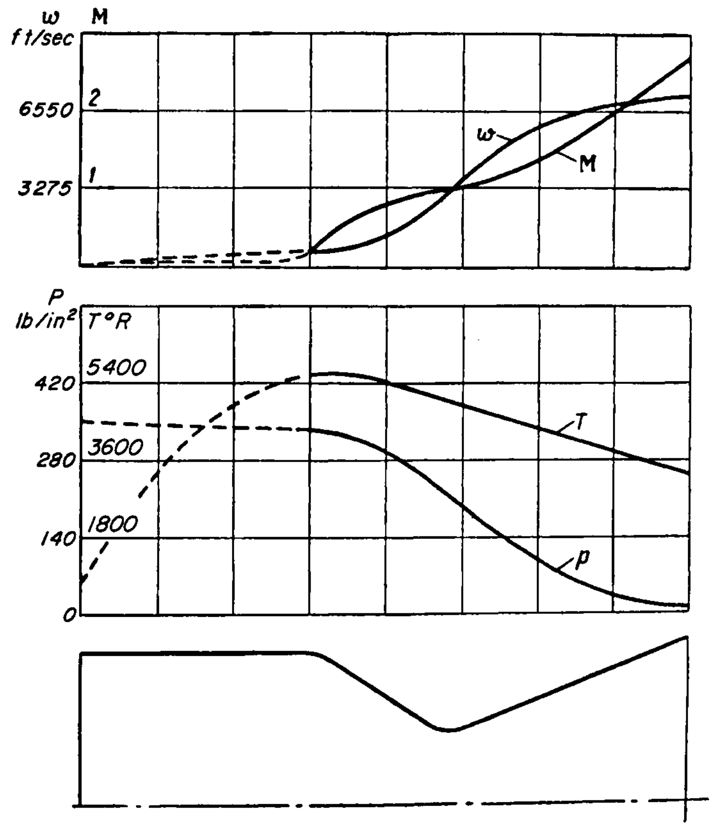


FIG. 6.11. Variation in gas flow parameters along the rocket motor operating on nitric acid and kerosene ($k = 1.2$).

Equation (6.38) may be used for the subsonic as well as for the supersonic sections of the nozzle. In this manner, it is possible to determine the changes in parameters of gas flow along the entire length of the nozzle.

A typical graph of parameter variations of gas flow along the gas duct of a rocket motor is shown in Fig. 6.11. The variation in gas parameters along the length of the combustion chamber is shown by dashed lines, inasmuch as these variables are not sufficiently clear. From this graph it

is seen that the more rapid variation in parameters (with the exception of the Mach number) takes place in the region of the critical section of the nozzle. In a divergent portion of the nozzle, parameters vary at an ever-decreasing rate.

3. Divergence of a Nozzle. Thermal Efficiency of a Rocket Motor

The expressions obtained above for the gas flow parameters permit an evaluation of the degree of recovery of the gas heat content in a rocket motor. In moving along the nozzle, the velocity, and consequently the kinetic energy, of the stream increases. The increase in kinetic energy is accompanied by a corresponding decrease in the heat content of the gas; the higher the speed attained by the gas at nozzle exit, the greater is the part of its heat content that is converted into useful kinetic energy.

The complete conversion of the entire heat energy into kinetic energy, i.e., the attainment of maximum velocity w_{max}, at the exit plane of the nozzle is impossible, as we have already seen. This would require building a nozzle with infinitely large dimensious of the exit plane. Therefore, in actual motors, it is necessary to limit the conversion to only a part of the total heat energy. The greater the expansion ratio, i.e., the ratio S_e/S_{cr}, the lower the temperature and pressure of gas at the nozzle exit are, and the smaller the portion of available energy that is lost in the gas stream leaving the nozzle.

The degree of conversion of heat energy into kinetic energy is defined by the thermal efficiency of the motor, η_t. It is natural to define thermal efficiency as the ratio of kinetic energy of the gas stream to heat content of the gas at nozzle entrance

$$\eta_t = A(w_e^2/2g)/H_0.$$

Since, according to the energy equation,

$$A(w_e^2/2g) = H_0 - H_e$$

then

$$\eta_t = (H_0 - H_e)/H_0 = 1 - H_e/H_0.$$

Ordinarily, thermal efficiency is expressed as a ratio of pressures at the entrance and exit of the nozzle p_0/p_e, or its reciprocal, p_e/p_0

$$\eta_t = 1 - (p_e/p_0)^{(k-1)/k}. \tag{6.39}$$

The relationship of thermal efficiency of the motor to pressure drop and adiabatic exponent is shown in Fig. 6.12.

We must also note that thermal efficiency may be expressed as the ratio of the square of exhaust velocity to the square of maximum velocity.

The pressure ratio, p_0/p_e, is directly connected with the term S_e/S_{cr} in Eq. (6.38). The interrelationship of these terms is influenced by the value of the adiabatic exponent of the combustion products. Fig. 6.13 shows the variation of p_0/p_e with S_e/S_{cr} for two values of k.

Relationship p_0/p_e is one of the basic parameters in designing the nozzle. The calculation of thermal efficiency by formula (6.39) is possible only when the pressure at the nozzle exit plane is equal to atmospheric pressure. Otherwise, other, more complex relationships must be applied, which take into account the effect on the operation of the motor of excessive static pressure or rarefication at the exit plane of the nozzle.

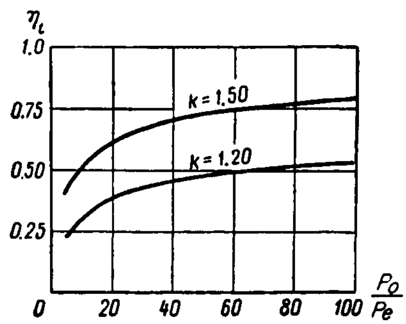


FIG. 6.12. Relationship of a rocket motor thermal efficiency to the pressure ratio of the nozzle p_0/p_e for the two values of k.

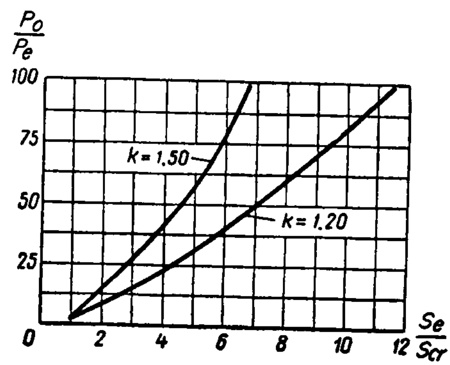


FIG. 6.13. Relationship of nozzle pressure ratio p_0/p_e to nozzle expansion ratio for two values of k.

The curves of Figs. 6.12 and 6.13 show that the quantity k has a pronounced effect on the dimensions of the nozzle and thermal efficiency. Thus, the increase in the adiabatic exponent of the combustion products results in a decrease of nozzle dimensions and increase in thermal efficiency. Therefore, an increase in k reflects advantageously on the dimensions of the postcritical part of the nozzle and on the degree of recovery of heat energy in the motor.

4. The Effect of Gas Recombination and Completion of Combustion on Gas Flow Along the Nozzle of a Rocket Motor

Until now, we have discussed the flow of gas of a constant composition along the nozzle. Partial recombination of dissociated gases and completion of combustion of fuel which was not burned in the chamber take place within the products of combustion during their movement along the nozzle, and is accompanied by heat release. Besides, the composition of gases changes in the nozzle. This directly affects the value of the gas constant

R and adiabatic exponent k. The change in the composition of gas, however, does not have a marked effect on the process of expansion. A most substantial effect is the release of an additional quantity of heat energy in the nozzle.

An energy equation derived earlier must be, in view of what has been said, correspondingly revised. We must introduce, into the over-all reserve of energy, chemical energy as well. In order to do this we must substitute for the heat content of the combustion products H in Eq. (6.5), the total heat content, E_t, and assume as before

$$E = E_t + Aw^2/2g = \text{constant}$$

or, for the two states corresponding to sections 0–0 and a–a

$$E_0 + Aw_0^2/2g = E_e + Aw_e^2/2g.$$

Equation (6.8) for calculating velocity will have the form

$$w_e = \sqrt{w_0^2 + 2g(E_0 - E_e)/A}.$$

Specifically, for exhaust velocity from a nozzle of a rocket motor, we will get an expression

$$w_e = \sqrt{2g(E_0 - E_e)/A}.$$

The total heat content of the combustion products in the chamber, $E_0 = E_g$, is determined from combustion process calculations. In the absence of heat losses

$$E_g = E_f.$$

The total heat content of the combustion products at nozzle exit plane, E_e, is determined in the same manner as the total heat content in the chamber, according to temperature and composition.

It is quite obvious that the exhaust velocity, taking into account recombination and completion of combustion, will be greater by comparison with the value of w_e, calculated on the assumption that the gas mixture flowing along a nozzle is unchanged. Under these conditions the thermal efficiency of a motor may be found by the formula

$$\eta_t = (E_0 - E_e)/K_G,$$

where K_G is the gravimetric heating value of the fuel.

5. Rocket Motor Nozzle Configurations

Liquid rocket motors known at the present time are characterized by the following basic parameters.

Combustion chamber pressure is within the limits of 25–50 lb/in.^{2}. Increase in the chamber pressure, all other conditions being equal, leads to the increase of the specific impulse and to the decrease in the dimensions of the nozzle critical section, and consequently all other dimensions, and to some decrease in the dimensions of the combustion chamber. Therefore, in the future, we may expect pressure increase in the combustion chamber of rocket motors.

The pressure ratio p_0/p_e, in contemporary motors, equals 20–40. Therefore, the expansion ratio S_e/S_{cr} equals 4–7. A further increase in p_0/p_e may be expected, both as a result of increased chamber pressure and of the reduction of pressure at the exit plane of the nozzle, which is desirable for long range motors and certain types of aircraft engines.

The elementary theory of the supersonic nozzle, examined above, shows that the velocity of the exhaust gas from the nozzle and the pressure at the exit plane do not depend on its configuration. It is necessary only that the nozzle first converge and then diverge. The change in nozzle profile, i.e., variation of section area with respect to length, reflects only on the change in velocity along the length. Pressure and velocity at exit are determined only by the ratio of the exit area S_e to the area of the critical section S_{cr}.

The geometric configuration of the nozzle must be chosen so as not to result in large velocity losses in the nozzle. At the same time, its surface area must be as small as possible. With an increase in surface area there is an increase both in the weight of the nozzle and the amount of heat transferred to the cooling liquid.

Losses in the nozzle result from friction, shocks, and vortices in the gas stream. Because of this, part of the kinetic energy of the directed motion is converted into unusable, for generation of thrust, heat energy. In this connection, let us examine separate sections of the nozzle profile.

In order to decrease friction losses, the nozzle entrance must be made as short as possible and, at the same time, smooth, to eliminate shock and turbulence.

In order to decrease surface area, the postcritical section should be made as short as possible, with a correspondingly increased expansion angle.

However, it may so happen that, with a large expansion angle, the gas stream flowing with high velocity will not be able to expand and fill the cross section of the nozzle. Flow separation from the walls will take place, which will result in sharply increased losses due to formation of vortices. Therefore, the expansion angle of a nozzle must be limited.

The allowable nozzle expansion angle, in liquid propelled rockets, is quite large. At the beginning of the nozzle divergent section it may be as high as 35°–40° (Fig. 6.14). Intensive reactions of combustion and recombination, which are accompanied by heat release, still take place here. For

this reason, the radial expansion of the stream is promoted and flow separation from the wall will not take place even at these large expansion angles.

At the exit of the nozzle the expansion angle must be reduced. Here, the reactions of recombination and combustion are considerably less intense, the supply of heat is reduced, and the conditions for expansion are less favorable. Besides, it is desirable to reduce the expansion angle at nozzle exit in order to reduce the radial component of stream velocity (Fig. 6.14), which does not contribute to thrust. It is desirable that the entire flow have a direction parallel to the axis of the rocket.

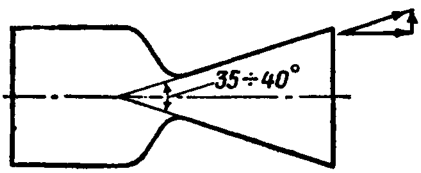


FIG. 6.14. Loss of kinetic energy due to radial component of gas flow.

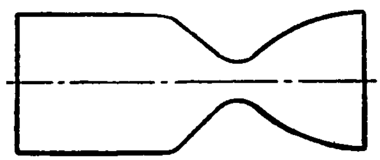


FIG. 6.15. A reasonable nozzle configuration with large expansion.

For technological considerations it is more convenient to make the divergent portion of the nozzle in the shape of a cone. However, in this case, in order to reduce losses, the nozzle expansion angle selected must be smaller (20°–25°), and the resulting nozzle will be long with a large surface area.

A more desirable nozzle is of the configuration shown in Fig. 6.15. Transition from a large expansion angle at the critical section to a small angle at the exit is accomplished by a circular arc. The nozzle length and losses due to radial flow are reduced to the absolute minimum. Such nozzle configuration, however, does not exclude the possibility of the occurrence of local losses. In order to eliminate losses completely, it is necessary for a nozzle profile to correspond to the trajectory of gas particles flowing near the walls. This trajectory is completely defined if the expansion ratio of the nozzle and distribution of velocities at the critical section and at the nozzle exit plane are given. Nozzles designed in this manner are called dynamically profiled nozzles.

Methods of profiling, known from aerodynamics, yield very long nozzles and, consequently, large wall surface areas. Because of this, dynamically profiled nozzles are not used in rocket motors.

D. Properties of Supersonic Nozzle. Characteristics of Rocket Motors

1. Properties of the Supersonic Nozzle and Regimes of its Operation

A supersonic nozzle has a series of interesting properties which reflect on the operation of a rocket motor.

The first of these properties is the independence of the parameters of combustion products in the chamber and nozzle of the motor from external atmospheric conditions. This is explained by the fact that at supersonic exhaust, atmospheric disturbances cannot penetrate into the nozzle against the flow which moves with a velocity greater than the speed of sound. Therefore, with the rise of the rocket to an altitude, the pressure at the nozzle exit plane (during constant chamber pressure) remains unchanged. On the other hand, if the motor operates with a variable fuel consumption, the pressure at the nozzle exit plane will vary following a change in chamber pressure, independently of atmospheric conditions.

The velocity at nozzle exit, with a given expansion ratio, will remain constant under any flow conditions if only the temperature and the gas composition in the combustion chamber remain constant. The exhaust velocity is also not affected by chamber pressure.

Let us examine the so-called design and nondesign regimes of nozzle operation.

When the conditions are such that the pressure at the nozzle exit is equal to atmospheric pressure, the nozzle is said to be operating in the design regime. If the pressure at the nozzle exit is not equal to atmospheric, then it is said to be operating in the nondesign regime.

There are two distinct nondesign regimes of nozzle operation: regime of underexpansion and regime of overexpansion.

When operating in the underexpansion regime, the pressure in the gas jet at nozzle exit is greater than atmospheric pressure. Underexpansion can take place, for instance, when the rocket reaches some altitude where the atmospheric pressure becomes less than the pressure at the nozzle exit plane. Gas emerging from the nozzle with an excess of pressure expands in the atmosphere and mixes with the external medium. During this external expansion, the flow velocity does not increase, for all practical purposes, due to formation of vortices along the flow boundary.

In the case of overexpansion, the pressure at the nozzle exit plane is less than external pressure. Under these conditions there is, naturally, a deceleration of flow aft of the nozzle, and the velocity is reduced to subsonic. The change to subsonic velocity, as will be shown below (page 229), is accompanied by a sudden change in pressure. As the external pressure increases, the pressure shocks approach the nozzle exit and, in the presence of a sufficiently large excess of atmospheric pressure, enter into the nozzle.

In this case, the normal operating regime of the nozzle is disturbed, since the formation of shocks within the nozzle leads to the flow separation at the walls and gives rise to strong vortices and large losses in kinetic energy. According to some data, the formation of shocks within the nozzle takes place if the external pressure is 2.5–5.5 times the pressure at the nozzle exit.

The overexpansion in the motor occurs when the motor operates at such altitudes and such regimes that the pressure of the surrounding atmosphere is greater than the pressure at nozzle exit (for instance, during the decrease

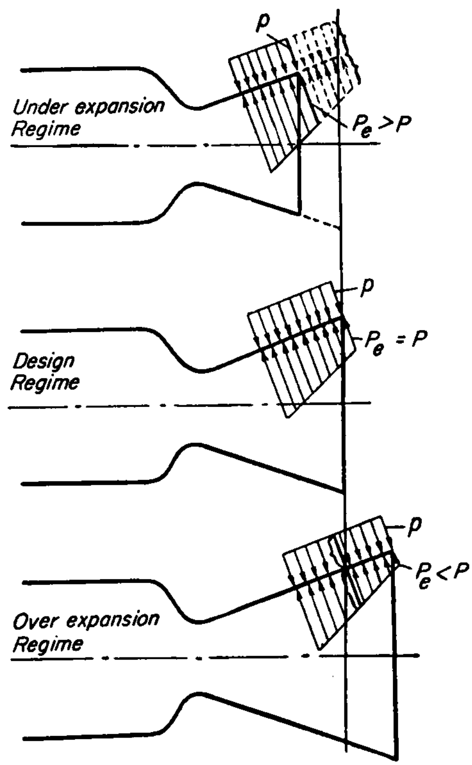


FIG. 6.16. Comparison of operating regimes of a nozzle.

of fuel consumption in the motor and the corresponding reduction in chamber pressure).

More interesting in the evaluation of nozzle operating regimes is the comparison of these regimes according to the magnitude of the thrust developed by the motor.

Let us follow through a comparison of three nozzles shown in Fig. 6.16. The first nozzle is shortened and the pressure, p_e, at its exit, will be greater then the pressure of the surrounding medium, p. The nozzle operates in a regime of underexpansion. The second nozzle has larger dimensions and operates in a design regime (pressure p_e is equal to the external pressure). Finally, the third nozzle has even larger dimensions, and the pressure at nozzle exit becomes less than the pressure of the surrounding medium. Here we have a regime of overexpansion.

If we return to formula (1.6)

$$P = mw + S_e(p_e - p),$$

then it is difficult to say in which of the three cases we will get greater thrust. In the first case, the exhaust velocity, w, will be less, but then the difference $p_e - p$ will have a positive value. In the third case, the exhaust velocity will be greater, but the difference $p_e - p$ becomes negative.

The posed problem, however, is solved rather simply, if it is remembered that the thrust is the resultant of pressure forces distributed along the internal and external surfaces of the motor chamber. Fig. 6.16 shows the distribution of pressures near the exit plane of the nozzle in the three cases under discussion. As we see, the enlargement of the nozzle during underexpansion will lead to the increase of the axial component of the pressure forces. These forces are shown in the first sketch by dashed lines. In the regime of overexpansion, the atmospheric pressure dominates over the pressure of the exhaust gases and at the right section of the nozzle yields a force directed opposite to thrust. Thus it becomes obvious that the optimum operating regime of the nozzle, with constant consumption, is the design regime.

2. Characteristics of Rocket Motors

The relation of thrust to the operating conditions of the motor is called the characteristic of the rocket motor. The basic characteristics are altitude and throttling characteristics.

The altitude characteristic is the dependence of the motor thrust on the altitude at which it operates with constant fuel consumption. In order to obtain the altitude characteristic, let us examine the formula for the thrust of the motor, Eq. (1.7)

$$P = P_0 + S_e(p_0 - p_h).$$

During constant fuel consumption G, the value P_0 also remains constant. Therefore, the only variable in the thrust formula is the pressure p_h. The total variation (increase) of the motor thrust with ascent from the ground, where $p_h = 14.7$ lb/in.^{2}, to an altitude where the atmospheric pressure can be considered as equal to zero, is $14.7\ S_e$ lb, if the area of the nozzle exit plane is expressed in in.^{2}. Under these conditions (in a vacuum),

$$P_{\text{vac}} = P_0 + S_e p_0$$

and thrust at sea level, $(p_h = p_0)$

$$P_{\text{sl}} = P_0.$$

The altitude characteristics of motors will look like the curves shown in Fig. 6.17.

It will be interesting to determine the thrusts that motors will have with the same fuel consumption, G, the same area of the nozzle critical sections, and the same chamber pressures, but different areas of the nozzle exit planes, S_e, and, consequently, different exit pressures, p_e.

From considerations presented in the previous section, it is obvious that a motor with a high design altitude, h (for which $p_e = p_h$), i.e., having a large exit plane area S_e, will develop greater thrust in the upper regions

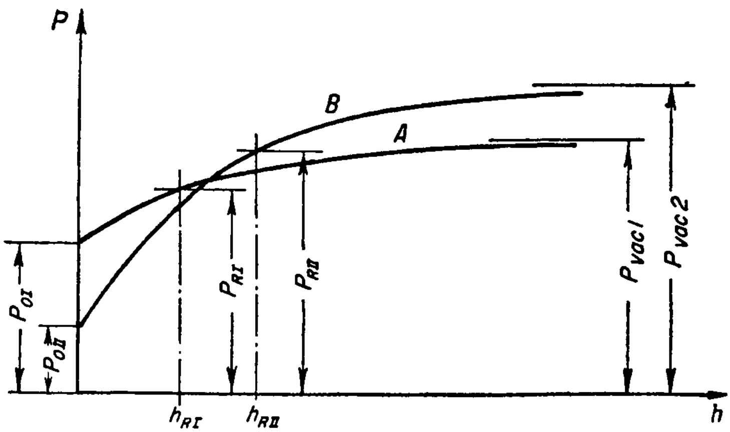


FIG. 6.17. Altitude characteristic of a rocket motor.

of the atmosphere (curve B in Fig. 6.17). A motor having lower design altitude will develop a greater thrust in lower regions of the atmosphere (curve A).

That is why many motors of the long range and anti-aircraft rockets, which operate for the most part at high altitudes, and also aircraft engines, are designed in such a way that the pressure at the nozzle exit is lower than atmospheric pressure at sea level (10–11 lb/in.^{2}). Nozzles of such motors are called high altitude nozzles. The relative change in thrust during ascent to an altitude depends on the design altitude of the nozzle, and for contemporary motors constitutes 12–15%.

The other characteristic of a rocket motor is the throttling characteristic.

The dependence of thrust on the variation of fuel consumption at constant altitude is called the throttling characteristic. Generally speaking, variation in fuel consumption in the combustion chamber of a motor varies the temperature and the value of the gas constant. However, these variations usually are not taken into account when determining the throttling characteristic.

The equation for throttling characteristic can be easily obtained from

the equation of thrust of a rocket motor. We take into account that for a given motor the exhaust velocity w_e does not depend (at constant R and T_0 in the chamber) upon consumption G, and the pressure at nozzle exit drops in proportion to consumption. The last statement stems from the fact that the pressure ratio, p_e/p_0, for a nozzle of given dimensions, i.e., with a given ratio S_e/S_{cr}, is constant. In this connection, the pressure at nozzle exit can be expressed as a constant percentage of chamber pressure

$$p_e = xp_0.$$

The chamber pressure itself, with accepted allowable constancy of T_0 and R, is proportional to fuel consumption G. Consequently,

$$p_e = xG$$

from which we get

$$p_e = p_{e\text{nom}} G/G_{\text{nom}}$$

where G_{nom} is the nominal (design) fuel consumption for a given motor; and $p_{e\text{nom}}$ is the nozzle exit pressure at this consumption.

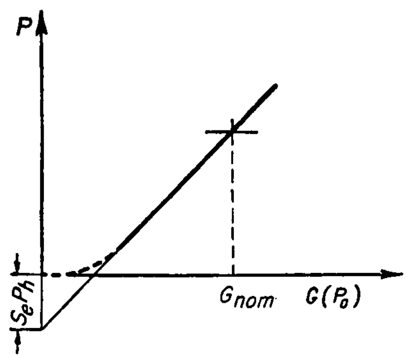


FIG. 6.18. Throttling characteristic of a rocket motor.

Taking into account the remarks made above, expression (1.6) for the thrust of a motor

$$P = Gw_e/g + S_e(p_e - p_h)$$

can be written in the following manner:

$$P = G(w_e/g + S_e\, p_{e\,\text{nom}}/G_{\text{nom}}) - S_e p_h. \tag{6.40}$$

The obtained equation of the throttling characteristic represents an equation of a straight line, intersecting the ordinate at a distance—$S_e p_h$ (Fig. 6.18). The slope of this line is determined basically by the value w_e, and increases with an increase in S_e.

In certain cases, making use of the fact that the chamber pressure is proportional to fuel consumption, the throttling characteristic is based not on consumption but on the value of chamber pressure. This is particularly convenient in determining experimental characteristics obtained during motor testing, since the chamber pressure is much easier to measure than fuel consumption.

The linear property of the throttling characteristic determined by Eq. (6.40) is well confirmed experimentally, with fuel consumptions greater than nominal, and with lower consumptions when the overexpansion is not pronounced enough to allow pressure shocks to enter the nozzle. Therefore, the throttling characteristic is linear down to consumptions equal to 30–40% of nominal. At lower consumptions, the relationship in Eq. (6.40) becomes invalid and the throttling characteristic takes the approximate form shown in Fig. 6.18 by a dotted line. At low chamber pressure a considerable increase begins in the degree of dissociation of combustion products and there is a corresponding change in chamber temperature, which also reflects on the throttling characteristic of a motor.

E. Cooling of Liquid Rocket Motors

1. Heat Exchange in Rocket Motors

The high temperature of the combustion products and the high velocity of flow in the chamber, especially along the nozzle, lead to rather intensive heat transfer from the gas to the walls of a rocket motor.

This would seem to contradict what has been said before with respect to the processes taking place in the rocket motor. Before, we have considered gas expansion as adiabatic, i.e., taking place without heat exchange between the gas and the walls. Now we speak of intensive heat transfer to the walls. However, the crux of the matter is that heat energy, transferred to the walls, constitutes only a small part of the total heat content of the gas and cannot substantially affect the character of flow. However, the absolute value of this small part constitutes a very large quantity of heat which, without suitable cooling, cannot be absorbed by the walls without burning through and destruction of the walls.

The heat transfer from gas to the wall takes place in two ways; by convection and by radiation.

In contemporary rocket motors heat exchange by convection plays a basic role. In substance, it is that particles (molecules and atoms) of gas, during their random motion and flow along the walls, exchange heat energy with the walls. In this way, only a layer of gas flowing next to the wall, the so-called interface layer, participates in convection heat exchange. The dimensions of this layer are close to those of the boundary layer.

The amount of heat transferred to the wall depends on the number of molecule collisions against the wall and the difference in temperatures between the gas and the wall. When the temperatures are equal the gas, on the average, receives as much heat from the wall as it transfers to it.

The effect of the number of molecular collisions with the wall on heat exchange is determined by the coefficient of heat transfer, α, which represents the amount of heat transferred to a unit wall surface in a unit time with a temperature difference between the wall and the gas of one degree. The number of molecular collisions of a moving gas against the wall is proportional to the number of molecules in a volume of gas passing by the wall (i.e., density of the gas ρ) and gas velocity w.

Therefore, the number of collisions, which means the amount of heat transferred to the wall, is proportional to the product ρw. This last term represents the specific mass discharge, which is equal to the ratio of the mass rate of discharge to the area of the passage section

$$\rho w = G/gs.$$

The amount of heat transferred by a gas to a unit wall surface in a unit time, i.e., unit heat rate, is

$$q = \alpha_g(T_{\text{gas}} - T_w), \tag{6.41}$$

where T_w is the temperature of the wall surface wetted by the gas; T_{gas} is the gas temperature; and α_g is the heat transfer coefficient from gas to the wall.

Let us note here that since the gas flow near the wall is decelerated, it has at all sections of the nozzle temperatures close to the temperature of stagnation and the chamber temperature T_0. The wall temperature is considerably less than the gas temperature. As a result of this the temperature difference $T_{\text{gas}} - T_w$ is considerable. On the other hand, because of high pressures and high velocities the specific mass discharge of gas is high and the coefficient of heat transfer α_g becomes very high in liquid propelled rockets. All of these circumstances result in a condition in which the walls of rocket motors absorb a considerably greater quantity of heat then the walls of any other heat engine.

Both coefficient α_g and the temperature difference $T_{\text{gas}} - T_w$ vary along the length of the motor. The coefficient of heat transfer α_g is the more strongly affected of the two, due to change in specific mass discharge ρw along the length of the motor. The highest values of ρw and α_g are reached in the critical section of the nozzle. Here, also, is the maximum value of the heat flow q (Fig. 6.19). In contemporary motors the value of convection unit heat rate in the critical section reaches $37 \cdot 10^5$ Btu/ft^2/hr.

During the increase in the chamber pressure and temperature, the heat flow toward the nozzle walls increases considerably.

The heat flow due to radiation is added to the convection heat flow. The amount of radiation heat flow is considerably less than heat flow by convection. The greatest amount of radiation takes place where gas temperature is high, i.e., in the combustion chamber. Here, radiation heat flow is equal to (55–75) · 10^4 Btu/ft^2/hr. The amount of radiation is less in the critical section because of the lowered gas temperature.

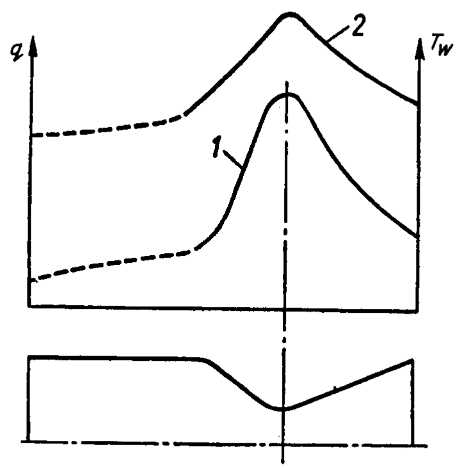


FIG. 6.19. Distribution of heat flow and temperature in a hot wall along the length of the motor: *1*, heat flow across the wall; and *2*, temperature of a hot wall.

The heat flow received by the walls within the chamber must be transmitted to the surface of the wall in contact with the cooling liquid. At the same time, the wall surfaces absorbing and giving up heat have temperatures related by the equation of thermal conductivity

$$T_w - T_{wc} = q(\delta/\lambda) \tag{6.42}$$

where T_{wc} is the temperature of the wall surface wetted by cooling liquid; δ is the wall thickness; and λ is the coefficient of thermal conductivity of the wall material.

The quantity δ/λ, by its sense, is the thermal resistance of the wall. The thicker the wall and the poorer the heat transmission of the material, the higher the thermal resistance of the wall.

The heat flow must then be transmitted to the cooling liquid, having a temperature T_c. The amount of heat flow is related to the temperatures of the surfaces wetted by the coolant and the liquid by the equation

$$q = \alpha_c(T_{wc} - T_c) \tag{6.43}$$

where α_c is the coefficient of heat transmission from the wall to the cooling liquid.

Eliminating T_{wc} from Eqs. (6.24) and (6.43) we will get, for the wall surface temperature in contact with the gas, the following expression:

$$T_w = T_c + q(1/\alpha_c + \delta/\lambda). \tag{6.44}$$

Temperature variation during heat transfer from gas to wall and then to the cooling liquid is shown in Fig. 6.20.

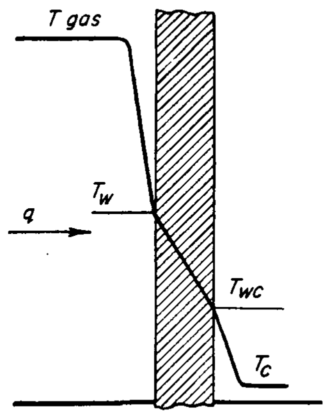


FIG. 6.20. Temperature distribution in the boundary gas layer, in the wall, and in the cooling liquid.

The temperature of the wall, T_w, must not exceed that upper limit at which the wall material noticeably loses its structural properties. In this sense, it is desirable for the value T_w to be as low as possible. Temperature T_w may be limited by increasing the cofficient of heat transfer and by reducing the thermal resistance of the wall.

In addition, the coefficient of heat transfer to the liquid must be large, because the excessive rise of T_{wc} above the boiling point of the cooling liquid cannot be allowed in the cooling system of the motor. A slight increase above the boiling temperature is allowable, and even desirable, since it increases heat transfer coefficient α_c. The heat transfer is intensified as a result of steam bubble formation at the wall surface. The bubbles are continually washed away by the flow of the cooling liquid and are condensed in the cooler liquid, giving up the heat received from the wall.

The values of heat transfer coefficients α_c and α_g depend on the mass flow of the wall wetting liquid. However, since the density of the liquid is practically constant, the basic factor determining the value α_c is the flow velocity of the cooling liquid. The intensive cooling of chamber walls

and nozzle in liquid propelled rockets requires high values of coefficient α_c. In this connection, the velocity of liquid in a cooling tract must be considerable.

2. Motor Cooling Design

The problem of motor cooling consists in preventing the wall temperature from exceeding a certain limit during the entire operation of the motor. This problem is solved differently in various types of rocket motors.

In powder motors there is no liquid which could cool the motor. In this case, therefore, the operating time of the motor is limited in order not to overheat its walls to a temperature at which they lose structural integrity. In order to reduce the rate of wall heating and increase possible operating time of a powder motor, heat insulating liners, which are deposited on the inner surfaces of the motor, are used. A similar method is used when it is desirable to prevent overheating of the walls of short duration liquid rocket motors.

In liquid rocket motors, however, reliable cooling is made more difficult by higher combustion chamber temperatures. On the other hand, cooling design is facilitated by the fact that protection of the walls may be obtained by means of a plentiful supply, to the internal wall surfaces, of one of the fuel components. It is true that such use of the fuel components reduces the efficiency of combustion and consequently lowers the impulse of the motor. However, it is still possible to use it in designs requiring the utmost simplicity.

Most often the cooling of a liquid propelled rocket is accomplished by one of the fuel components, circulated through the cooling jacket. This method of cooling is quite advantageous. It does not require any additional liquids and location of containers for their storage in the rocket, i.e., it permits a motor installation of minimum weight. Besides, the heat absorbed by the cooling liquid is returned to the combustion chamber and is not uselessly lost.

However, there are certain difficulties in designing motor cooling in this manner. They are determined, primarily, by the properties and the weight consumptions of components. It may be that one of the components, for instance, liquid oxygen, is unsuitable as a cooling liquid. There may not be enough of the second component for absorption of the total heat transferred by the wall of the motor during its operation.

Another difficulty consists in the fact that in order to obtain the necessary value of heat transfer coefficient α_c it is necessary to create high velocities of the cooling liquid in the jacket. Therefore, the gap between jacket walls must be very small. It is especially difficult to insure the required velocity of the cooling liquid in motors of small dimensions. For

instance, in a motor of the booster rocket shown in Fig. 5.6 the width of the gap at the critical section had to be made only 0.25 in. Because of this, a machined insert had to be added into the cooling jacket and the space maintained by the installation of spacers. In large motors, the size of the necessary gap increases and technological difficulties are partly removed.

It must be said that the high velocities in the cooling tract are accompanied by hydraulic losses, and necessitate an increase in the delivery pressure of the cooling component. This, in turn, complicates the operation of the delivery system and increases its weight.

Returning to expression (6.44), we see that the lowering of T_w may be achieved not only by increasing the velocity of the cooling liquid, with a corresponding increase in α_c, but also by decreasing the thermal resistance of the wall δ/λ. From this, it follows that the thickness δ must be as small as possible, and the coefficient of thermal conductivity of the wall material λ, as great as possible.

In increasing the thickness δ, temperature T_w is increased so sharply that the wall strength is reduced. With the increase of heat flow to the wall, the wall must be made thinner and thinner. Therefore, the environment of the wall material is peculiar in that at high rates of heat transfer it is impossible to increase the strength of the wall by increasing its thickness.

In order to decrease the temperature of a gas surface, the inner wall may be fabricated from a material with high heat conductivity, for instance, copper or aluminum. However, it must be considered that heat conductive materials, as a rule, are less heat resistant.

The indicated conditions, especially the necessity for making a thin inner wall, lead to great difficulties in fabricating the motor chamber and the nozzle.

The inner wall of the motor is subjected to considerable force due to the difference in pressures between the cooling liquid in the jacket cavity and the gases in the chamber and nozzle. This difference in pressures is especially large at the end of the nozzle. Therefore, in order to obtain a sufficiently strong inner wall, it is necessary to reinforce it by some means. Serving as an example may be the layout of a motor in which the inner jacket has been designed in the form of spirally wound copper tubes of a small diameter for the passage of the cooling liquid.

Small diameter tubes can withstand the difference in liquid and gas pressures well. The internal chamber pressure is taken by a thick steel jacket, protected from the high temperature by a ring of tubes. Difficulties of motor design connected with the presence of large heat flows force a search for methods of reducing them. The basic method consists of supplying an excess of combustible into a layer adjacent to the wall. For instance, in long range rockets, the combustible is supplied into the interface layer

by several rows of ports (Fig. 5.8). Something like a curtain of combustible, protecting the wall, is thereby created. The action of the combustible curtain consists not only in heating and evaporating, by which it takes heat away from the wall, but also in lowering the temperature of that layer of gas which is directly in contact with the walls of the motor. The protective action of the layer overrich with combustible is maintained for some time. Gradually, however, the boundary layer is diluted by the main flow of the combustion products and its temperature is raised. Therefore, in large motors it is necessary to have several rows of ports supplying excess combustible.

The introduction of combustible through radial ports is not expeditious, since the greater part of it would pass into the gas stream at some distance from the walls. A more reasonable method of combustible supply is in a motor whose chamber is shown in Fig. 5.6. Here, the combustible, through small openings, enters a narrow gap between the head and the chamber and moves along the walls without immediately mixing with the main gas mass.

The expenditure of combustible for the creation of the curtain certainly lowers the specific impulse of the motor, since part of the combustible is not fully burned and carries chemical energy with it away from the motor. Therefore, a combined system of cooling is often used in motors in which, by moderate internal cooling, the heat flow to the wall is reduced, and the heat is removed by external cooling with comparatively small hydraulic losses in the cooling tract.

A considerable reduction in the expenditure of combustible for internal cooling may be expected if a porous material is used for the internal chamber walls to make the so-called sweating wall,* which permits a uniform passage of a very small amount of liquid along its entire surface, forming a continuous curtain.

There are also proposals involving the cooling of the motor by means of water, which would not only be boiled, but completely evaporated into superheated steam. This steam may be used in the turbine of the turbopump and then condensed in the heat exchanger, cooled by one of the components. Such cooling systems can find applications only in very large motors.

* Translator's note: evaporative cooling.

VII. Forces and Moments Acting on the Rocket in Flight

A. System of Forces Acting on the Rocket in Flight and the Differential Equation of Motion

1. Coordinates Determining the Position of the Rocket in Space

The position of a rocket in space is determined, first of all, by the three coordinates of its center of gravity x, y, and z, in the so-called earth coordinate system. The origin of this system is taken at the launching point of

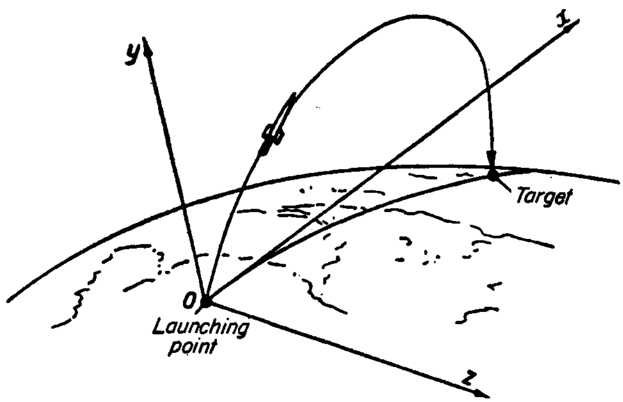


Fig. 7.1. Ground system of coordinates.

the rocket. For long range rockets the x axis is considered to be a straight line, tangent to the arc of the great circle connecting start with the target (Fig. 7.1). Axis y is directed upward, and axis z is perpendicular to the first two axes.

Let us also introduce the so-called coupled or ballistic coordinate system. We will place its origin at the center of gravity of the rocket and relate the axes x', y', and z' to the characteristic elements of a rocket. We will direct axis x' along the axis of the rocket and will call it the longitudinal axis. We will place axis y' in a plane of symmetry of the rocket and will direct it perpendicular to axis x', and axis z' we will direct from the left stabilizer to the right. We will call axis y' the normal and axis z' the transverse axis (Fig. 7.2).

For long range rockets, the plane x'–y' of the rocket in a trajectory

coincides or, more exactly, almost coincides, with the plane xy of the earth system.

In order to orient the position of a rocket fully as a solid body in space, in addition to coordinates x, y, and z, we will also introduce three angles

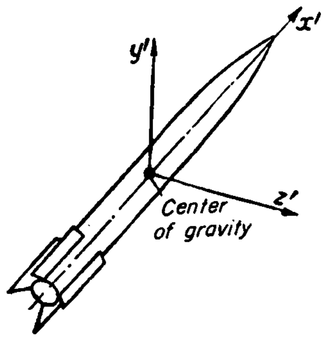


Fig. 7.2. Flight system of coordinates.

which determine the direction of the ballistic and the earth systems of coordinates.

The angle between rocket axis x' and plane x–z, i.e., the angle of inclination of the rocket axis with respect to the initial horizon, we will designate

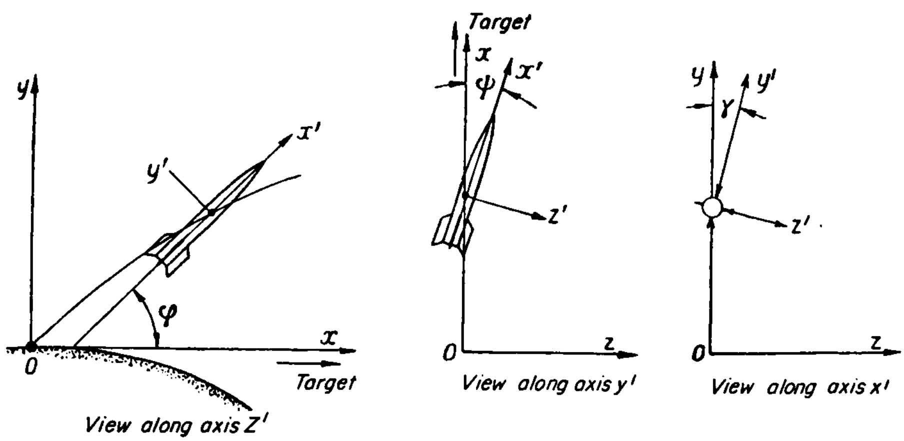


Fig. 7.3. Angles of pitch φ, yaw ψ, and roll γ.

by φ. This angle is called the pitch angle of the rocket (Fig. 7.3). The angle between the axis of the rocket and the plane x–y we will designate by ψ. This is called the yaw angle. It represents the deviation of the rocket axis from the verticle plane passing through the target.

Finally, let us introduce the last angle determining the rotation of the rocket frame about the longitudinal axis; roll angle γ between the axis y' and the plane x–y.

In this manner, three coordinates, x, y, and z, and three angles, φ, ψ, and γ (see Fig. 7.3) completely define the position of a rocket in space.

2. Forces Acting on a Rocket

Let us examine the forces acting on the rocket in flight.

In Fig. 7.4 is shown a rocket and the trajectory of its center of gravity. The vector of flight velocity, v, is directed along the tangent to the trajectory.

The rocket axis, generally speaking, does not coincide with the velocity vector v, but forms with it angle α, called the angle of attack. Ordinarily, in a guided trajectory, angle α is comparatively small ($\alpha < 6°$).

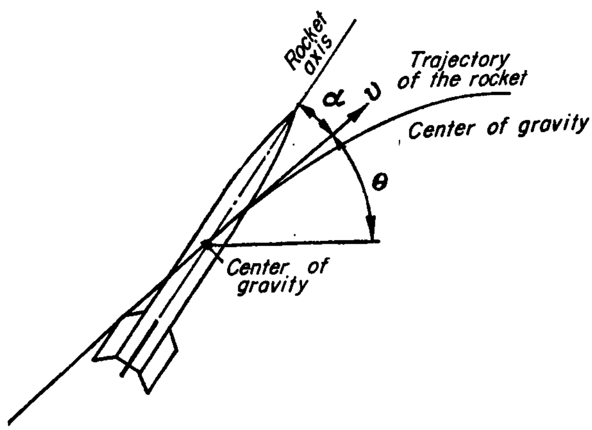


Fig. 7.4. Rocket angle of attack α and trajectory angle with respect to the horizon θ.

Any body moving through air is subjected to the action of a system of forces distributed along its surface. The resultant of these forces is called the total aerodynamic force. The component of aerodynamic force along the tangent to the trajectory of the center of gravity of the body (or the projection of the total force onto the direction of the velocity vector) is called the frontal resistance force or, simply, drag. This force is always opposite to the direction of motion.

The drag force is usually designated by D.

The component of the total aerodynamic force along the normal to the trajectory (or its projection on a normal to the direction of velocity) is called the lift force and is designated by L. Fig. 7.5 shows forces D and L; also the thrust, P, and gravity weight force, Mg.

The system of aerodynamic forces distributed along the surface of a rocket, just like any system of forces, may be reduced to a force (in this case, the total aerodynamic force) according to the laws of mechanics, and a moment whose magnitude depends upon the point of force application.

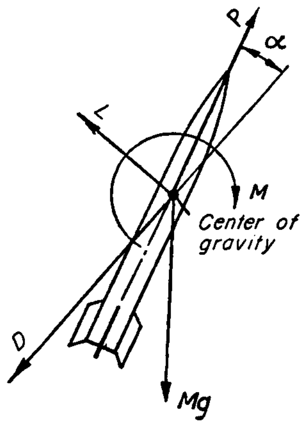


FIG. 7.5. Forces and moments acting on a rocket (with the aerodynamic forces referred to the center of gravity).

Referring the force system to the center of gravity of the rocket we will get, aside from the mentioned forces D and L, the resultant moment M (Fig. 7.5). Ordinarily, it is considered as a summation of stabilizing and damping moments

$$M = M_{st} + M_{d}.$$

These quantities will be discussed in more detail later.

If the rocket has steering mechanisms (aerodynamic fins, jet vanes, or gimbaled motor chamber) a system of forces due to steering mechanisms must be added to the system of forces shown in Fig. 7.5. These forces, in the form of two force components and a moment, are applied at the hinge of the steering mechanism located at a distance x'_v from the center of gravity of the rocket.

One of the components, D_v (Fig. 7.6), is directed along the rocket axis; the second, L_v, is perpendicular to it.

The first force, by analogy with aerodynamic forces, is called the vane drag and is not a steering force. The second force, L_v, is called the vane lift and constitutes the steering force.

Moment M_h is called the vane hinge moment.

Fig. 7.7 shows the full force system acting on the rocket.

In order to derive the equation of motion let us use D'Alembert's principle and introduce inertial forces.

Tangential acceleration of a rocket (acceleration along the tangent to the trajectory) will be

$$dv/dt = \dot{v}.$$

The corresponding inertial force has a direction opposite to the acceleration and is equal to $M\dot{v}$.

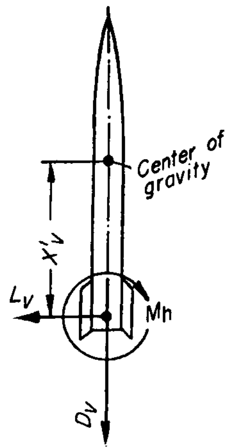


FIG. 7.6. Steering forces.

The normal acceleration, depending on the curvature of the trajectory, is equal, as is known, to v^2/R, where R is the radius of the trajectory curvature.

However,

$$1/R = d\theta/ds = d\theta/dt \times dt/ds = \dot{\theta}/v,$$

where θ is the angle of inclination of the trajectory (Fig. 7.3).

Therefore, normal acceleration directed toward the center of curvature becomes equal to $v\dot{\theta}$. The interial force directed in an opposite direction is equal to $Mv\dot{\theta}$. Force $Mv\dot{\theta}$ is shown in Fig. 7.7.

Finally, let us introduce the moment of inertia, equal to the differential, with respect to time, of the moment of momentum of the rocket, and opposed to the angular acceleration $\ddot{\varphi}$ i.e., clockwise (Fig. 7.7),

$$[J(\theta + \alpha)^{\cdot}]^{\cdot} = (J\dot{\varphi})^{\cdot}.$$

The moment of inertia J of the rocket about the central axis perpendicular to the plane of the trajectory is also a function of time. Therefore, moment $(J\dot{\varphi})^{\cdot}$ can be written as a sum

$$(J\dot{\varphi})^{\cdot} = J\ddot{\varphi} + \dot{J}\dot{\varphi}.$$

The first addend is the moment of inertia. The second addend is the result in inertial moment change of the rocket with time. This addend, proportional to the angular velocity, may be included in the aerodynamic

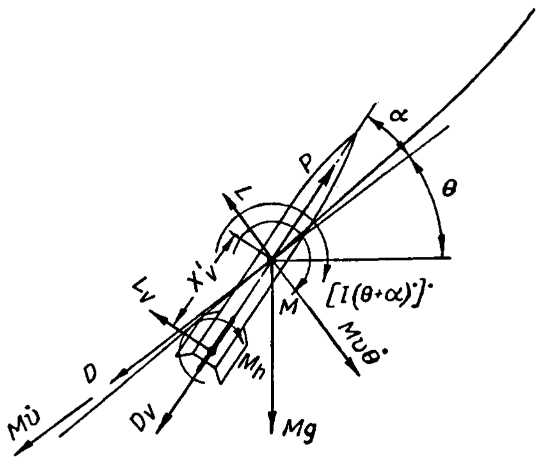


FIG. 7.7. Derivation of equations of motion.

damping moment. In derivation of the equation of motion we will add it to the previously examined moment M. In analyzing the damping moments further on, we will return to this question again.

3. Equation of Motion

Projecting all forces applied to the rocket onto the tangent to the flight trajectory we will get

$$M\dot{v} + D - (P - D_v) \cos \alpha + Mg \sin \theta + L_v \sin \alpha = 0.$$

Because of the small angle α, $\cos \alpha \approx 1$. Neglecting $L_v \sin \alpha$, we will get

$$\dot{v} = (P - D - D_v)M^{-1} - g \sin \theta. \quad (7.1)$$

Now we will project all forces onto the normal to the flight trajectory

$$L + L_v \cos \alpha - Mv\dot{\theta} + (P - D_v) \sin \alpha - Mg \cos \theta = 0.$$

With small values of α

$$\dot{\theta} = (1/Mv)[(P - D_v)\alpha + L + L_v] - (g/v) \cos \theta. \quad (7.2)$$

In these equations the term g is the acceleration due to earth's gravity at flight altitude. Finally, summing up the moments about the center of gravity we get

$$J\ddot{\varphi} + M + L_vD_v + M_h = 0. \quad (7.3)$$

The system of equations (7.1), (7.2), and (7.3) describes the motion of a rocket in one plane only and does not account for the possibility of three-dimensional motion. Nevertheless, these equations can be freely used in any case for determining the parameters of a rocket trajectory, especially a ballistic rocket.

In cases in which the flight trajectory is a spatial curve, as, for instance, that of an anti-aircraft guided rocket pursuing a maneuvering aircraft, the system of equations becomes considerably more complex. In developing this system it is necessary to examine forces and moments acting along three coordinate axes and introduce additional coordinates determining the position of a rocket. Instead of three equations we will get six.

It is important to note that to the equations of motion of a guided rocket, equations of guidance which relate steering forces and vane angles to time, or motion parameters, must be added. The number of these equations and their form depend on the steering system and the method of guidance and steering of the rocket.

In an ideal case for a long range ballistic rocket, the steering equation looks very simple. To Eqs. (7.1) and (7.3) we must add the condition

$$\varphi = \theta + \alpha = f(t).$$

where $f(t)$ is the given time function.

The steering condition is determined by the fact that during the guided portion of flight the rocket is oriented in a definite manner relative to a gyroscope whose axis is fixed in direction. We will discuss the question of steering more fully in Chapter IX.

In order to be able to integrate the equations of motion it is necessary to know how the variables change with time and, in general, on what they depend.

It is very clear that, for a given rocket, the aerodynamic forces depend on the velocity and flight altitude. The motor thrust does not remain constant and changes (if for no other reason than that along the flight trajectory the external atmospheric pressure, which enters into the thrust expression, changes). The rocket mass diminishes with time, corresponding to the expenditure of fuel. Finally, even the acceleration of gravity, g (if we are speaking of high altitudes), must be considered as a variable quantity.

All of these questions we will examine below. Let us start with the properties of the atmosphere.

B. Earth's Atmosphere and Its Properties

Let us examine the properties of the atmosphere, keeping in mind that the magnitudes of aerodynamic forces acting on the rocket in flight depend on these properties.

The basic parameter of the atmosphere, affecting aerodynamic forces, is the air density, ρ. Temperature T also has some effect, inasmuch as a change in temperature, as we already know, changes the speed of sound

$$c = \sqrt{kgRT},$$

and, depending upon the ratio of flight velocity to sound velocity (Mach number), changes the character of the flow, which changes the magnitude of aerodynamic forces, as will be shown below.

It may also be said that the velocity of sound depends on the suspected variation in the chemical composition of the atmosphere with altitude. This will reflect on the value of the gas constant R and adiabatic exponent k. However, within the altitude limits for which the aerodynamic forces must be taken into consideration in calculating rocket trajectories, i.e., altitudes of 60–75 miles, there is no evidence of any substantial change in the composition of the atmosphere.

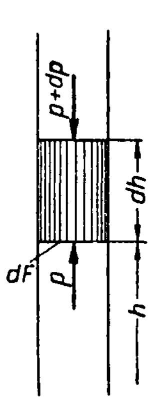


FIG. 7.8. Change in air pressure with altitude.

Therefore, the magnitude of aerodynamic forces is primarily affected by parameters of the air state—density ρ, temperature T, and, related to them, pressure p. Let us examine the dependence of these quantities on altitude.

Pressure p must decrease with increase in altitude, inasmuch as the amount of pressure is determined by the weight of a column of air. Let us develop an equilibrium equation of an elemental column with the base area equal to dF, and a height dh, isolated from the atmosphere (Fig. 7.8).

Pressure p acts on the column from below. Pressure $p + dp$ acts on the column from above, corresponding to the addition of height increment dh. Let γ be the specific weight of air at altitude h. The equilibrium condition will be as follows:

$$dpdF + \gamma dhdF = 0$$

from which

$$dp/dh = -\gamma$$

or

$$dp/dh = -g\rho. \tag{7.4}$$

The derivative dp/dh is negative, which indicates reduction in pressure with increase in altitude. It is easy to realize that in the absence of an external heat supply the temperature will also drop.

During an upward displacement of some mass of air, an expansion takes place with a corresponding lowering of gas temperature. Conversely, air being displaced downward compresses and its temperature is raised. In this manner a temperature equilibrium is established in the atmosphere in which the lower layers will have a higher temperature then the upper layers. Certainly, this assertion is correct only to the extent that the heat supply to the upper layers from without (by means of solar radiation) may be considered small.

The temperature distribution with altitude may be considered as depending on a thermodynamic process which corresponds to the above-mentioned expansion and compression of the gas during its slow vertical mixing with conservation of thermal equilibrium. Suppose that this is a polytropic expansion with an exponent n which is independent of the altitude h

$$p/p_0 = (\rho/\rho_0)^n, \tag{7.5}$$

where p_0 and ρ_0 are the pressure and density of the air at sea level.

Having accepted this law we have the means of establishing, in the first approximation, the law of change of pressure, density, and temperature of air with altitude.

Let us find, with the aid of Eq. (7.5), an expression for density ρ, and let us substitute this expression into the equilibrium equation, Eq. (7.4). We will get

$$dp/dh = -g\rho_0(p/p_0)^{1/n}.$$

Separating the variables

$$p^{-1/n}dp = -g\rho_0 p_0^{-1/n}dh$$

from which, after integration, we find

$$[n/(n-1)]p^{(n-1)/n} = -g\rho_0 p_0^{-1/n}h + C.$$

The constant C is determined from the condition that when $h = 0$ the value of $p = p_0$. Consequently,

$$C = [n/(n-1)]p_0^{(n-1)/n}$$

and

$$p^{(n-1)/n} = p_0^{(n-1)/n} - g\rho_0 p_0^{-1/n} \times [(n-1)/n]h$$

or

$$p = p_0\left[1 - g\frac{\rho_0}{p_0}\left(\frac{n-1}{n}\right)h\right]^{n/(n-1)}.$$

Since

$$p_0/\rho_0 = gRT_0$$

the final variation of pressure with velocity will be in the form

$$p = p_0\left[1 - \left(\frac{n-1}{nRT_0}\right)h\right]^{n/(n-1)}. \tag{7.6}$$

Density is related to height by the relationship

$$\rho = \rho_0\left[1 - \left(\frac{n-1}{nRT_0}\right)h\right]^{1/(n-1)}. \tag{7.7}$$

According to the accepted polytropic process

$$T = T_0(p/p_0)^{(n-1)/n}$$

or, considering Eq. (7.6)

$$T = T_0 - [(n-1)/nR]h. \tag{7.8}$$

Therefore, temperature drop with altitude is a linear function. Temperature drop gradient is equal to $(n - 1)/nR$.

If for air we take $n = k = 1.4$, i.e., assuming that in a state of temperature equilibrium the expansion and compression of air, during vertical mixing, is adiabatic, we will get

$$(n - 1)/nR = (1.4 - 1)/(1.4 \times 53.3) = 0.0054 \text{ deg/ft}.$$

i.e., the air temperature drops approximately 5.4° for each 1,000 ft altitude. Actually, in the lower layers of the atmosphere the temperature drops an average of 3.57° per 1,000 ft, which corresponds to $n = 1.23$, and addition of heat during expansion.

The quantity n, however, does not remain constant and changes with altitude. Therefore, relationships (7.6) through (7.8) must be considered as approximations. Investigations of the atmosphere show that up to the altitude $h = 35{,}000$–$40{,}000$ ft, the developed laws of air parameter variations with altitude agree with observations sufficiently well. At high altitudes there are sharp discrepancies from the developed laws, primarily in temperature distribution.

Starting with an altitude of approximately seven miles, the temperature no longer falls and remains approximately constant and equal, on the average, to −67°F. Then, starting at an altitude of 12 miles, a rise in temperature is observed, to a maximum at a 30 mile altitude. Further on, there is another drop in temperature. At an altitude of 50–60 miles, the temperature is approximately equal to −76°F. Therefore, the temperature varies with altitude, approximately as is shown in Fig. 7.9.

The first temperature rise at altitudes of 12–30 miles is explained, at present, by the presence of an ozone layer at these altitudes which intensely

absorbs the short wave (ultra-violet) solar radiation. The subsequent lowering of the temperature may be explained in the same manner as the first. Finally, the temperature rise in the uppermost layers is tied in with the bombardment of the earth's atmosphere by cosmic particles and solar radiation. Therefore, the temperature of these atmospheric layers must

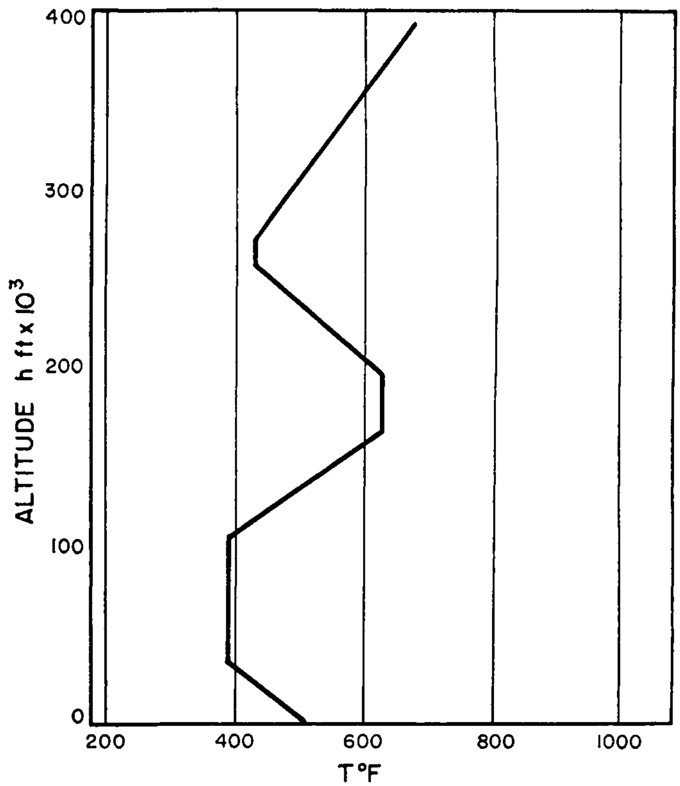


FIG. 7.9. Curve of air temperature variation with altitude. Based on NACA TN 1200.

fluctuate within a 24-hr period. At night it is lower, and during the day, higher; according to hypothesis it reaches a value of the order of hundreds of degrees.

It must not be supposed, however, that such high temperature will in any way cause complications during penetration of this region of the atmosphere. The temperature of the aircraft remaining a prolonged period of time at these altitudes will remain considerably less then the temperature of the surrounding medium. Due to the unusual rarefication of the gas, the heat transfer from the air to the aircraft will be very insignificant, and the balance between the heat supplied by the gas and the heat lost by the

body through radiation will be established at a relatively low temperature of the aircraft structure. In this sense an incomparably greater danger is represented by temperatures resulting from high velocities of rocket motion in the atmosphere. We shall dwell on this question later.

The lower layers of the atmosphere, below seven miles, where the above-developed equations hold, are usually called the troposphere. The layers above seven miles are called the stratosphere. The uppermost layers, starting at 37 to 44 miles, are called the ionosphere. Here, the air is strongly ionized.

In calculating rocket trajectories, it is extremely important to know the law of density and pressure variation of air with altitude.

TABLE VII.1. PROPERTIES OF THE ATMOSPHERE*

Altitude h, ft	Temperature T, °R.	Pressure ratio, p/p_0	Density ratio, ρ/ρ_0	Altitude h, ft	Temperature T, °R.	Pressure ratio, p/p_0	Density ratio, ρ/ρ_0
0	518.4	1.000	1.000	35,000	393.6	0.235	0.3098
1,000	514.8	0.957	0.9710	40,000	392.4	0.185	0.2447
2,000	511.2	0.930	0.9428	45,000	392.4	0.146	0.1926
3,000	507.8	0.896	0.9151	50,000	392.4	0.115	0.1517
4,000	504.2	0.865	0.8881	65,000	392.4	56.12×10^{-3}	74.14×10^{-3}
5,000	500.6	0.832	0.8616	80,000	392.4	27.41	36.21
6,000	497.0	0.801	0.8358	95,000	392.4	13.39	17.69
7,000	493.4	0.772	0.8106	110,000	412.6	6.58	8.27
8,000	490.0	0.744	0.7859	125,000	472.9	3.49	3.82
9,000	486.4	0.714	0.7619	140,000	533.3	1.99	1.94
10,000	482.8	0.687	0.7384	160,000	613.7	103.50×10^{-5}	87.42×10^{-5}
11,000	479.2	0.660	0.7154	180,000	630.0	56.98	46.89
12,000	475.6	0.636	0.6931	200,000	619.4	31.40	26.28
13,000	472.0	0.610	0.6712	220,000	552.4	16.56	15.54
14,000	468.6	0.587	0.6499	240,000	485.3	8.03	8.58
15,000	464.9	0.564	0.6291	260,000	432.0	3.51	4.21
20,000	447.1	0.460	0.5327	280,000	447.4	1.51	1.66
25,000	429.3	0.371	0.4480	300,000	487.4	0.72	0.69
30,000	411.3	0.294	0.3740	320,000	527.5	0.38	0.32

* Based on NACA No. 218 and TN 1200.

In the troposphere and, to a considerable extent, in the upper atmospheric layers, the parameters of the air state vary with time of day, time of year, latitude, and finally with respect to the general meteorological conditions. However, all parameters fluctuate about certain mean values which were determined as a result of atmospheric observations over a period of some years. The average parameters are taken as the parameters of the so-called international standard atmosphere (ISA), i.e., a certain arbi-

trarily accepted atmosphere whose parameters are not considered to vary with time of year, day, or latitude.

For troposphere and stratosphere, specifically, the following constants and relationships are accepted, tying in the air parameters with the altitude:

1. Air–ideal gas with the gas constant $R = 53.3$.

2. Pressure at sea level $p_0 = 29.92$ in. Hg; density $\rho_0 = 0.002378$ slugs/ft^3; temperature $T_0 = 518.4°\text{R}$.

3. Temperature gradient in the troposphere is constant and is equal to 0.003566°F/ft.

4. Temperature in the lower layers of the stratosphere is constant and is equal to −67°F.

With these standards for the troposphere, the variation in pressure, density and temperature with altitude according to expressions (7.6), (7.7), and (7.8) have the form

$$p = p_0(1 - h/147{,}500)^{5.256}$$
$$\rho = \rho_0(1 - h/147{,}500)^{4.256}$$
$$T = T_0 - 0.003566 \cdot h.$$

Ordinarily, a table of variation of air parameters with altitude, which was obtained by extrapolation of the international standard atmosphere data to high altitude region, is used for ballistic calculations; see Table VII.1.

C. Aerodynamic Forces

1. Aerodynamic Force Coefficients

Let us suppose that the rocket has two planes of symmetry, forming the rocket axis by their intersection.

If the direction of a rocket flight coincides with its axis, the lift force L is absent. This obviously follows from the symmetry of flow. The lift force will appear only as the rocket axis begins to deviate from the tangent to the trajectory (from the direction of velocity) and form with it an angle of attack α.

Therefore, the value of the lift force L becomes zero when $\alpha = 0$. As opposed to the lift force, the resistance force is present at all angles of attack.

We will begin the examination of aerodynamic forces with the resistance force D, inasmuch as in calculating flight trajectories of rockets the role of this force, compared to other aerodynamic forces, is more substantial. The first and more natural attempt to determine the resistance or drag was an attempt to express it in terms of ram pressure.

If we examine the reversed motion and, as a first approximation, consider the gas incompressible, then for the simplest bodies having a shape approaching that of a plate, the value of resistance force D can seemingly be obtained in the following manner.

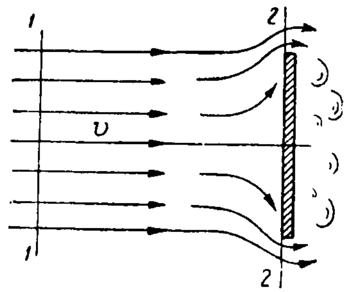


FIG. 7.10. Transverse flow around a plate.

Designating by v the velocity of undisturbed air flow, let us write Bernoulli's equation (see page 168) for a gas stream at section 1–1, sufficiently removed from the body ($v_1 = v$), and at section 2–2, at the front surface of the body (Fig. 7.10).

$$\tfrac{1}{2}\rho v_1^2 + p_1 = \tfrac{1}{2}\rho v_2^2 + p_2.$$

But $v_2 = 0$. Therefore

$$p_2 - p_1 = \tfrac{1}{2}\rho v_1^2 = \tfrac{1}{2}\rho v_2^2.$$

If we assume that the pressure at the back of the plate is equal to the pressure of the surrounding medium, i.e., p_1, then the drag force can be obtained by multiplying $(p_2 - p_1)$ by the area of the plate S

$$D = (p_2 - p_1)S = \tfrac{1}{2}\rho v^2 S. \tag{7.9}$$

Experience, however, does not confirm the obtained relationship even in this simplest case. This is understandable. A simplifying assumption that $v_2 = 0$ was made in the derivation. This is true only at the forward critical point while the air flowing away from that point has velocity other than zero. Further, it was assumed that directly behind the plate the pressure was equal to the pressure of the undisturbed flow. This is also incorrect. The pressure here would be somewhat lower.

From experiment, the value of D for a round plate is about 11% higher than indicated by formula (7.9). Also, it so happens that this deviation at relatively low velocities remains constant independently of flow velocity and plate dimensions.

Therefore, if we introduce the correction coefficient 1.11, then for a round plate the drag force is

$$D = 1.11 \times \tfrac{1}{2}\rho v^2 S.$$

In the general case, introducing the correction coefficient C_D, which is called the drag coefficient, we can write

$$D = C_D \times \tfrac{1}{2}\rho v^2 S \tag{7.10}$$

where C_D depends on the shape of the plate.

The drag of any body is determined according to formula (7.10). For axially symmetric bodies, the area S is the projected area on the plane perpendicular to the axis of symmetry (the median section), or any other so-called characteristic area.

The freedom in the choice of area S is reflected in the value of C_D. Therefore, in each case when the numerical values of C_D are applied, it is necessary to define the area taken as the characteristic.

The formula for the lift force is derived in a manner analogous to Eq. (7.10)

$$L = C_L \times \tfrac{1}{2}\rho v^2 S \tag{7.11}$$

where C_L is the coefficient of lift force and S is the same characteristic area as in expression (7.10).

At relatively low flight velocities, corresponding to Mach numbers not exceeding 0.5–0.6, coefficients C_D and C_L may be considered as independent of velocity. The indicated velocity limit until recently corresponded exactly to velocities attained by aviation, and C_D and C_L were defined as constant values for a given shape of an aircraft or some of its parts.

Formerly, these coefficients were considered as coefficients of configuration. Later, however, it was discovered that they also depend on velocity. As the flight velocity approaches the velocity of sound the values of C_D and C_L increase sharply. At high supersonic velocities the values of C_D and C_L decrease, asymptotically approaching some constant value. Variation of C_D and C_L with respect to Mach number and the angle of attack α is shown in Figs. 7.11 and 7.12. At the same time it must be pointed out that, in practical cases, C_D and C_L also depend to some degree on other factors—linear dimensions of the body, density of the air, etc.

In one way or another, in rocket technology, where it is necessary to deal with a full range of velocities from low to high supersonic, the aerodynamic coefficients C_D and C_L are considered as functions of velocity v, or more exactly, Mach number M. In this sense, formulas (7.11) and (7.12) lose their basic content, which established the proportionality between aerodynamic forces and the velocity head. However, due to their simplicity,

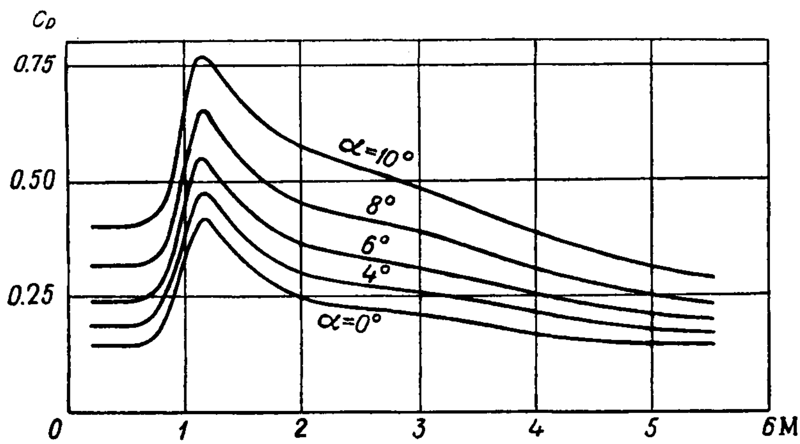


FIG. 7.11. Dependence of drag coefficient C_D of a ballistic rocket on Mach number M and angle of attack α (coefficient C_D is referred to the median section of the rocket; rocket jet is absent).

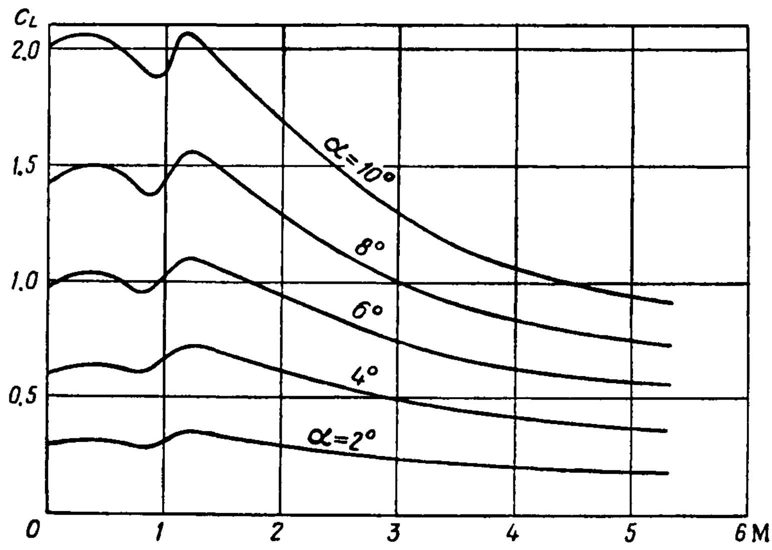


FIG. 7.12. Dependence of lift coefficient C_L on Mach number M and angle of attack α (coefficient C_L is referred to the median section of the rocket).

convenience, and certain traditions established in aviation, they remain basic design formulas regardless of whether or not low or high velocities are considered.

The mechanism of formation of aerodynamic forces is quite complex. Determination of D and L forces by computation, even in the simplest

cases, often constitutes an insoluble problem. Therefore, currently, the determination of aerodynamic forces (values of C_D and C_L) is done by approximate calculations which are used as a guide and which later on are corrected by wind tunnel tests.

Since wind tunnel tests of full scale rockets are possible only for small rockets, the tests are made with models geometrically identical to the rocket being investigated. Inasmuch as we are speaking about high flight velocities, it is necessary during tests to maintain the condition of similarity

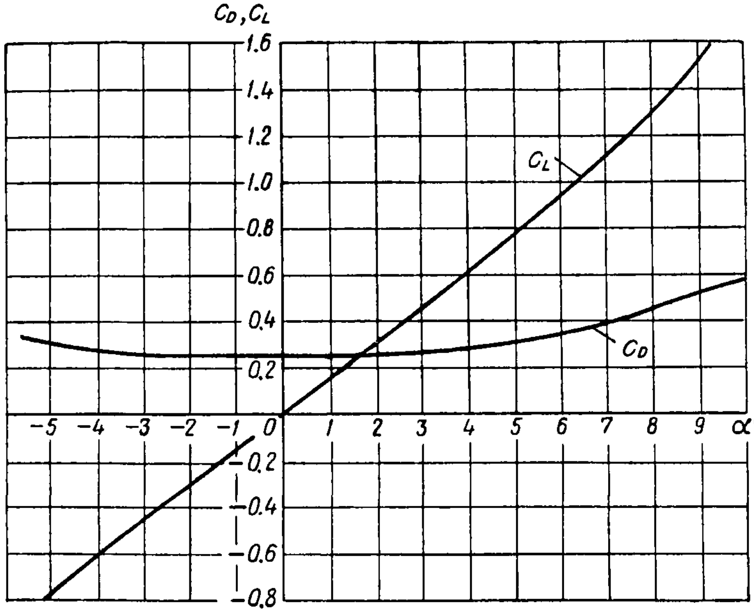


FIG. 7.13. Dependence of drag and lift coefficients of a ballistic rocket on the angle of attack at $M = 2$.

of Mach numbers. In other words, the model must be tested in the wind tunnel at the same Mach number for which it is necessary to obtain the actual C_D. It is also desirable to maintain the simllarity in Reynold's number Re, which takes into account the simultaneous action of viscosity and inertial forces

$$\mathrm{Re} = \rho v l / \mu$$

where μ is the viscosity coefficient, and l is some characteristic dimension of a rocket.

The fulfillment of the last condition, i.e., the equality of the Re for the model and the prototype, meets with considerable difficulties and, in wind tunnel practice, as a rule, it is not complied with.

The dependence of aerodynamic forces on the angle of attack α, is expressed by coefficients C_D and C_L as functions of α for constant S. For a symmetrical rocket it is obvious that this relationship will be symmetrical for C_D and unsymmetrical for C_L

$$C_D(-\alpha) = C_D(\alpha)$$
$$C_L(-\alpha) = -C_L(\alpha).$$

Dependence of C_D and C_L on angle of attack α is shown in Fig. 7.13 for a V-2 rocket at $M = 2$, and also by the curves of Figs. 7.11 and 7.12.

It is important to note that at small angles of attack the drag coefficient may be considered as independent of the angle of attack.

2. Components of Aerodynamic Forces and Subsonic Flow

The character of flow and the mechanism of the formation of aerodynamic forces differs depending on whether the flight velocity is subsonic or

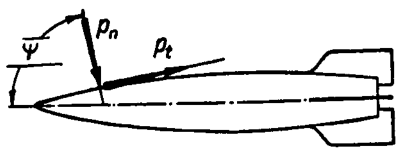


Fig. 7.14. Normal and tangential aerodynamic forces acting on the surface of a rocket.

supersonic. However, in both cases, the force of the frontal resistance can be split up into two basic components; component of friction and component of pressure

$$D = D_t + D_n.$$

Force D_t is the resultant of resistance forces, p_t, tangential to the surface of the body, and force D_n is the resultant of forces, p_n, normal to the surface (Fig. 7.14).

$$D_t = \int_s p_t \sin \psi dS$$
$$D_n = \int_s p_n \cos \psi dS$$

where dS is an area element of a rocket and ψ is the local angle of the normal.

Forces p_t and p_n are referred to the unit area of the surface of the body.

Further, we can write

$$D_t = C_{Dt} \times \tfrac{1}{2}\rho v^2 S$$
$$D_n = C_{Dn} \times \tfrac{1}{2}\rho v^2 S$$

and get from this the expression for the over-all drag coefficient in the form

$$C_D = C_{Dt} + C_{Dn}.$$

In the simplest cases the boundary layer theory enables the calculation of the quantity C_{Dt} based on Reynold's number $\mathrm{Re} = \rho v l/\mu$. For instance, for longitudinal flow around a plate (Fig. 7.15), coefficient C_{Dt} for turbulent flow may be determined according to the formula

$$C_{Dt} = 0.074/\sqrt[5]{\mathrm{Re}}$$

which holds true up to values $\mathrm{Re} = 10^7$.

The plate formula for C_{Dt} enables the evaluation of the magnitude of the coefficient C_{Dt} for any body, if it is assumed that the amount of the "wetted" surface determines the friction force, independently of the configuration of that surface.

The effect of viscosity on drag is not limited to simple friction, which was just discussed. The existence of viscosity substantially affects the distribution of normal pressures, which form the second component of drag D. Depending on the value of viscosity, especially with poorly streamlined bodies, separation of flow occurs in the boundary layer at higher or lower velocities, which directly reflects on pressure distribution.

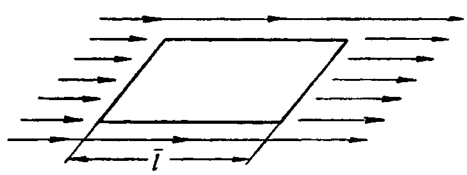


FIG. 7.15. Longitudinal flow around a plate.

The theory which assumes an ideal liquid having no viscosity leads to unexpected results. For instance, there is the so-called Euler-D'Alembert paradox: a symmetrical body (sphere, cylinder) during slow motion in an ideal fluid should not experience drag. For instance, a cylinder with transverse flow (Fig. 7.16,a) is subjected to pressure forces symmetrical with respect to a plane perpendicular to the direction of flow. These forces are balanced and do not yield a resultant. The existence of viscosity leads not only to the appearance of tangential forces p_t, but, what is even more important, changes the character of flow itself. In some area the flow separates from the surface of the body resulting in the appearance of strong vortices behind the cylinder. The pressure, acting on the cylinder from behind, becomes less than the pressure of the surrounding medium (Fig. 7.16,b). The pressure forces form a resultant directed against the motion of the cylinder.

Generation of drag in this case, and its absence in the first case, is understandable from the energetic point of view as well. During the motion of the cylinder through an ideal fluid, the streams separating in front of the

cylinder merge behind it and remain motionless. Therefore, the force imparting motion to the cylinder in the liquid does not perform any work and does not transmit any kinetic energy to the fluid. This is not true for

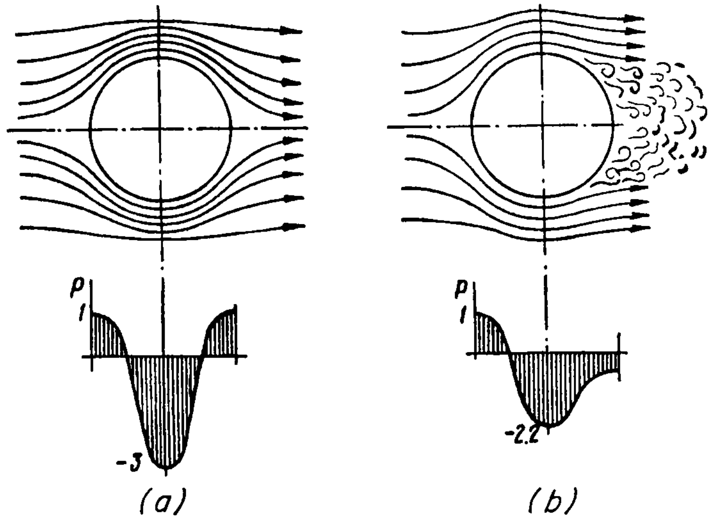


FIG. 7.16. Development of drag at subsonic velocities.

the second case. In this case, a wake of vortices is formed behind the cylinder which possesses kinetic energy. This energy is imparted to the particles of fluid or gas by the expenditure of work of the force which moves the body in a viscous medium.

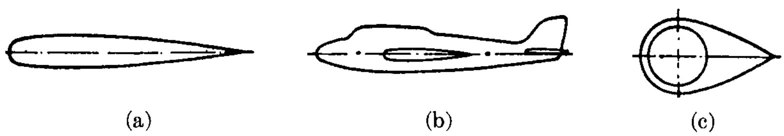


FIG. 7.17. Examples of streamline shapes: (a) profile of an aircraft wing for subsonic flight; (b) aircraft fuselage; and (c) a faired cylinder.

From what has been said, it follows that the boundary layer separation and formation of vortices increase drag. In order to reduce drag forces in subsonic flow, it is necessary to have a profile of smooth contour which would facilitate flow without separation, i.e., create a so-called streamlined configuration. Fig. 7.17 shows some streamlined shapes.

It is important to note that all streamlined bodies have smoothly converging contours at the aft part which do not permit flow separation, and rounded forward portions. The sharp-nosed shapes are not advantageous

in subsonic flow. The sharp edge disturbs the flow and increases the danger of its separation.

At moderate flight velocities, the friction drag of a streamlined body comprises the main portion of the total drag; whereas the shape drag (pressure) plays a secondary role. For poorly streamlined bodies, such as a sphere or a cylinder, the form drag has considerably greater value than friction drag. At high velocities, however, even for well-streamlined bodies, the pressure drag becomes a deciding factor.

In examining the pressure diagram shown in Fig. 7.16,b, we notice that the pressure on the surface of a body may be positive or negative. In this connection the pressure drag for bodies of revolution (rocket air

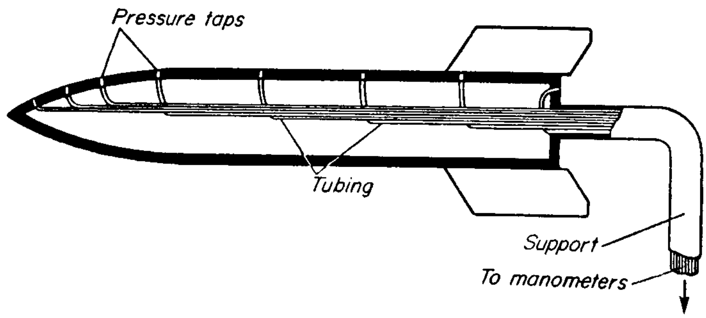


FIG. 7.18. Rocket model for pressure measurement.

frame) is divided into two parts: frontal drag (drag due to pressure distributed along the air frame) and base drag arising as a result of rarefication at the base (base suction). The drag is divided in the same manner at supersonic speeds.

During flight, the air ahead of the rocket is compressed. An increased pressure is created here, whose magnitude determines "the pressure drag." This drag increases without limit with increase of velocity. A rarefication is formed behind the rocket—the base vacuum, which determines base drag. This component of drag also increases with velocity, not without limit, however. It cannot be greater than the value corresponding to the absolute base vacuum.

The separation of pressure drag into frontal and base is usually done on the basis of wind tunnel tests. The model is pressure tapped; that is, holes are drilled in it at many points. Through these holes the pressure is ducted to monometers (Fig. 7.18). As a result of wind tunnel tests a complete picture of pressure distribution along the surface of the model is obtained. After this, it is not difficult to determine the frontal and base drags.

Fig. 7.19 shows the pressure distribution along the surface of the rocket body (without considering fins). In the presence of the angle of attack α, the character of flow around the rocket will differ from that at an angle of

attack $\alpha = 0$. In this case there arises a side flow of air from the zone of high pressure into the zone of lower pressure which results in additional

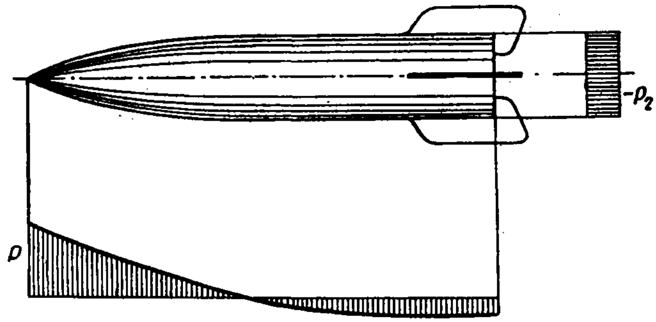


FIG. 7.19. Distribution of aerodynamic pressure along the surface of a rocket.

formation of vortices and an additional expenditure of energy for their formation. The corresponding additional drag force is called the induced drag. The additional air flow and vortices formed by it are shown by arrows in Fig. 7.20.

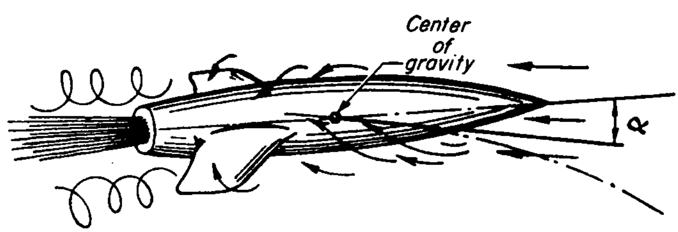


FIG. 7.20. Formation of vortices resulting in induced drag at a rocket flight angle of attack.

The indicated division of drag forces into separate components is valid both for subsonic and supersonic flow. However, during a transition to supersonic flow, additional components of drag forces appear which sharply change the flow character and correspondingly change the approach toward a selection of the more rational aerodynamic profiles.

3. The Nature of Supersonic Flow

Let us return to the derivations which were evolved in Chapter VI [formula (6.19)] for determination of sound velocity.

An expression was obtained for the propagation velocity of a compression wave.

$$c_s = \sqrt{\left(\frac{\rho + \Delta\rho}{\rho}\right)\frac{\Delta p}{\Delta\rho}}.$$

For very weak (sound) disturbances, when $\rho + \Delta\rho \approx \rho$, this expression becomes the formula for the velocity of sound

$$c = \sqrt{\Delta p/\Delta\rho}$$

from which, with the assumption of adiabatic process for gas compression during acoustical vibration, we get

$$c = \sqrt{kp/\rho} = \sqrt{kgRT}.$$

From the comparison of expressions for the propagation velocity of a shock wave and velocity of sound, it follows that strong disturbances always propagate faster than sound.

Let us assume that in a still gas medium there is a constant source of weak disturbances. Let us imagine that at some time this source created a local compression of gas. As a result of this a spherical sound wave is created which begins to travel uniformly in all directions with the speed of

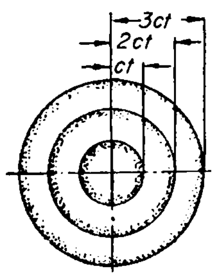


FIG. 7.21. Propagation of weak wave fronts in a stationary medium

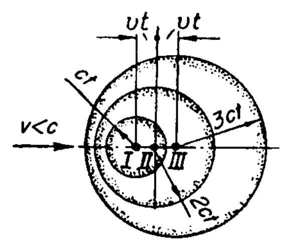


FIG. 7.22. Propagation of weak wave fronts in a medium which moves with velocity $v < c$.

sound c. After time t, the wave front would be at distance ct from the source of the disturbance. If the source of disturbances sends out periodic signals, the fronts of all waves will be arranged in the form of concentric spherical surfaces with their centers at the source of disturbances (Fig. 7.21). The propagation of these waves may be pictured as being analogous to the propagation of waves on the surface of the water from a pebble thrown into the water.

Let us imagine now that the source of disturbances is immobile, while the gas medium moves relative to it with velocity v, and $v < c$. During the time t, between two successive signals from the source of disturbances, the first wave will be displaced by quantity vt and the centers of the spherical waves will be displaced relative to each other by the same quantity vt. At the same time, inasmuch as $v < c$ the center of each subsequent sphere will remain within the confines of the preceding sphere (Fig. 7.22).

The picture is different when $v > c$, i.e., in the case when the flow moving past the body is supersonic. In this case the center of each succeeding sphere will be outside the surface of the preceding sphere (Fig. 7.23), and we will get a family of spheres having a conical envelope.

In this way the character of propagation of disturbances differs, depending upon the velocity of flow.

While the flow velocity was less than the velocity of sound, the disturbances propagated in all directions, even though not uniformly but with the flow as well as against it. However, when the flow velocity became greater

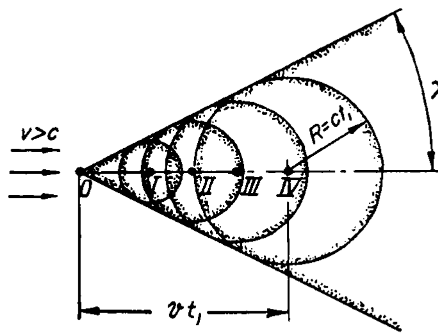


FIG. 7.23. Propagation of weak wave fronts in a medium which moves with velocity $v > c$.

than the speed of sound, the disturbances began to propagate in one direction only; with the flow. The conical envelope—the so-called Mach cone—constitutes a boundary separating the region of points which can be reached by the sound disturbances from the region which cannot be reached by them.

While outside the boundary of this cone we cannot receive signals being sent by the source of disturbances. The angle at the apex of the cone (Mach angle) depends on the ratio of flow velocity to velocity of sound. From Fig. 7.23 it may be seen that

$$\sin \chi = c/v = 1/M.$$

The angle χ increases with a decrease in flow velocity, and when $v = c$, becomes equal to 90°. This means that the cone of disturbances becomes a plane. When this happens the signals transmitted by the source of disturbances reach any point located behind the source. Let us suppose now that the source of disturbances is capable of continually transmitting not only the weak, acoustical waves, but also strong shock waves. Let us see in what manner these waves will be propagated in supersonic flow.

A shock wave, as we already know, propagates with velocity c_s, greater

than the velocity of sound. Therefore, initially, the wave traveling away from the source of disturbances and having sufficient initial force can propagate against supersonic flow. As it propagates, this wave will become weaker and its velocity will be decreasing, approaching, as the limit, the velocity of sound. Under these conditions, the envelope of the family of spherical waves will not be a simple conical surface. It will be a surface resembling at its head a hyperboloid, and then changing into a cone of weak disturbances. Fig. 7.24 shows such an envelope. Where the strength of the shock wave is greatest (directly in front of the source), the envelope is shown in

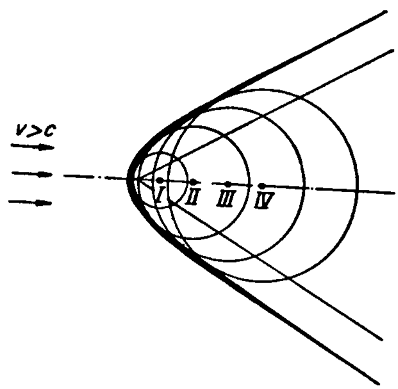


FIG. 7.24. Envelope of a family of spherical shock wave fronts moving at a velocity $v > c$.

heavier outline. The concept of "disturbance source," used until now, includes more than a special device for transmitting sound signals. Each point, each projection, of a body moving in the stream may be a source transmitting to the stream continuous disturbances. There may be sources of either weak or strong disturbances.

Let us examine a supersonic flow around some blunt-nosed body (Fig. 7.25). Each point on the body may be considered as a source of weak disturbances. Signals from this source may be propagated only within the cone of weak disturbances.

The disturbances in the vicinity of points located on the front flat part of the body form a zone of increased pressure in front of the body. However, if a zone of increased pressure with a free forward surface was formed in front of the body, then a shock wave was also formed, which has the property of propagation with velocity greater than the speed of sound. A wave appears in this manner which moves against the flow. However, it does not have the opportunity to move too far forward. As soon as the distance between the wave front and the forward edge increases, the gas is able to flow outward from the region of higher pressure (as is shown by

arrows in Fig. 7.25). During this time, the strength of the shock wave is reduced and the velocity of its propagation diminishes. Thus, there will always be a shock wave in front of a body in a supersonic stream which moves forward with the velocity of the supersonic flow. The distance between the wave and the body depends on the configuration of the body, which determines the amount of gas spillover, and on the velocity of flow.

Outside the boundary of the nose part of the body the shock wave front becomes inclined and the intensity of the wave diminishes. As a limit, the inclination angle of the shock wave relative to the direction of velocity becomes equal to Mach angle and the wave itself becomes an ordinary acoustical wave.

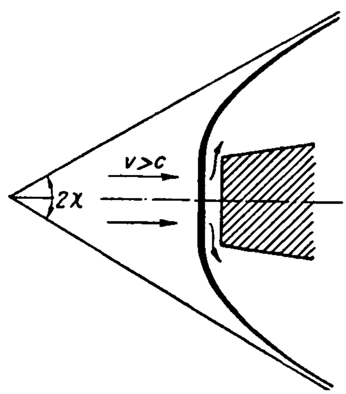


FIG. 7.25. Shock wave ahead of a blunt-nosed shell at supersonic flight velocity.

The presence of shock waves is a characteristic peculiarity of supersonic flow. In supersonic aerodynamics, shock waves are ordinarily called pressure shocks.

Shock waves develop not only at the front but also at the back of a body in the air flow.

Let us examine the aft portion of a body in a supersonic stream or, what is the same, a body flying with supersonic velocity (Fig. 7.26).

Let us take a corner of the body base as a source of weak disturbances.

Let the flow, approaching point O, have the velocity v_1, with $v_1 > c$. The approaching flow will remain undisturbed up to line OA. The location of line OA is determined by the angle $\chi_1 = \arcsin (c/v_1)$. Going around the corner the flow must expand. For a supersonic flow the expansion, as is known, is accompanied by an increase in velocity. Therefore, beyond the line OA the flow velocity will be increasing and the flow will be deflected. Beyond a certain line OB the flow will be completely deflected and we move along a new direction with velocity v_2. Angle $\chi_2 = \arcsin (c/v_2)$; since $v_2 > v_1$, then $\chi_2 < \chi_1$.

The shown schematic is somewhat simplified, inasmuch as stream boundaries of the turned flow will not be parallel, but an inescapable conclusion is that with a flow past an external corner we will not get pressure shocks. Lines OA and OB represent waves of weak disturbances.

Let us follow the subsequent behavior of the turned flow. We will note that the flow must turn again by the same angle but now in the opposite direction, having met an identical flow which has passed around the lower side of the symmetrical body.

The point of merging of the flows on the axis of symmetry we will designate by O', and will take it as a new source of disturbances.

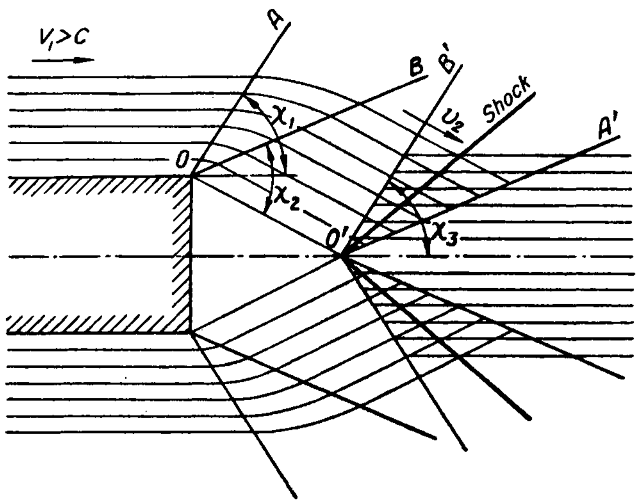


FIG. 7.26. Schematic of a formation of a tail shock wave.

The flow approaching the angle has a velocity v_2. The source of disturbances O' makes itself felt only to the right of the line $O'A'$, which is parallel to OB. Beyond the line $O'A'$, the flow compresses. For a supersonic flow, this means reduction in velocity, to value v_3, for instance. The Mach angle corresponding to this velocity is

$$\chi_3 = \text{arc sin } (c/v_3).$$

Since $v_3 < v_2$, then $\chi_3 > \chi_2$. In Fig. 7.26 line $O'B'$ is located to the left of line $O'A'$.

In this manner, we arrive at a contradiction: the boundary, where the curvature of the flow should start, is located beyond the line where this curvature should end. This contradiction can be explained only by the sudden change in gas velocity, across the shock, from v_2 to v_3, with the abrupt turn of the flow. During all this, pressure and other flow parameters

also experience change across the shock. The shock boundary is located between lines $O'A'$ and $O'B'$.

The formation of the tail shock was presented in schematic terms. In actuality, directly behind the flat base there is a turbulent wake, and the source of disturbances, O', is purely arbitrary. The shock is formed at some distance from the axis. Fig. 7.27 is a schematic of the actual location of a shock wave.

The appearance of shock waves during supersonic flow gives rise to an additional resistance, called the shock resistance. Actually, the force moving a body with supersonic velocity must perform additional work to maintain shock waves. It is important to emphasize that work is not expended on

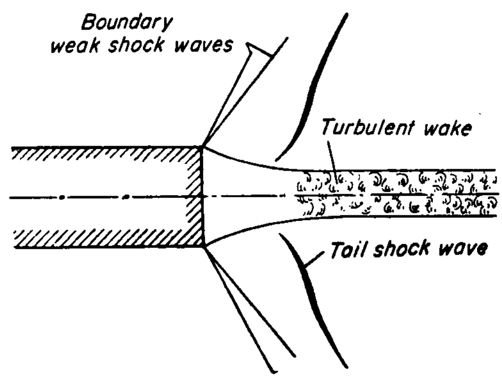


FIG. 7.27. Tail shock wave and turbulent wake aft of a flat tail of a shell flying with supersonic speed.

transporting the waves (they move of their own accord) but on maintaining the constant intensity of the shock waves. If additional energy were not supplied so the waves at the expense of the moving force, their velocity would drop below that of sound and they would be left behind a rocket flying at supersonic velocity.

Inasmuch as a zone of increased pressure is formed behind a compression wave, the shock resistance may be treated as the consequence of additional pressure which acts on the surface of the rocket in the presence of nose shock. Obviously, the stronger the shock the greater the losses, and the greater the shock resistance will be.

We have already noted (Fig. 7.24) that the strength of the shock (pressure drop across the shock) depends upon the angle which the wave front makes with the flow vector. It is clear that for a wave to travel directly against the flow (the so-called normal shock), it must be stronger than when it moves at an angle to the flow (oblique shock). In the limiting case, when the wave front forms a Mach angle with the flow, its strength

may be negligibly small. Since the shock wave drag at supersonic speed represents considerable, and at high velocities even the major, part of the total drag, it is necessary to take special measures for its reduction. In order to reduce the over-all drag in supersonic flow, the rocket must be given such a form that shocks, since they are inescapable, be oblique and the shock angle approach the Mach angle as closely as possible. The least acceptable are shapes which give rise to normal shocks.

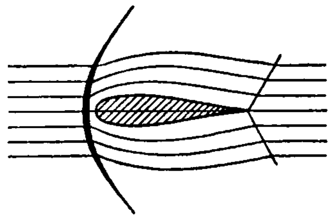


FIG. 7.28. Flow around a subsonic profile in supersonic flow.

After what has been said it becomes clear that the so-called streamlined bodies, with the least frontal drag in subsonic flow, become poorly streamlined in supersonic flow. Fig. 7.28 shows a representation of supersonic flow around a subsonic profile of a wing. A curved shock, approaching a normal, is formed in front of the wing as a result of which the drag on such a wing becomes considerable.

Thin, elongated, pointed profiles are most advantageous at supersonic flight velocities.

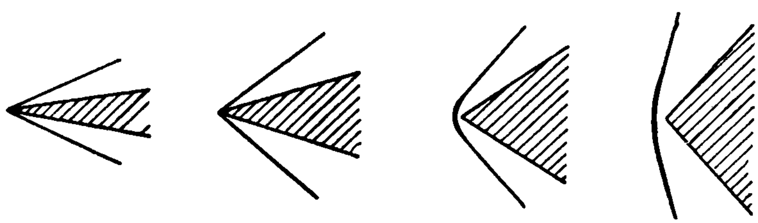


FIG. 7.29. Dependence of the shock shape on the nose cone angle, at constant flight velocity.

For a pointed profile, or a cone-shaped rocket head, the leading shock becomes oblique, and for very acute angles of the leading edge, approaches the wave of weak disturbances. Energy loss in such a shock will be minimum.

The shock angle increases with an increase in the thickness of the leading edge, or the cone angle. When the angle of the cone or the wedge of the leading edge becomes equal to the so-called critical angle, the shock at the central partition will become normal and will detach itself from the nose. The losses due to this shock will be considerably greater.

Fig. 7.29 shows the change in the shock wave shape with respect to the angle of the leading edge at constant flow velocity. Therefore, the minimum included angle of the rocket head is a condition for the minimum

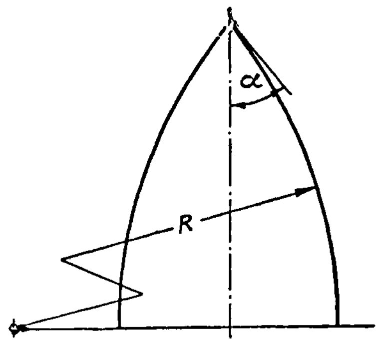


FIG. 7.30. Ogival shape.

shock resistance. However, excessive elongation of the nose leads to the increase in friction drag and structural difficulties. Ordinarily, the forward sections of rockets and shells are given the so-called ogival form which is described by arcs of circles or parabolas (Fig. 7.30).

4. Critical Mach Number

Shock wave drag which is the result of compression shock also occurs at subsonic velocities which are close to sonic velocity. Under these conditions the local stream velocity, while flowing over wings, fins, or other parts of the aircraft or rocket, may exceed the speed of sound and shocks will appear in corresponding places.

The flight velocity at which the local velocities approach or reach the speed of sound is called a critical Mach number and the appearance of the first shocks is called the critical shock wave point.

In order to understand the appearance of shocks in subsonic flow first let us convince ourselves that a supersonic flow changes into subsonic flow only across a shock front. Flow, in which a gradual transition from supersonic to subsonic velocity occurs, is unstable.*

Actually, in order to reduce the velocity of supersonic flow, the area of the section must be reduced. After the flow velocity becomes sonic, the area of the section must be increased in order to reduce the velocity of flow further. Thus we arrive at the scheme of the reversed Laval nozzle (Fig. 7.31). A flow through the Laval nozzle has been described earlier (see page 176).

* Translators note: This is called a transsonic region.

Let us assume that the flow velocity changes gradually from supersonic to subsonic (see Fig. 7.31). The flow velocity will equal the sonic velocity at the narrowest or the critical section of the nozzle.

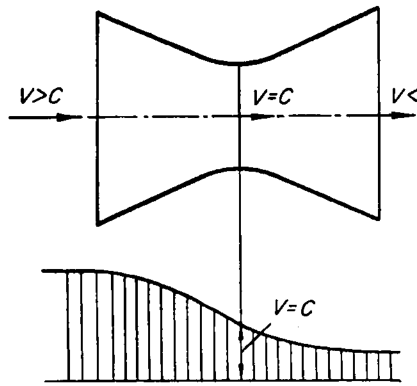


FIG. 7.31. Smooth deceleration of supersonic flow in a reversed Laval nozzle.

Each point on the nozzle wall, and even the gas itself, may be considered as a source of weak disturbances. Let us take any point located in the subsonic region of flow, for instance, point O in Fig. 7.32. The disturbances arising from this point will be propagated in all directions and to any dis-

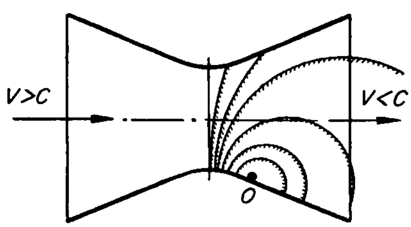


FIG. 7.32. Propagation of weak disturbances in a subsonic section of flow.

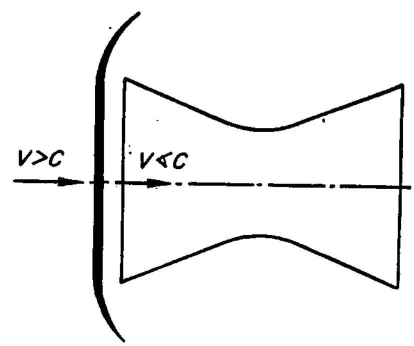


FIG. 7.33. Shock in front of diffuser entrance.

tance along the flow but only as far as the critical section against the flow. And this will be the case with regard to all disturbances from all points located in the subsonic region of flow. As a result, the disturbances from various sources will accumulate in the vicinity of the critical section and the wave so created will have the ability to move against the flow with the velocity greater than the speed of sound.

At some section, depending on the configuration of the supersonic flow, the velocities of flow and the wave (shock) will be equalized and the location of shock will be stabilized.

In practical cases, with a rigid diffuser, the described retardation of supersonic flow leads to the fact that the shock emerges from the confines of the diffuser and is located immediately before entrance, while subsonic velocity is established behind the shock (Fig. 7.33).

Therefore, the transition from supersonic to subsonic velocity takes place across shock and is explained by the fact that a sufficient number of disturbance sources in the flow is always present to form an integral wave capable of motion against a supersonic flow.

Let us turn now to the question of a critical Mach number.

During flow around a body some streamlines first converge, and then diverge, as is shown, for example, in Fig. 7.34. With sufficient pressure drop

FIG. 7.34. Change in streamline section during flow past a body (wing profile).

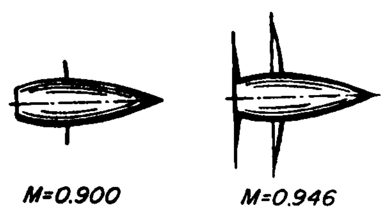


FIG. 7.35. Shocks formed at the surface of a bullet at high subsonic velocities.

across these streamlines, there is a possibility of velocity increase to supersonic, as in a Laval nozzle. Further on, however, inasmuch as the free stream velocity is below sonic, there must be a reverse transition to a velocity less than the speed of sound. However, we already know that this transition takes place through a shock. It is in this manner that shocks appear at subsonic flight velocities.

Fig. 7.35 shows shocks formed at the surface of a bullet in flight. These shocks are formed at velocities close to the speed of sound ($M = 0.900$ and $M = 0.946$).

In contemporary aviation, which attains subsonic and sonic flight velocities, the question of critical Mach number is quite serious. At the appearance of the first shocks, there is a sharp increase in aerodynamic loading and there is qualitative change in the flow characteristic with an accompanying redistribution of pressure forces along the surface of the aircraft. All this results in high structural overloading and a change in the degree of stability and controllability of the aircraft.

The difficulties thus arising are overcome by investigation and solution of stability and controllability problems in the region of the critical Mach

number, on one hand, and the application of design configuration which postpones the occurrence of the critical Mach number, on the other hand. An example of the last, for instance, is the trend toward thin, symmetrical wing profiles, and swept back and delta wings.

The postponement of the critical Mach number with respect to rockets whose trajectory, for the most part, takes place at supersonic velocities (and such rockets are in the majority) does not make sense. Here it is necessary to determine the optimum configuration from the point of view of minimum shock drag.

5. Rocket Jet Effect on Aerodynamic Forces

The presence of the jet stream of an operating motor introduces certain specific peculiarities into rocket aerodynamics.

Let us examine the jet effect on each of the drag components. The motor jet cannot influence pressure drag to any appreciable extent. At supersonic flight velocity such an effect is simply impossible and at subsonic speed, even if any is present, it is not noticeable.

Friction drag at supersonic flight velocity does not change in the presence of the jet, either. At subsonic velocity it is somewhat increased due to the fact that the jet stream aspirates (sucks in) surrounding air and accelerates the flow along the surface of the rocket (Fig. 7.36). The accel-

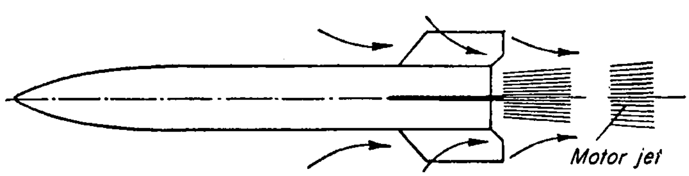


Fig. 7.36. Aspiration of external flow by the motor jet.

erated flow near the rocket tail is at a lower pressure, which tends to increase the frontal drag, especially in the case where the rocket has a conical aft section or boattail.

The main influence of the jet is on the base drag. There is no rarefied zone behind the nozzle opening of an operating motor. Therefore, the total rocket drag at subsonic as well as supersonic flight velocities includes only that part of the base drag which corresponds to the base area surrounding the nozzle. As an example, in calculating the frontal drag for a rocket of the V-2 type with an operating motor, only 50% of the base drag is included.

At low flight velocities the indicated condition lowers the frontal drag by a considerable amount. The change in drag due to the aspiration of the

surrounding flow is neglected. At high velocities, the reduction of the frontal drag due to the presence of the jet becomes unnoticeable, since the base drag plays a small part, and the shock drag becomes the main contributor.

Therefore, in calculating rocket trajectories, the frontal drag varies depending on whether, all other conditions being equal, we are concerned with powered or coasting portions of the trajectory, i.e., those portions of the trajectory with an operating oı nonoperating motor. It must be noted that in general the thrust of a rocket exceeds many times the frontal drag and that in evaluating aerodynamic forces acting on the rocket there is no need for great refinement. In this sense the rocket differs considerably from an aircraft whose propeller thrust is equal to dıag at level cruising speed.

6. Aerodynamic Heating of Rockets at High Flight Velocities

The aerodynamic effect of the medium on the air frame of a rocket in flight is not manifested by force alone. At high velocities there develops a thermal effect which manifests itself in the heating of the rocket surface.

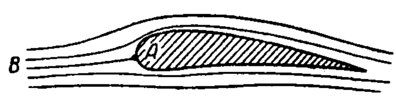


Fig. 7.37. Determination of stagnation temperature.

From the above-developed energy equation (in Chapter VI), it follows that during gas flow in a pipe, or around any body, there is a conversion of energy from one type to another. Kinetic energy is converted into heat and conversely. Reduction in kinetic energy (reduction in speed) necessarily leads to a rise in gas temperature and consequently in the temperature of the body.

The gas velocity may be decreased either by contact with a body around which it flows, or as the result of boundary layer friction. It is obvious that the rise in gas temperature is independent of the source of the velocity loss.

Let us examine a body in a gas flow (Fig. 7.37).

At the leading edge of this body the flow divides into two parts. A portion of the streamlines goes up and another portion, down. It is obvious that at some point A at the leading edge of the body, the flow velocity is equal to zero. Here, there is a complete flow stagnation and all of the kinetic energy of the gas is converted into its heat content. This point is known as stagnation point, or critical point. It is not difficult to determine the gas temperature at this point.

Let us separate out of the total flow a gas streamline AB, where point B

is located in the free stream, and let us write the energy equation (6.10) for this streamline.

$$\frac{v_B^2}{2} + \frac{k}{k-1}\left(\frac{p}{\rho}\right)_B = \frac{v_A^2}{2} + \frac{k}{k-1}\left(\frac{p}{\rho}\right)_A.$$

Since

$$p/\rho = gRT,$$

then

$$\frac{v_B^2}{2} + \frac{k}{k-1}(gRT)_B = \frac{v_A^2}{2} + \frac{k}{k-1}(gRT)_A.$$

But $v_B = v$ and $v_A = 0$; therefore, the increase in temperature in a decelerating flow from point B to point A will be

$$\Delta T = T_A - T_B = \tfrac{1}{2}v^2(k-1)/kgR.$$

The temperature at the critical point T_A represents the stagnation temperature.

For air, $k = 1.4$, $R = 53.3$, and

$$\frac{k-1}{2kgR}v^2 \approx \frac{v^2}{12 \times 10^3}.$$

for this the stagnation temperature is

$$T_{\text{stag}} = T + v^2/12 \times 10^3.$$

At low velocities, the stagnation temperature is not high. However, at supersonic speeds attainable by rockets it may turn out to be considerable. For instance, at flight velocity of 5000 ft/sec

$$T_{\text{stag}} = T + 5000^2/12 \times 10^3 = T + 2080°.$$

At an ambient temperature of $T = 495°$R (35°F), this amounts to 2575°R (2115°F).

The rocket temperature will be, naturally, noticeably below the stagnation temperature due to the partial heat transfer into the structure of the body and radiation to the atmosphere. However, it remains sufficiently high to require suitable design measures. In particular, long range ballistic rockets have nose tips made of heat resistant material (quartz, graphite).

If the stagnation of the gas is not complete, then, understandably, the temperature of the stream will be lower than the complete stagnation temperature. For well-streamlined bodies, stream temperature in the boundary layer equals 40–70% of the full stagnation temperature.

The heating of the rocket airframe, due to air stagnation near the rigid skin, is the basic obstacle in attaining supersonic flight velocities in the

atmosphere. This obstacle may be overcome by the development of highly heat resistant materials and the introduction of artificial cooling of the more critical places, as is done at the inner walls of the combustion chamber and nozzle of a liquid propelled rocket. However, the logically necessary step, and of the first order of importance, is, in all cases, to conform to the proper aerodynamic shapes which do not permit stagnation to occur.

If we return to the problem of the ejection of combustion products from the nozzle of a rocket motor, it must be pointed out that stagnation takes place there too, with a resultant increase in temperature in the boundary layer along the nozzle walls. This, as has been mentioned earlier, leads to an increased heat flow across the wall.

In case of a complete gas stagnation in the nozzle of a liquid propelled rocket, and in the absence of heat losses, the kinetic energy of the flow is converted back into the heat content which the gas had in the combustion chamber. Consequently, in the absence of heat losses, the stagnation temperature of the stream in the nozzle of a liquid propelled rocket will be equal to the temperature in the combustion chamber. Such gas stagnation, and corresponding heating, take place, for example, at the leading edges of jet vanes.

D. Stabilizing and Damping Moments

1. Stabilizing Aerodynamic Moment

It already has been mentioned above that a system of aerodynamic forces, distributed along the surface of a rocket, can be referenced to any point on the rocket in the form of a resultant force and a moment whose magnitude depends on the location of the point to which a system of forces is referred.

If the system of aerodynamic forces is referred to the center of gravity of a rocket, then at an angle of attack other than zero, we will get, besides the L and D forces discussed above, a resulting moment—the so-called restoring or stabilizing moment M_{st}, acting to reduce the angle of attack α (Fig. 7.38).

The magnitude of the stabilizing moment is determined by a formula analogous to the formulas used to determine drag force D and lift force L

$$M_{st} = C_m \times \tfrac{1}{2}\rho v^2 S l \qquad (7.12)$$

where ρ, v, and S are air density, flight velocity, and characteristic area respectively; l is some characteristic linear dimension (ordinarily rocket length); and C_m is the dimensionless moment coefficient.

At low subsonic flight velocities, the term C_m is independent of velocity,

At high velocities C_m, like coefficients C_D and C_L, is considered as a function of velocity.

Coefficient C_m, in a first approximation (at small α), is proportional to the angle of attack

$$C_m = C_m{}^\alpha \alpha.$$

The value of $C_m{}^\alpha$ unlike other aerodynamic characteristics, does not depend on flight velocity alone. Inasmuch as aerodynamic forces are referred to the center of gravity of the rocket, and the center of gravity itself moves along the axis as the fuel is expended, the value of $C_m{}^\alpha$ also

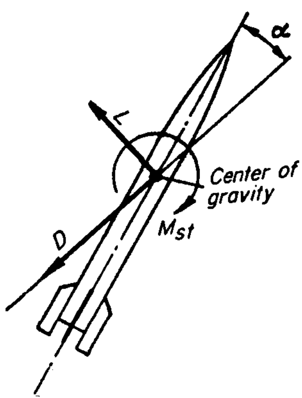


FIG. 7.38. Reference of aerodynamic forces to the rocket's center of gravity.

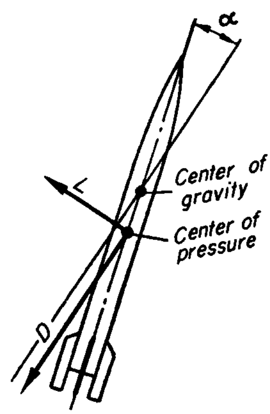


FIG. 7.39. Reference of aerodynamic forces to the rocket's center of pressure.

depends on the internal distribution of the rocket mass. Because of this, it is more convenient to select some other characteristic instead of $C_m{}^\alpha$ as a measure of the stabilizing moment.

Along the rocket axis a point exists so located that, when the system of aerodynamic forces is referred to it, we obtain a moment equal to zero. This point is called the center of pressure.

The center of pressure may be considered as a point of intersection of resultant aerodynamic forces with the rocket axis (Fig. 7.39). The location of the center of pressure depends exclusively on the external aerodynamics of the rocket.

The relative location of the center of pressure and the center of gravity is important for stabilizing the rocket in flight.

If the center of pressure is located behind the center of gravity, then, during deviation of the rocket axis from direction of flight, the aerodynamic forces create a moment which reduces the angle of attack and restores the

initial direction of the axis. If the center of pressure is located ahead of the center of gravity, the aerodynamic forces, during axis deviation from direction of flight, will create a moment increasing the angle of attack. In the second case it is said that the rocket is not stabilized or is statically unstable. In order to displace the center of pressure aft, the rocket is equipped with fins. An unfinned rocket, as a rule, is statically unstable.

Fig. 7.40 shows the relationship of the center of pressure location of a

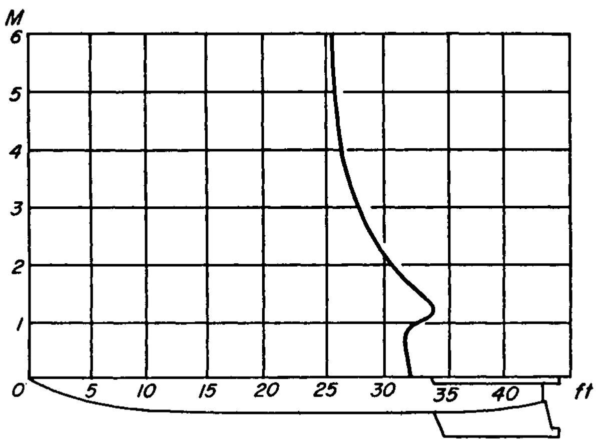


FIG. 7.40. Shift of center of pressure location on the rocket with flight Mach number.

long range ballistic rocket to flight velocity (Mach number). This graph, however, is not very informative, since it does not give a concept of rocket stabilization. It is preferable to show the relationship of the locations of center of gravity and center of pressure together with the function of time of rocket flight. Fig. 7.41 shows this relationship for the same rocket.

It is seen from the included curves that the center of pressure always remains behind the center of gravity of the rocket during the guided flight (t = 64 sec). Therefore, the rocket is statically stable during its entire guided flight.

The stability of a rocket is evaluated by the degree of stabilization, or, as it is called, the margin of stability—the ratio of the distance between the center of pressure and the center of gravity to the total length of the rocket. This value for finned rockets varies within the limits of 5 to 15%.

It must be noted, however, that stabilization by means of fins cannot be effective during flights beyond the atmosphere. Fins retain their usefulness only during the atmospheric portion of the rocket trajectory, and, in some cases, may be altogether discarded.

In constructing a graph similar to the one shown in Fig. 7.41, the loca-

tion of the center of gravity is determined by the simple calculations of weight and distribution of a fuel remaining in the rocket at a given moment.

The location of the center of pressure is determined by approximate calculations with the aid of model tests in wind tunnels.

It is possible to determine the distribution of pressure forces along the surface of a rocket with the aid of a model equipped with pressure probes. Referring the system of forces thus obtained to some point, it is possible to

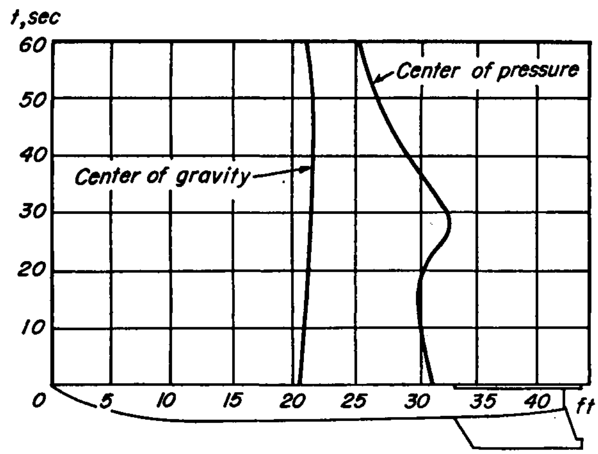


FIG. 7.41. Shift in location of center of gravity and center of pressure of a rocket with time during powered phase.

find the moment and the total aerodynamic force, and then the location of the center of pressure. The moment and the aerodynamic forces may also be determined directly with the aid of an aerodynamic balance. Then the location of the center of pressure may be found by so selecting the point of model support as to have a zero overturning moment.

2. DAMPING MOMENT

Damping moments are moments whose magnitude depends on the angular velocity of rocket rotation about some axis. Damping moments may be considered relative to the longitudinal and the two transverse axes. Damping moments are opposed to the direction of rotation, and, in the first approximation, are proportional to the angular velocity.

The development and magnitude of the damping moments depend on the conditions of flow around the rocket by an external stream, and on the conditions of flow of liquids and gases within the rocket and motor. Correspondingly, the moments are differentiated as external aerodynamic, and internal damping moments.

The external damping moment is obviously the result of a simple resistance of the air to the turning of a rocket.

During the rotation of a rocket with an angular velocity ω, the flow past a point located at a distance x from the center of gravity will take place with a change in the local angle of attack by the amount

$$\alpha_l = \omega x/v$$

(Fig. 7.42). Because of this, a local aerodynamic moment develops in the opposite direction from rotation. The damping moment is determined by the summation of the elemental moments along the surface of the rocket.

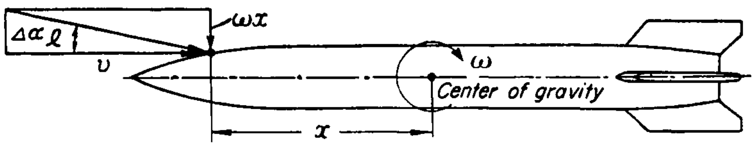


FIG. 7.42. Development of an additional local angle of attack during rotation of a rocket.

An expression for calculation of the damping moment can be derived by analogy with the expressions which were derived earlier for the aerodynamic forces D and L and the moment M_s. We will consider the total damping moment as being proportional to the velocity head $\frac{1}{2}\rho v^2$, characteristic area S, and ratio ω/v. Also, introducing from dimensional considerations the square of the rocket length l, we get

$$M_d = C_d \frac{\rho v^2}{2} \times \frac{\omega}{v} Sl^2. \tag{7.13}$$

The dimensionless damping moment coefficient C_d, like all the other aerodynamic coefficients so far encountered, is independent of flight velocity at low speeds. At high velocities this quantity is considered as a function of velocity. Fig. 7.43 shows, as an example, a graph of the relationship of coefficient C_d to Mach number M for a V-2 rocket during rotation about its transverse axis.

The internal damping moment is due to the Coriolis acceleration which occurs during the change in direction of the liquid flow through tanks and tubing of the rocket, and gas flow through the chamber and the nozzle of the motor. These moments are easily determined if we assume that the above-mentioned flows completely follow the turning of the rocket. Let us take an element of flow with the length dx' and cross section S_x, at a distance from the center of gravity x', and, consequently, with a mass $\rho S_x dx'$.

As is known, the Coriolis acceleration equals $2w\omega$, where w is the flow velocity in the rocket; and ω, the angular velocity of the rocket.

The elemental moment of the Coriolis force is

$$dM = 2w\omega\rho S_x dx'x.$$

The damping moment is determined by integrating this expression between the limits of x'.

$$M_d = 2\omega \int_{x_2'}^{x_3'} \rho w S_x x' dx'. \tag{7.14}$$

But, from the conditions of constant discharge

$$\rho w S_x = m = \text{constant}.$$

Therefore

$$M_d = 2\omega m \int_{x_2'}^{x_3'} x' dx' = \omega m(x_3'^2 - x_2'^2).$$

For this expression, limits of integration x_2' and x_3' in a liquid fuel rocket must be established between the surface of the component in the lower

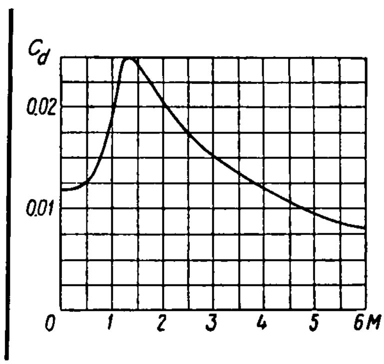


FIG. 7.43. The relationship of damping moment coefficient to Mach number during rotation of a rocket about its transverse axis.

tank and the exit plane of the nozzle (see Fig. 7.44). In this interval the sum of the discharges of both components is equal to m. To the obtained expression it is necessary to add a similar one in which the total discharge m is replaced by the mass discharge of the upper component m_1, and the integration performed between x_1' and x_2' (see Fig. 7.44), i.e., between the surfaces of the upper and the lower components. Finally, we will get

$$M_d = \omega[m(x_3'^2 - x_2'^2) + m_1(x_2'^2 - x_1'^2)].$$

Part of the mass moves toward the center of gravity and not away from it. Because of this the damping moment will be partially reduced.

To the obtained expression may be added moment $J\dot{\varphi}$, proportional to

the angular velocity $\dot{\varphi}$, which is isolated from the moment of inertia (see page 204).

The rate of change of the moment of inertia $\dot{J}$ of a rocket is determined from simple considerations.

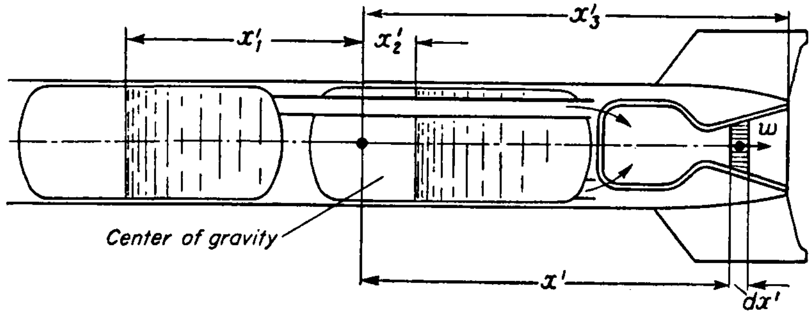


FIG. 7.44. Determination of the damping moment developed by the motor jet.

During time Δt, the distribution of mass in a rocket will change only in the vicinity of the surfaces of the first and second fuel components (see Fig. 7.44). Consequently

$$\Delta J = (-m_1 x_1'^2 - m_2 x_2'^2)\Delta t$$

and

$$\dot{J} = -m_1 x_1'^2 - m_2 x_2'^2,$$

where m_1 and m_2 are the discharge rates of the first and second components.

Adding $\dot{J}\omega$ to the above-obtained expression for the moment, and taking into consideration that $m = m_1 + m_2$, we obtain

$$M_d = \omega[m_1(x_3'^2 - 2x_1'^2) + m_2(x_3'^2 - 2x_2'^2)].$$

During rocket flight in dense regions of the atmosphere, and with the usual relationship between φ and $\dot{\varphi}$, the damping moment of a finned rocket constitutes approximately 10% of the stabilizing moment. Furthermore, the greater part of this 10% is due to the damping moment of the external aerodynamic forces and only a small part is due to the internal moment determined by the last equation. Therefore, in calculating rocket trajectories in the dense atmospheric regions the damping moment due to internal forces may be neglected. Only during rocket flight beyond the limits of the atmosphere, where aerodynamic forces are nonexistent, does this moment become independently significant.

E. Steering Forces

Let us examine the gas (jet) vanes as the basic functional steering members.

The jet vane in the stream of a motor is acted upon by the gas dynamic forces of the flow in the form of the frontal force D_v, the lift force L_v, and the moment M_v (Fig. 7.45).

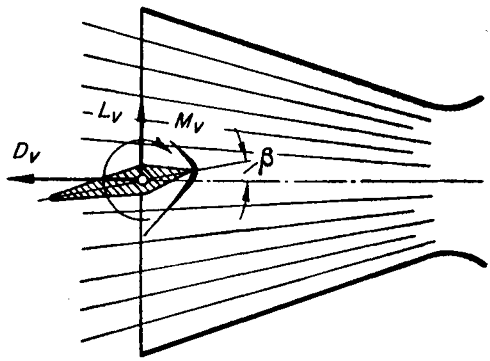


FIG. 7.45. Jet vane in the stream of a motor.

A vane in the gas stream behaves in a manner similar to an aircraft rudder in the airflow.

The frontal and lift forces on the vane, and also moment M_v, may be determined by the usual aerodynamic formulas

$$\begin{aligned} D_v &= C_{Dv} \times \tfrac{1}{2}\rho w^2 S; \\ L_v &= C_{Lv} \times \tfrac{1}{2}\rho w^2 S \\ M_v &= C_{Mv} \times \tfrac{1}{2}\rho w^2 S l \end{aligned}$$

where w is the gas flow velocity past the vane; ρ, the density of gas in the stream; S, the characteristic vane area; and l, the characteristic vane length.

Coefficients C_{Dv}, C_{Lv}, and C_{Mv} in this case may be considered as values independent of the flow velocity w, inasmuch as the flow is considerably supersonic (M $\approx$ 2.4), and at these velocities aerodynamic coefficients vary insignificantly with change in velocity. Finally, the exhaust gas velocity through the nozzle for all liquid fuel motors is approximately the same, and usually equals 6500–7200 ft/sec.

Coefficients C_{Dv}, C_{Lv}, and C_{Mv}, as do all aerodynamic coefficients, depend on the shape of the vanes and the angle of attack, i.e., on the vane turn angle β in the stream (see Fig. 7.45). It must be said that this angle varies through a considerable range. For instance, jet vanes in long range ballistic rockets are capable of turning in the stream up to $\pm 25°$, while the working range of the vanes is within the interval $\pm 15°$. This leads to the fact that the forces acting on the vane during the guided portion of rocket

flight vary between considerable limits. The typical dependence of coefficients C_{Dv}, C_{Lv}, and C_{Mv} on the vane angle of attack β is shown in Fig. 7.46.

The magnitude of the operating angle β in flight may be considered as consisting of two addends. The first addend is determined by the programmed flight of the rocket to the target, i.e., by that trajectory which must be imparted to the rocket under the ideal conditions of undisturbed flight. This vane angle of attack may be calculated beforehand. The necessary steering forces may be calculated according to the shape of the trajectory and, from them, the necessary angles may be calculated.

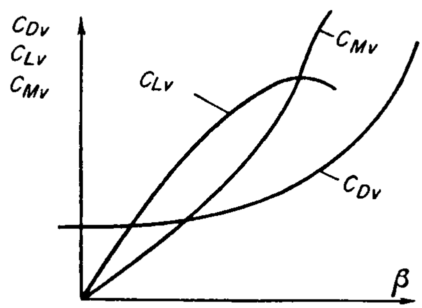


FIG. 7.46. Relationship of coefficients C_{Dv}, C_{Lv}, and C_{Mv} with the vane angle of rotation β.

The second addend is an angle, resulting from the reaction of the stabilizing mechanism and, consequently, of the vanes, to random disturbances. They include factors which constantly act during the trajectory, such as noncoaxiality of the aerodynamic and reactive forces and variable wind gusts. Their exact magnitude cannot be determined beforehand. It is determined on the basis of allowances for fabricating malalignment and from statistical data of observed atmospheric conditions. Allowance is made for additional vane motion, above that which is determined from programmed conditions, to compensate for possible wind gusts.

The first of the two mentioned addends of the operating angle β is basic, and results in systematically acting forces taken into consideration in ballistic calculations. Therefore, the steering forces D_v, L_v, and M_v are examined, first of all, as definite functions of time, inasmuch as angle β, as a function of time, is known. Concerning the second addend of angle β, it leads to random, unsystematically acting forces, which are not considered in ballistic calculations.

Let us examine the forces D_v and L_v, and the moment M_v separately.

The total jet vane force D_v is not a steering force and is considered as thrust loss due to jet vanes. This loss is considerable. For instance, for the V-2 rocket, when $\beta = 0$, it constitutes approximately 1400 lb (350 lb per

vane) and increases with vane rotation. A factor which somewhat reduces the magnitude of D_v, for graphite vanes, is the erosion of vanes at the surface during the operation of the motor.

The moment M_v consists of the sum of the stabilizing and damping moments proportional to the vane angle of rotation β and angular velocity $\dot{\beta}$, respectively. If, to this moment, we also add the moment of inertia of the vane $J_v\ddot{\beta}$, then we will get the magnitude of the hinge moment M_h, (see page 205):

$$M_h = a_0\beta + a_1\dot{\beta} + a_2\ddot{\beta}, \tag{7.15}$$

where a_0, a_1, and a_2 are the coefficients of proportionality.

For graphite vanes, these coefficients change somewhat during the operation of the motor due to erosion of graphite at the surface. The most substantial change is in the coefficient a_0, since the graphite erosion takes place essentially at the leading edge, which is subjected to the strongest flow (Fig. 7.47,a). This displaces the vane center of pressure considerably

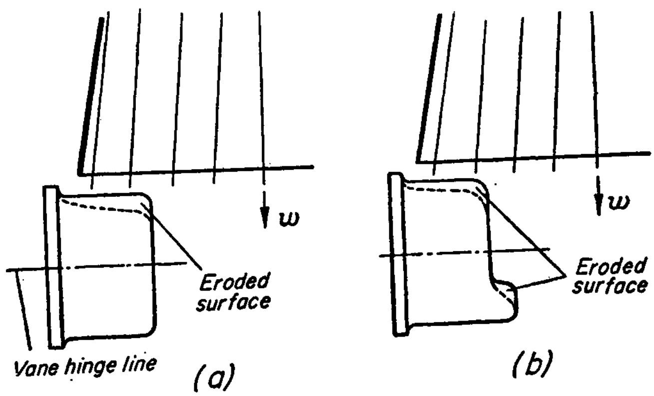


FIG. 7.47. Variation in the graphite vane configuration during the operating time of the motor.

aft (coefficient a_0 increases). In order to avoid this, the graphite vane is given the shape shown in Fig. 7.47,b. Here, the erosion takes place on both sides of the vane hinge line; therefore, the center of pressure is not displaced.

The hinge moment included in Eq. (7.3) is considerably smaller than the magnitude of $L_v x_v'$, and it may be neglected in the equation of motion. The magnitude of the hinge moment is important only for evaluating the necessary power requirements of the steering mechanisms which rotate the vanes.

The jet vane lift force L_v creates the main steering moment $L_v x_v'$, which turns the rocket in the required direction; see Eq. (7.3). The lift

force L_v itself is usually included in the aerodynamic lift force L; see Eq. (7.2). This is done in the following manner.

Let us assume the rocket flies with a constant programmed angle φ. Then, from Eq. (7.3), neglecting the term M_h, we get

$$M + L_v x_v' = 0.$$

But the moment

$$M = Le$$

where e is the distance from the center of pressure to the rocket center of gravity. Therefore

$$L_v = -Le/x_v'.$$

In this manner Eq. (7.2) is converted to the form

$$\dot{\theta} = (1/Mv)[(P - D_v)\alpha + L(1 - e/x_v')] - (g/v)\cos\theta. \qquad (7.16)$$

If the steering of the rocket is performed by the rotation of the combustion chamber (Fig. 7.48), then the thrust loss D_v will be very small, inas-

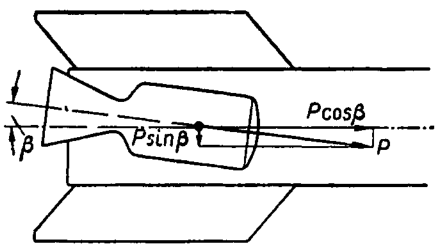


FIG. 7.48. Development of the steering force with the canting of the motor chamber.

much as the angle of chamber rotation β is small while the magnitude

$$D_v = P(1 - \cos\beta) \approx P(\beta^2/2).$$

The force L_v in this case is determined according to the formula

$$L_v = P\sin\beta \approx P\beta.$$

F. Change in Thrust and Rocket Weight Along a Trajectory

Of all the forces entering into the equations of motion (7.1), (7.2), and (7.3), we have left only two which were not examined: the thrust of the motor, and the weight of the rocket. The main reason for the change of thrust during flight is the pressure change in the surrounding medium. In gaining altitude, the atmospheric pressure changes and, together with it, the axial component of the external static pressure. This follows from expression (1.6)

$$P = mw + S_e(p_e - p).$$

In other words, as the rocket gains altitude, the thrust increases to correspond to the drop in atmospheric pressure.

For liquid fuel rockets, a second reason exists for some change in thrust during flight. This is the change in the discharge of fuel components due to variable conditions of delivery.

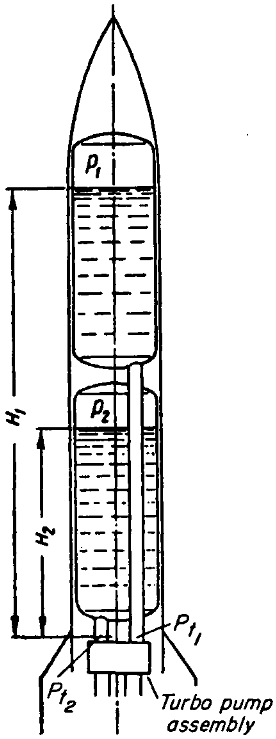


FIG. 7.49. Pressure determination at the intake of the turbopump assembly.

The pressure of fuel components at the entrance to the turbopump assembly is made up of pressures p_1 and p_2, above the liquid surfaces in the tanks (due to pressurization), and the pressure head of the liquid columns H_1 and H_2 (Fig. 7.49).

In a stationary rocket, the pressure at the entrance to the turbopump assembly for a component designated by subscript "1" will be

$$p_{t_1} = p_1 + \gamma_1 H_1$$

where γ_1 is the specific weight of the component.

If the rocket moves with an axial acceleration a, and its axis forms an angle φ with the horizon (Fig. 7.50), then

$$p_{t_1} = p_1 + \gamma_1 H_1 (a + g_h \sin \varphi)/g$$

where g_h is the acceleration of gravity at an altitude h.

In this expression, the terms p_1, a, g_h, H_1, and φ are variables along the trajectory with time; a increases, g_h, H_1, and φ diminish, while p_1 varies depending on pressurizing conditions. Sometimes, for instance, pressurization is accomplished by ram pressure. In this case, pressure p_1 equals the atmospheric pressure at a given altitude plus the ram pressure, with a correction for compressibility of air. With this system of pressurization, the pressure at the entrance to the turbopump first rises and then drops. Analogous reasoning applies to the second fuel component.

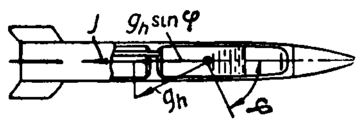


FIG. 7.50. Development of axial overloading resulting from axial acceleration a.

Increase in pressure p_t will cause approximately the same increase in component pressures at the entrance to the combustion chamber, inasmuch as the centrifugal pump creates approximately constant pressure drop. The picture is the same for the displacement delivery, for which p_t is simply the pressure at the entrance to the jets.

The thrust of the motor is a complex function of pressures p_t for each of the components. During change in the delivery pressure, the total fuel consumption varies, as well as the percentage ratio of the components.

It must be said, however, that the described complex dependence of thrust on conditions of delivery results qualitatively in comparatively small variations, considerably smaller, in any case, than those which arise in the static component of thrust as a result of the changes in the external pressure. In long range ballistic rockets the change in atmospheric pressure results in thrust variation of approximately 15%, whereas delivery conditions vary thrust by only 1–2%.

In speaking of thrust, it is impossible not to mention its variation during starting and stopping of the motor.

Some motors develop their full power not immediately upon fuel ignition, but after a passage of a more or less prolonged period of time, which constitutes some small fraction of the total operating time. Similarly, during cut-off of a liquid motor the thrust does not disappear immediately, but the so-called aftereffect is observed. After cut-off, there is still some thrust, due to the burning of the remains of the fuel. The magnitude of this thrust remains somewhat indefinite, which results in a noticeable range dispersion for ballistic long range rockets. In order to reduce the indicated dispersion, it is possible to shift the operation of the motor to a preliminary stage with reduced fuel supply before cut-off. Fig. 7.51 shows the typical graph of thrust variation for a motor of a long range ballistic rocket.

The weight of the rocket Mg_h also changes along the trajectory. This occurs because of the change in mass M and the variation in acceleration of gravity g_h.

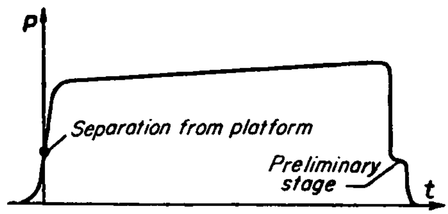


FIG. 7.51. Variation of thrust with time for a long range ballistic rocket.

The mass of a rocket at a given time during flight equals the initial mass M_0 minus the mass of the burned fuel.

$$M = M_0 - \int_0^t m dt.$$

If the consumption remains constant, then

$$M = M_0 - mi.$$

The acceleration of gravity at an altitude h above sea level is

$$g_h = gR^2/(R + h)^2$$

where g is the acceleration of gravity at sea level, and R is the earth's radius.

VIII. Rocket Flight Trajectory

A. Trajectory Phases

From the instant of initial motion, and ending at the time of impact or explosion, the flight trajectory of a rocket is usually divided into two phases: the powered phase, i.e., flight with an operating power plant, and a free, coasting phase with inoperative power plant (Fig. 8.1). Therefore, from the

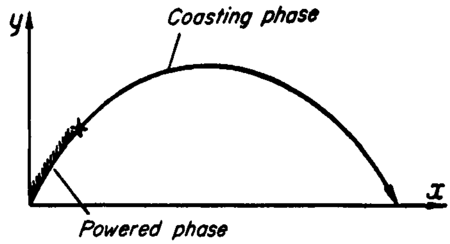


FIG. 8.1. Powered and coasting phases of a rocket trajectory.

point of view of flight dynamics the difference between trajectory phases is that, during the coasting phase, the thrust and the steering forces are excluded from the forces acting on the rocket.

The powered phase in its turn may be divided into phases.

In those cases in which the rocket is launched from trainable launchers, the motion of the launcher is examined separately. This region is primarily characteristic of unguided rockets; in particular, short range powder rockets. During motion along the launcher, coupling forces caused by launcher guide rails are added to the external forces.

For guided anti-aircraft rockets, the powered phase can be divided, depending on the system of guidance, into unguided and guided phases, and in the case of a rocket equipped with a seeking or homing system, into a homing phase also.

Long range ballistic missiles are launched vertically from a launching pad, and during the first few seconds rise straight up. This phase of the flight may be called the launching phase. Further on, the rocket starts its entry into the trajectory. The rocket deviates from the vertical and, describing an arc in the entry stage, enters the last, inclined phase (cut-off phase), where the power plant is switched off. Further on, the coasting phase of flight begins (free flight).

The coasting phase of the long range ballistic rocket is divided into two parts. The first part of the coasting phase takes place in such a rarefied region of the atmosphere that the aerodynamic forces have an insignificant effect, compared to the force of the rocket weight. During the second part of the coasting phase, in moving through denser atmospheric strata, a noticeable deceleration and stabilization of the rocket takes place, which up to that time moved without a definite angular orientation (Fig. 8.2).

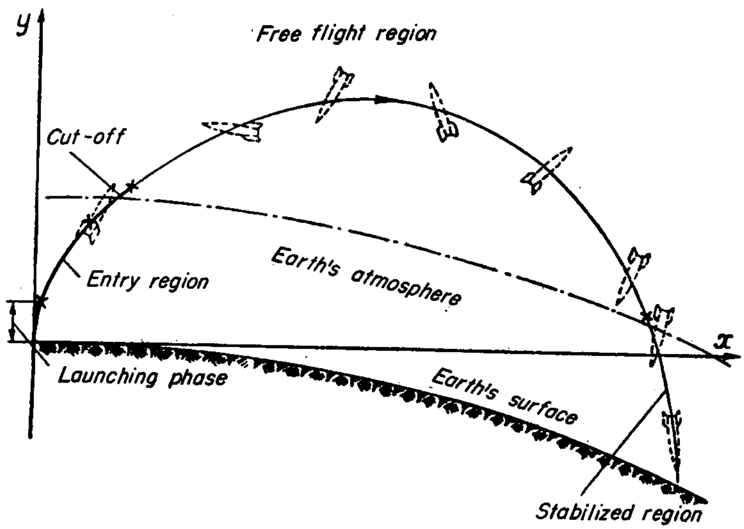


FIG. 8.2. Trajectory of a long range ballistic rocket.

In summarizing the above, it must be noted that the division of the trajectory into phases is arbitrary. It is done, primarily, according to the presence of various forces determining the motion of a rocket which dictate, to a considerable degree, the approach to the integration of differential equations of motion.

Now, let us return to the equations of motion (7.1), (7.2), and (7.3). Let us rewrite them, keeping in mind that Eq. (7.2) is reduced to the form of Eq. (7.16) and that in Eq. (7.3) the quantity M_h amy be omitted and moment M replaced by the sum $M_{st} + M_d$

$$\dot{v} = (P - D - D_v)M^{-1} - g \sin \theta \tag{8.1}$$

$$\dot{\theta} = (1/Mv)[(P - D_v)\alpha + L(1 - e/x_v')] - (g/v) \cos \theta \tag{8.2}$$

$$J(\theta + \alpha)^{\cdot\cdot} + M_{st} + M_d + L_v x_v' = 0. \tag{8.3}$$

Analytical integration of these equations in the general form is impossible. Therefore, we will dwell on the simplest specific cases.

B. Powered Phase

1. Powered Phase of a Long Range Ballistic Rocket

It is simplest to determine the law of motion of a long range ballistic rocket during the powered phase under the conditions of ideal steering, i.e., steering which completely assures conformity to the given regime of entry into the free flight trajectory.

For a given set of coordinates, at the end of powered phase, a definite velocity and flight direction must be imparted to the rocket at the termination of the powered phase in order for it to reach a given point. This is provided, first of all, by programmed steering.

The rocket, as has been mentioned above, is launched vertically. Then its axis deviates from the vertical. This deviation takes place gradually, and stops shortly before the end of power plant operation. The turning of the rocket is performed by steering mechanisms which receive their signal from the programming device. The rocket turns relative to a stable vertical reference, fixed by a gyroscope, according to a given function of time. The angle of inclination of the rocket axis to the horizon $\varphi = \theta + \alpha$ (see page 201) is called the programmed angle, and its variation with respect to time, the rocket program

$$\varphi = \theta + \alpha = f(t).$$

The long range ballistic rocket program has the shape of a curve, as shown in Fig. 8.3.

At the end of powered phase, angle φ is maintained constant, in order to reduce as much as possible the deviation from a given flight direction, in case the cut-off of the power plant takes place a bit sooner or later. The

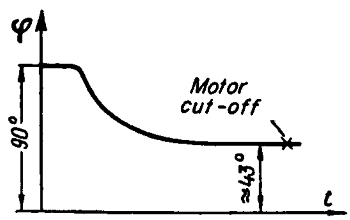


FIG. 8.3. The program curve of a V-2 rocket.

attainment of a given velocity is controlled by a special device, the so-called velocity integrator, which at the time of reaching the regimed velocity generates a signal to switch off the power plant.

Let us return to the equations of motion. Suppose that the rocket program has been established and that the steering is ideal.

For trajectory calculations in the first approximation, it may be assumed that the angle α is small, and that the lift force L, because of this, is also small compared to the projection of the weight force vector onto the normal to the trajectory, $Mg \cos \theta$. At the same time

$$\theta \approx \varphi = f(t)$$

i.e., angle θ turns out to be the given function of time. In this case Eq.(8.1) can be quite simply integrated numerically, independently of Eqs. (8.2) and (8.3). Let us rewrite this equation in the following form:

$$\Delta v = [(P - D - D_v)M^{-1} - g \sin \theta]\Delta t \tag{8.4}$$

Two more obvious ones are added to this equation in order to determine the coordinates of a moving rocket

$$\left.\begin{aligned} \Delta x &= v \cos \theta \Delta t \\ \Delta y &= v \sin \theta \Delta t \end{aligned}\right\} \tag{8.5}$$

where Δx and Δy are displacements of the rocket center of gravity along the axes x and y during the time Δt (Fig. 8.4). The variable coordinates x

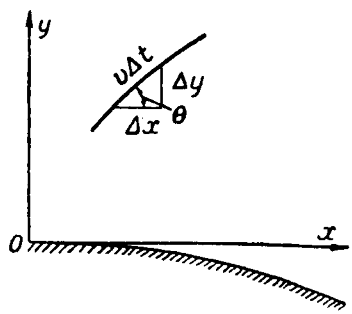


FIG. 8.4. Calculation of a rocket trajectory.

and y determine the position of the rocket along the trajectory at any instant.

During integration it is necessary, first of all, to select an integration interval Δt, for instance $\Delta t = 1$ sec. The smaller the interval the higher the precision of integration will be.

The integration is done in the following manner.

We assume that for the initial instant, the rocket velocity is equal to zero, and $\theta = 90°$. According to formula (8.4) we find the velocity after a time interval Δt after launching

$$v_1 = \Delta v = \left(\frac{P_0 - D_v}{M_0} - g\right) \Delta t.$$

Here, the drag D is assumed to be zero. Furthermore, it is obvious that

$$x = 0$$

and

$$y_1 = \Delta y = v_1 \Delta t.$$

Therefore, after time Δt after launching, the rocket is at an altitude y_1. In the standard atmosphere table we find the air density ρ corresponding to this altitude; from the graph $C_D = f(M)$, we find the coefficient C_D; and from velocity v_1, we calculate the drag

$$D_1 = C_{D1} \times \tfrac{1}{2}\rho_1 v_1^2 S$$

During time Δt the thrust P also changes: with increase in altitude, the pressure of the surrounding medium is reduced, and the thrust is increased. Besides, the thrust may change because of variation in the delivery conditions.

The mass of the rocket will decrease by the amount of fuel expended in time Δt, and will have a value M_1.

Angle θ, according to the program, may have a value other than 90° and will be equal to θ_1.

Therefore, at the end of the second step of integration, with the aid of formula (8.4) we get

$$v_2 = v_1 + \left(\frac{P_1 - D_1 - D_{v1}}{M_1} - g \sin \theta_1\right) \Delta t$$

and then according to formula (8.5) we find

$$x_2 = x_1 + v_1 \cos \theta_1 \Delta t$$
$$y_2 = y_1 + v_1 \sin \theta_1 \Delta t.$$

Furthermore, we determine parameters at the end of the third step

$$v_3 = v_2 + \left(\frac{P_2 - D_2 - D_{v2}}{M_2} - g \sin \theta_2\right) \Delta t$$
$$x_3 = x_2 + v_2 \cos \theta_2 \Delta t$$
$$y_3 = y_2 + v_2 \sin \theta_2 \Delta t.$$

Similar calculations are performed for all of the given trajectory phase.

As always, in numerical integration of equations, the results of calculations should be tabulated and reduced to a system requiring a minimum expenditure of effort.

The relationship of velocity, rocket acceleration, aerodynamic forces, and inertial forces to time and to rocket coordinates x and y along the trajectory may be obtained from these calculations.

Fig. 8.5 shows the trajectory of the V-2 rocket in a powered phase, and Fig. 8.6, variation of velocity, acceleration, and drag with respect to time for the same rocket. As can be seen from the curves, the drag initially increases sharply, due to increase in rocket velocity. Further on, the drag decreases due to reduction of air density ρ along the trajectory.

Rocket acceleration increases during the powered phase. This occurs because the mass of the rocket decreases and the thrust increases. Only in a small region (in a region of velocities close to the velocity of sound) does acceleration decrease somewhat, since the drag here increases sharply.

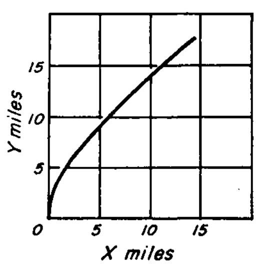


FIG. 8.5. V-2 rocket trajectory during the powered phase.

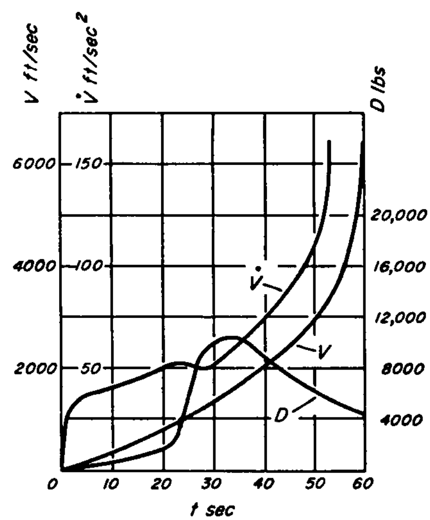


FIG. 8.6. Relationships of velocity, acceleration, and drag with respect to time of a V-2 rocket in a powered phase of the trajectory.

The angle of attack α can be determined as a function of time from Eq. (8.2), knowing the variation of the angle φ.

Fig. 8.7 shows, as a sample, the change in the angle of attack for a long range ballistic rocket. The shape of this curve is almost entirely determined by the rocket program.

Initially, while the rocket, according to the program, rises vertically, the angle of attack is equal to zero. With deviation of the rocket from the vertical, the angle of attack necessarily assumes a negative value (Fig. 8.8). If the rocket deviates sharply from the vertical, then the absolute value of the angle of attack will be large. Ordinarily, the absolute value of α does not exceed 2.5–3°.

The rocket is programmed in such a way that, at the time when the ram

pressure is maximum, the angle of attack would be equal to zero. This is necessary for the elimination of transverse aerodynamic forces, under whose action the rocket would fail structurally.

Further on, the angle of attack becomes positive and, at the end of the powered phase, rapidly increases, inasmuch as the programmed angle $\varphi = \theta + \alpha$ remains constant and angle θ decreases, due to the action of the acceleration of gravity.

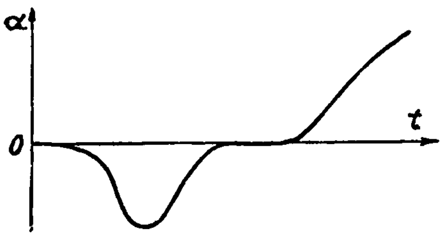


FIG. 8.7. An example of the relationship of the angle of attack to time of a long range ballistic rocket in a powered phase of the trajectory.

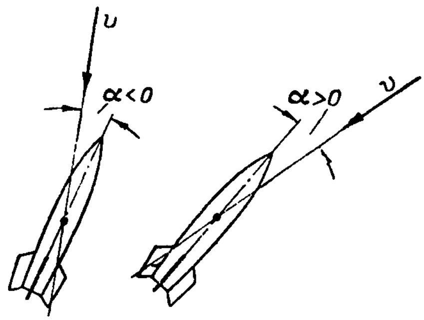


FIG. 8.8. Negative angle of attack at the start, and positive angle at the end, of the powered phase of the trajectory.

After the angle α has been determined as a function of time, the above-described ballistic calculation may be repeated with refined data, considering that

$$\theta = \varphi - \alpha.$$

The trajectory calculations are not performed for range determination alone. In integrating the equations of motion, the forces acting on the rocket with respect to time may be determined. This will enable the performance of structural analysis of rocket assemblies based on ballistic calculations. Specifically, knowing the values of acceleration and drag, it is possible to analyze the rocket air frame under the action of axial compressive forces. After determining the angle of attack α, transverse aerodynamic forces may be calculated, and the structural analysis may be performed to determine the significant bending moments on the air frame of the rocket.

2. The Unguided, Ideally Stabilized Rocket

As a second example of the integration of the equation of motion during the powered phase, let us examine the flight of an unguided, ideally stabilized rocket missile, launched from guide rails.

We will neglect the moments of inertia which develop during the rota-

tion of the rocket about transverse axis. This is equivalent to the assumptions that the stabilizing moment, if the rocket is statically stable (see page 237) will always so turn the rocket that its axis will coincide with the velocity vector, and the angle of attack α will become zero, and that the angular velocity of the velocity vector rotation θ is sufficiently small.

In this case, Eq. (8.3) will be satisfied for all points along the trajectory since, according to assumption, $\ddot{\theta}$ is vanishingly small, M_{st} and M_d become zero, and force L_s, for an unguided rocket, is absent. In Eq. (8.2) the force L will become zero, since the angle of attack is equal to zero.

Equations (8.1) and (8.2) are rewritten in the form

$$\left.\begin{aligned} \dot{v} &= \frac{P - D}{M} - g \sin \theta \\ \dot{\theta} &= -\frac{1}{v} g \cos \theta. \end{aligned}\right\} \tag{8.6}$$

These equations are most rationally integrated by the numerical method.

$$\left.\begin{aligned} \Delta v &= \left(\frac{P - D}{M} - g \sin \theta\right) \Delta t \\ \Delta \theta &= -\frac{1}{v} g \cos \theta \Delta t. \end{aligned}\right\} \tag{8.7}$$

Equations (8.5) are added to Eqs. (8.7).

$$\begin{aligned} \Delta x &= v \cos \theta \Delta t \\ \Delta y &= v \sin \theta \Delta t. \end{aligned}$$

During the motion of the rocket along the rails, the value of angle θ is constant and is equal to the launcher angle of elevation with respect to the horizon θ_0. In this case, the first Eq. (8.7) is integrated independently of the second.

If, while still on the launching rails, the rocket reaches such a velocity that the frictional resistance on the rails and drag D become significant, the integration of the first Eq. (8.7) should be done numerically. Assuming an integration interval Δt, we find

$$v_1 = \Delta v = \left(\frac{P_0}{M_0} - g \sin \theta_0\right) \Delta t.$$

Then, from the obtained velocity v_1, we determine drag D, find the new value of rocket mass $M_1 = M_0 - m\Delta t$ (m is the fuel consumption in a unit time), and take the next interval:

$$v_2 = v_1 + \left(\frac{P_1 - D_1}{M_1} - g \sin \theta_0\right) \Delta t.$$

The value of thrust P_1 may differ from the value of P_0. It is in this manner that the graph of change in rocket velocity on the launching rails is calculated as a function of time.

If the force D is negligible in comparison with the thrust P, Eqs. (8.6) are very simply integrated.

Assuming the mass discharge m and the value of thrust P as constant, we get with the aid of the first of Eqs. (8.6)

$$v = \int \left(\frac{P}{M_0 - mt} - g \sin \theta_0 \right) \Delta t$$

from which

$$v = -(P/m) \ln (M_0 - mt) - gt \sin \theta_0 + C.$$

The constant $C = (P/m) \ln M_0$, since only in this case is the initial condition, $v = 0$ at $t = 0$, satisfied. Therefore,

$$v = -(P/m) \ln (1 - mt/M_0) - gt \sin \theta_0.$$

Therefore, the velocity of the rocket leaving the launcher rails may be calculated in one way or another. Let us designate this velocity by v_0.

Subsequently, Eqs. (8.7) are integrated simultaneously. Again, the integration interval Δt is selected; and the velocity v_1, which the rocket will have in a time interval Δt after leaving the rails, and the angle θ_1, are determined

$$v_1 = v_0 + \left(\frac{P_0 - D_0}{M_0} - g \sin \theta_0 \right) \Delta t$$
$$\theta_1 = \theta_0 - \frac{1}{v_0} g \cos \theta_0 \Delta t.$$

It is understood that here P_0, D_0, and M_0 are values corresponding to the instant of leaving the rails, and not to the initial motion on the rails.

Furthermore, the values of x_1 and y_1 are determined by formulas (8.5)

$$x_1 = x_0 + v_0 \cos \theta_0 \Delta t$$
$$y_1 = y_0 + v_0 \sin \theta_0 \Delta t$$

and the second interval is performed.

$$v_2 = v_1 + \left(\frac{P_1 - D_1}{M_1} - g \sin \theta_1 \right) \Delta t$$
$$\theta_2 = \theta_1 - \frac{1}{v_1} g \cos \theta_1 \Delta t$$
$$x_2 = x_1 + v_1 \cos \theta_1 \Delta t$$
$$y_2 = y_1 + v_1 \sin \theta_1 \Delta t.$$

The behavior of rocket motion and the shape of its trajectory are determined in this manner.

The indicated integration of equations of motion may also be continued through the coasting phase of the trajectory, simply by assuming that at the time of motor cut-off $P = 0$ and $M =$ constant.

The form of the trajectory of a powder rocket missile is shown in Fig. 8.9. The disregard of the inertial moment during turn, to simplify the equa-

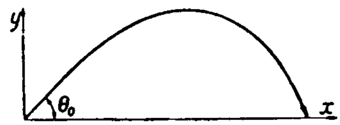


FIG. 8.9. Trajectory shape of an unguided powder rocket.

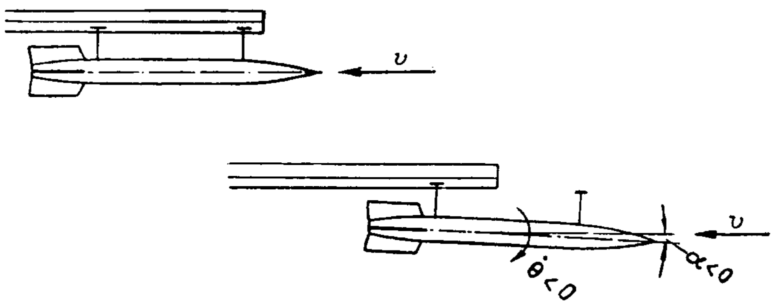


FIG. 8.10. Motion of a rocket upon leaving the launcher rails.

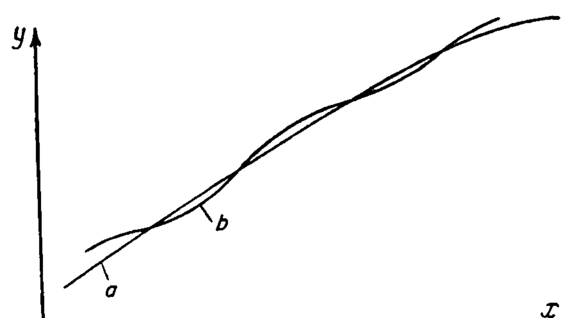


FIG. 8.11. Trajectory of the rocket center of gravity: a, in the absence of angular perturbations; b, in the presence of angular perturbations.

tions of motion, results primarily in the elimination from the obtained solution of the oscillatory motions of the rocket. Actually, the initial rotational impulse is imparted to the rocket immediately upon leaving the rails (Fig. 8.10). The rocket rotates in a direction reducing angle θ, and leaves the rails with a negative angle of attack. Further on, the rocket is stabilized,

performing an angular oscillatory motion. The center of gravity of the rocket also receives transverse oscillatory displacements due to the periodic change in the lift force L. The trajectory of the rocket center of gravity, taking into account angular oscillations, appears as shown in Fig. 8.11. As a first approximation, these oscillatory motions may be neglected, which has been done above.

C. Flight Beyond the Limits of the Atmosphere Within the Earth's Gravitational Field

1. Equations of Motion

The greater part of the trajectory of a long range rocket, or a high altitude meteorological rocket, occurs in such rarefied regions of the atmosphere that during this flight phase it is possible to neglect aerodynamic forces acting on the rocket. If, in addition to this, the flight is in the coasting phase, the equations of motion of a rocket, as a mass point, may be integrated for any rocket elevation above the earth, no matter how great.

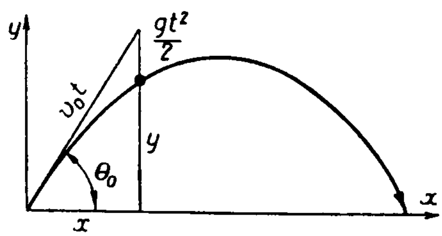


FIG. 8.12. Trajectory of a body projected at an angle to the horizon (parabola).

It is known in physics that any body thrown at an angle θ_0 with respect to the horizontal with velocity v_0, moves (if the air resistance forces are not considered) along a parabola. Indeed, at time t, after initial motion, the coordinates of a thrown body will be

$$x = v_0 t \cos \theta_0$$
$$y = v_0 t \sin \theta_0 - \tfrac{1}{2} g t^2.$$

By eliminating t, we find the equation of the parabola shown in Fig. 8.12:

$$y = x \tan \theta_0 - x^2 \frac{g}{2v_0^2 \cos^2 \theta_0}.$$

The obtained expression, however, is true only within limited boundaries. During its derivation it was assumed that the gravity acceleration vector g remains parallel to the axis y for all points on the trajectory, and does not change in value. Actually, this is not so. Acceleration due to gravity

g decreases with increase in altitude, proportionally to the square of the distance from the center of the earth. Besides, the force of earth's gravity is constantly directed toward its center, and g vectors at different points on the trajectory will not be parallel to each other. These properties of earth's gravitation cannot have significance if we are talking about low altitudes and short distances of the thrown body. However, if we are interested in long distances and high altitudes peculiar to long range ballistic and many meteorological rockets, then the indicated condition must necessarily be kept in mind.

The problem of determining the trajectory of a rocket flight under these conditions corresponds to the problem of determining the trajectory of an astronomical body in free flight (Kepler's problem). The theory of

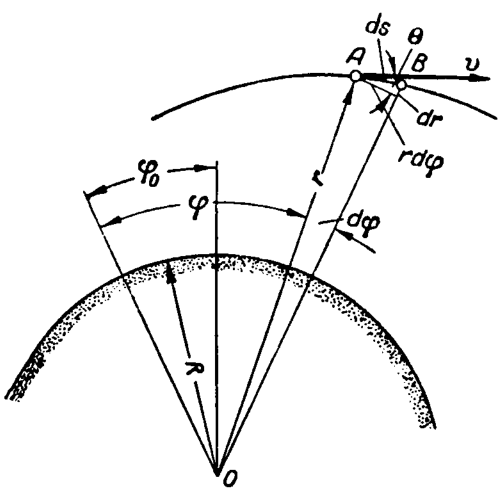


FIG. 8.13. Derivation of the equation of rocket motion in a polar system of coordinates.

body motion under these conditions is called "elliptical," as differentiated from the above-mentioned elementary problem of body motion along the parabola. The greatest interest of the elliptical theory lies in solving basic problems of astronautics, and other problems, such as the determination of the trajectories of a rocket earth satellite and of an interplanetary ship after its escape from the field of earth's gravitation.

Let us examine the motion of a rocket in a polar system of coordinates whose origin is at the center of the earth.

Let a rocket, represented in Fig. 8.13 by point A, move by inertia along some trajectory, and at a given time be located at a distance r from the center of the earth. During transition from point A to point B, the kinetic energy of a rocket $Mv^2/2$ changes by the amount $d(Mv^2/2)$. The potential

energy will change by the amount $d(Mg_r r)$. The acceleration of gravity g is not a constant quantity.

The change in kinetic energy is equal to the change in the potential energy, inasmuch as the rocket motor is not operating:

$$d(\tfrac{1}{2}Mv^2 - Mg_r r) = 0.$$

Since the mass M remains constant

$$\tfrac{1}{2}v^2 - g_r r = \text{constant}.$$

The acceleration of gravity is inversely proportional to the square of the distance to the center of the earth

$$g_r = gR^2/r^2$$

where R is the radius of the earth; and g, the acceleration of gravity at sea level.

Therefore

$$\tfrac{1}{2}v^2 - gR^2/r = \text{constant}.$$

At the initial point on the trajectory, where $r = r_0$, $v = v_0$. From this

$$\tfrac{1}{2}v^2 - gR^2/r = \tfrac{1}{2}v_0^2 - gR^2/r_0. \tag{8.8}$$

but the velocity

$$v = \frac{ds}{dt} = \frac{\sqrt{dr^2 + r^2 d\varphi^2}}{dt} = \sqrt{\dot{r}^2 + r^2\dot{\varphi}^2}$$

Then, instead of expression (8.8), we will get

$$\tfrac{1}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - g\dot{R}^2/r = \tfrac{1}{2}v_0^2 - gR^2/r_0. \tag{8.9}$$

Now let us derive the second equation. We can obtain it from the condition that the momentum of the rocket, relative to the center of the earth during coasting, is constant

$$Mrv \cos \theta = \text{constant}. \tag{8.10}$$

Angle θ is shown in Fig. 8.13; $v \cos \theta$ is a velocity component perpendicular to the radius.

It follows from expression (8.10) that

$$r\dot{v} \cos \theta = r_0 v_0 \cos \theta_0.$$

but it can be seen in Fig. 8.13 that

$$\cos \theta = rd\varphi/ds.$$

Therefore

$$rv \cos \theta = r(ds/dt)(rd\varphi/ds) = r^2\dot{\varphi}$$

and

$$r^2\dot{\varphi} = v_0 r_0 \cos \theta_0. \tag{8.11}$$

If Eqs. (8.9) and (8.11) are solved simultaneously, and if time t is eliminated, we get the relationship of r with respect to φ, i.e., the equation of rocket trajectory.

2. Trajectory of Flight

According to Eq. (8.11)

$$\dot{\varphi} = \frac{v_0 r_0 \cos \theta_0}{r^2}$$

but

$$\dot{\varphi} = d\varphi/dt = (d\varphi/dr)(dr/dt) = (d\varphi/dr)\dot{r}.$$

Taking this into consideration, we will get

$$\dot{r} = \frac{dr}{d\varphi} \cdot \frac{v_0 r_0 \cos \theta_0}{r^2}$$

or

$$\dot{r} = (dr/d\varphi)(k/r^2)$$

where

$$k = v_0 r_0 \cos \theta_0.$$

Substituting this expression for $\dot{r}$, and using the expression for $\dot{\varphi}$ obtained from (8.11) into Eq. (8.9), then

$$(dr/d\varphi)^2 k^2/r^4 + k^2/r^2 - 2gR^2/r = v_0^2 - 2gR^2/r_0.$$

From this

$$(dr/d\varphi)k/r^2 = \sqrt{v_0^2 - 2gR^2/r_0 - k^2/r^2 + 2gR^2/r}$$

or

$$d\varphi = \frac{-d(k/r)}{\sqrt{v_0^2 - 2gR^2/r_0 - k^2/r^2 + 2gR^2/r}}.$$

Under the square root sign let us add and subtract the constant term g^2R^4/k^2, and also introduce the term $-gR^2/k$ under the differential sign in the numerator, after which we will get

$$d\varphi = \frac{-d(k/r - gR^2/k)}{\sqrt{(v_0^2 - 2gR^2/r_0 + g^2R^4/k^2) - (k/r - gR^2/k)^2}}.$$

Integration of the last expression yields

$$\varphi - \varphi_0 = \text{arc cos} \frac{k/r - gR^2/k}{\sqrt{v_0^2 - 2gR^2/r_0 + g^2R^4/k^2}}$$

from which the required trajectory equation is obtained

$$r = \frac{k^2/gR^2}{1 + [(k/gR^2)\sqrt{v_0^2 - 2gR^2/r_0 + g^2R^4/k^2}] \cos(\varphi - \varphi_0)}. \tag{8.12}$$

The constant of integration φ_0 depends upon the reference angle φ (see Fig. 8.13).

Let us introduce the following notation

$$p = \frac{k^2}{gR^2} = \frac{v_0^2 r_0^2 \cos^2\theta_0}{gR^2} \tag{8.13}$$

$$e = \frac{k}{gR^2}\sqrt{v_0^2 - \frac{2gR^2}{r_0} + \frac{g^2R^4}{k^2}}$$

or

$$e = \sqrt{1 - \frac{2v_0^2 r_0 \cos^2\theta_0}{gR^2} + \frac{v_0^4 r_0^2 \cos^2\theta_0}{g^2R^4}}. \tag{8.14}$$

The final equation of the flight trajectory (8.12) will take on the form

$$r = p[1 + e\cos(\varphi - \varphi_0)]^{-1}. \tag{8.15}$$

Let us analyze the obtained expression.

It is known from a course in analytical geometry that Eq. (8.15) is an equation of the second order in a polar system of coordinates with the origin at one of the foci of the curve. Coefficient e is the eccentricity of the curve. When $e < 1$, Eq. (8.15) represents the equation of an ellipse; when $e > 1$, the equation of a hyperbola; and, finally, when $e = 1$, the equation of a parabola.

The parabola obtained from Eq. (8.15), when $e = 1$, should not be confused with that which was obtained earlier for a freely projected body moving in the earth's gravitational field under the influence of a gravitational vector, whose absolute value and direction are constant.

First, let us set $e = 0$. Equation (8.15) will take the form

$$r = p = \text{constant}.$$

In this case the body will move in a circle.

From Eq. (8.14) we will get

$$v_0^2 = (gR^2/r_0)(1 \pm \sqrt{1 - (1/\cos^2\theta_0)}.$$

It is not difficult to see that the velocity v_0 determined by the last formula has significance only in a case where $\cos^2 \theta_0 = 1$ and $\theta_0 = 0$. This means that the motion of a body along the circumference is possible only when the direction of the initial velocity vector is parallel to the horizon.

We get an expression for flight velocity

$$v_0 = \sqrt{gR^2/r_0} = v_1.$$

This is an expression for the orbital velocity with which we are acquainted from Chapter I. When $r_0 \approx R$, the numerical value of the orbital velocity:

$$v_1 \approx 26{,}000 \text{ ft/sec.}$$

Now, let us determine under what conditions the eccentricity e becomes greater than unity.

From expression (8.14) it can be established, without difficulty, that independent of the elevation angle θ_0 of the initial velocity vector, when

$$v_0 \geqslant \sqrt{2gR^2/r_0}$$

the value

$$e \geqslant 1.$$

In this case the trajectory of motion will be a hyperbola.

With

$$v_0 = \sqrt{2gR^2/r_0}$$

the motion is along a parabola, and with

$$v_0 < \sqrt{2gR^2/r_0}$$

along an ellipse.

The velocity

$$v_2 = \sqrt{2gR^2/r_0}$$

is called the parabolic velocity or, sometimes, the escape velocity.

Assuming that $r_0 \approx R$ we will get for the escape velocity the following numerical value:

$$v_2 = \sqrt{2gR} \approx 37{,}000 \text{ ft/sec.}$$

Figure 8.14 shows the variation in the shape of the trajectory of a body with respect to velocity at a constant projection angle $\theta_0 = 30°$.

At low velocity, a body released at an angle θ_0 with respect to the horizon will return to earth, having described an elliptical arc. The range of flight will be extended with an increase in initial velocity. The arc described by the flying body will also increase.

The trajectory becomes a parabola at velocity $v_0 = v_2$. A body projected from the earth with velocity $v_0 \geqslant v_2$ will not return to the earth. Hence, the name for v_2—escape velocity.

Up to the present time, such high velocities were not attained. However, with the velocities of the long range ballistic rockets which have been attained, the trajectory calculations according to the elementary "para-

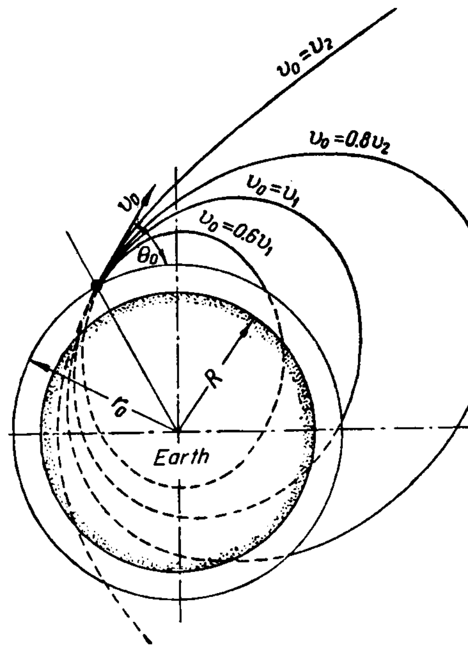


FIG. 8.14. Trajectories of a body related to the projection velocity at $\theta_0 = 30°$.

bolic" theory cannot yield sufficiently accurate results and it is necessary to perform the calculations according to the formulas of the "elliptical" theory.

The following procedure may be followed.

Up to a certain altitude, while the effect of the aerodynamic forces is still significant, the trajectory calculation may be carried out by the method of numerical integration described above. Starting with the time when it becomes possible to neglect drag, the calculation should be performed according to formulas (8.13), (8.14), and (8.15). In this way, it is possible to get the full picture of the variations in velocity and acceleration for the entire trajectory, depending on the duration of flight.

Figure 8.15 shows the graph of velocity and tangential acceleration of the ballistic rocket for its entire flight trajectory.

The velocity increases up to the time of motor cut-off. After cutoff, the velocity decreases, while the rocket gains altitude due to its momentum. After the rocket passes the trajectory apex, the velocity again starts to increase. Afterward, due to the action of air resistance forces, the velocity decreases.

Initially, acceleration increases in the powered phase, shown in Fig. 8.6. During the switch over of the motor to the preliminary stage, the acceleration sharply decreases. Following this, at full cut-off of the motor, the accel-

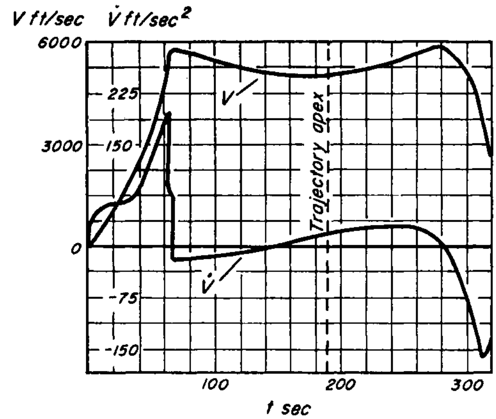


FIG. 8.15. Velocity and tangential acceleration along the trajectory of a ballistic rocket.

eration becomes negative. Obviously it equals $-g \sin \theta$. Further on, the tangential acceleration varies as the result of variation in θ and g. At the apex of the trajectory it equals zero. During the descending phase of the trajectory the force of earth's gravity accelerates the motion of the rocket. At reentry into the upper regions of the atmosphere, there begins a retardation of the rocket due to air resistance. At the very end of the trajectory the rocket velocity drops noticeably and the drag force lessens. The absolute value of the acceleration also falls at that time.

3. RANGE

At low velocities the range of a body projected at an angle θ_0 to the horizon is determined by an elementary formula derived from expressions on page 259

$$L = (v_0^2/g) \sin 2\theta_0.$$

The maximum range is at $\theta_0 = 45°$. This result is well known.

Things are more complicated at high projection velocities.

In this case, it is more expedient to express the range of the rocket by

the angle 2β, formed by the polar radius which passes through points A and C, the initial and final points, respectively, on the trajectory (Fig. 8.16). Due to the symmetry of the trajectory, it is sufficient to determine

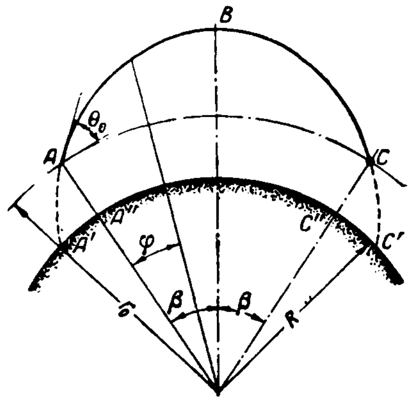


FIG. 8.16. Range determination of a ballistic rocket.

half of this angle—angle β (from point A to point B on the trajectory).

Let us find angle φ_0 in Eq. (8.15), setting $\varphi = 0$ and $r = r_0$

$$\varphi_0 = -\text{arc cos}\,\frac{p - r_0}{er_0}.$$

Equation (8.15) now takes the form

$$r = p\left[1 + e\cos\left(\varphi + \text{arc cos}\,\frac{p - r_0}{er_0}\right)\right]^{-1}.$$

Let us consider the eccentricity $e < 1$, which corresponds to the present velocity capabilities.

When the expression contained in brackets in the last equation becomes equal to π, the radius r will, obviously, reach its greatest value

$$r_{\max} = p/(1 - e).$$

At the same time, angle φ will be equal to β. Therefore,

$$\beta + \text{arc cos}\,[(p - r_0)/er_0] = \pi.$$

Substituting expressions (8.13) and (8.14) for p and e we get

$$\beta = \pi - \text{arc cos}\,\frac{(v_0^2 r_0/gR^2)\cos^2\theta_0 - 1}{[1 - (2v_0^2 r_0/gR^2)\cos^2\theta_0 + (v_0^4 r_0^2/g^2R^4)\cos^2\theta_0]^{1/2}}.$$

If this expression is differentiated with respect to $\cos^2\theta_0$, and the derivative is equated to zero, then, without undue difficulty, it is possible to find

that angle of projection θ_0 at which the value β will be maximum for a given velocity v_0

$$\cos^2 \theta_0 = \frac{1}{2 - v_0^2 r_0/gR^2}$$

or

$$\cos^2 \theta_0 = \frac{1}{2 - v_0^2/v_1^2} \tag{8.16}$$

where v_1 is the orbital velocity.

If velocity v_0 is low

$$\cos^2 \theta_0 \approx \tfrac{1}{2}.$$

This result has already been shown above; in order to obtain maximum range, the vector of the initial velocity must have an angle of 45° with the horizon.

When $v_0 = v_1$, as has been noted, $\cos^2 \theta_0 = 1$ and $\theta_0 = 0$. This case corresponds to the constant rotation of a body about the earth with an orbital velocity.

Figure 8.17 shows a graph of the optimum projection angle as a function of the ratio v_0/v_1. It is clear from the curve that the optimum angle de-

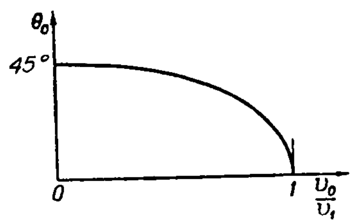


FIG. 8.17. Relationship of the optimum projection angle θ_0 to the initial velocity v_0 (v_1 is the orbital velocity).

creases with an increase in the projection velocity. At velocities of the order of 6500 ft/sec, the angle θ_0 only differs slightly from 45° (it is equal to approximately 44°). At high velocities it diminishes more and more noticeably.

At an optimum angle θ_0

$$\beta = \pi - \text{arc cos}\, \frac{-2\sqrt{1 - v_0^2/v_1^2}}{2 - v_0^2/v_1^2}. \tag{8.17}$$

If the square of the ratio v_0/v_1 is small compared to unity, then it is possible to obtain

$$2\beta \approx v_0^2/v_1^2.$$

For instance, at $v_0 = 6500$ ft/sec,

$$2\beta \approx \left(\frac{6500}{26{,}000}\right)^2 = \frac{1}{16}.$$

The range along the arc AC (see Fig. 8.16)

$$2\beta r_0 \approx 2\beta R = \frac{1}{16} \cdot 4000 = 250 \text{ mi.}$$

The presented range calculation is applied to the region of the trajectory which lies above the dense atmospheric layers (region ABC in Fig. 8.16). The actual range will be somewhat greater due to trajectory sectors AA' and CC'. The first of these (AA') is determined, as has already been mentioned, by the programming of the rocket into its basic trajectory. The region CC' may be considered, because of its shape, as the continuation of the basic elliptic curve. Here, the shape of the trajectory cannot noticeably vary from an ellipse. Only the rocket velocity changes noticeably, due to air resistance. The error in range determination due to substitution of an elliptical arc for region CC' is small, since the region CC' itself is small compared to total range. Therefore, the additional range in region CC' may be approximately calculated by formulas (8.13), (8.14), and (8.15).

The existence of section CC' affects the optimum angle of projection θ_0. The condition of maximum range from point A to point C' will differ from condition (8.16) postulated for the maximum range from A to C. Due to the difference in altitudes of points A and C', the optimum angle θ_0 is actually less than the one determined by formula (8.16). For instance, for a long range ballistic rocket, traveling at the end of powered phase with a velocity of 5000 ft/sec, the optimum projection angle θ_0, calculated according to formula (8.16), is equal to 44°30′. Actually, due to the existence of section CC', this angle must be approximately 42°.

4. Some Basic Problems in Mastering Interplanetary Space

In concluding this chapter, devoted specifically to the problems of flight with velocities commensurate with interplanetary velocities, let us pause briefly on the means of mastering interplanetary space. First of all we must agree which mastery we mean: rocket flights into interplanetary space with people as passengers, or without them. The first problem is immeasurably more complex than the second and cannot be the order of the day. At the same time, rocket flights into interplanetary space without passengers seem to be entirely possible in the immediate future.

The first, basic, and still unsolved, problem in this direction is the construction of an artificial earth satellite in the form of a rocket, equipped with transmitting instruments. The solution of this problem would mean, first

of all, an attainment of a definite and quite high degree in the development of rocket technology. At the same time, with the aid of an artificial satellite, it would be possible to collect extensive scientific data which would enable us to check and refine our concepts of the physical conditions beyond the limits of the earth's atmosphere.

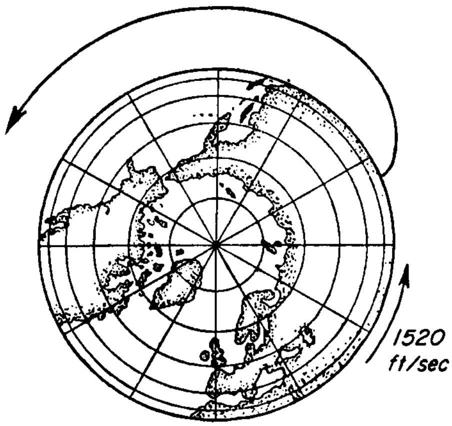


FIG. 8.18. Direction of a rational rocket path for launching an artificial earth satellite.

In order to put an artificial rocket satellite beyond the limits of the atmosphere it is necessary to impart to it a velocity of the order of 26,000 ft/sec. The velocity of the earth's rotation may be partially used for that. At the equator the earth's surface has a circumferential velocity of about 1500 ft/sec, directed from west to east. Therefore, in putting an artificial satellite into orbit, it is more reasonable to launch the rocket from the equator in an easterly direction (Fig. 8.18).

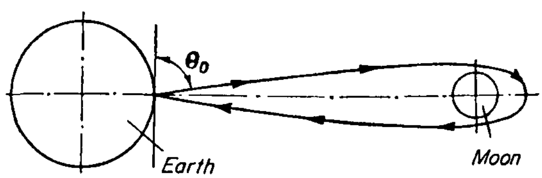


FIG. 8.19. Symmetrical trajectory around the moon.

The second interesting problem in mastering interplanetary space is the launching of a rocket to circle the moon, with return to earth (Fig. 8.19). With this flight it would be possible to obtain a photograph of the part of the moon invisible from the earth. In order to solve this problem, attainment of a considerably greater velocity is necessary; specifically, velocity close to escape velocity. But this is not the only difficulty. Calculations show that for a rocket to return to earth after flying around the moon

it is necessary to maintain the magnitude and direction of rocket velocity very strictly at the end of powered phase.

With a symmetrical trajectory shape as shown in Fig. 8.19, it is necessary that the angle θ_0 be within limits of from 81° to 89°. It is not difficult to satisfy this condition. At the same time it is necessary that the velocity v_0 be within limits of from 0.99 v_2 to 1.0001 v_2, where v_2 is the escape velocity. If velocity v_0 is less than 0.99 v_2, the rocket will fall on the moon. Should the velocity be greater than 1.0001 v_2, the rocket, having flown around the moon, will not return to the earth. To maintain such a narrow velocity margin represents quite a difficult problem at the present time. It seems

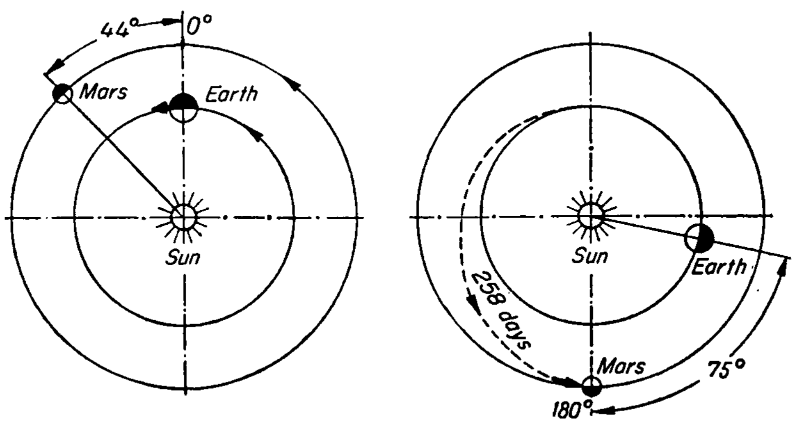


FIG. 8.20. Trajectory of a rocket flight to the planet Mars with a minimum expenditure of energy.

that for its solution it may not be necessary to increase the precision of steering mechanisms, but to introduce auxiliary motors which, when required, will correct the rocket flight along a trajectory.

In launching a rocket to the moon, just as in launching a satellite, the rotational velocity of the earth may be partially exploited.

Problems even more complicated than the two described are involved in reaching the nearest planets, Mars and Venus.

If it were possible to place a rocket on these planets, even without passengers but equipped with powerful transmitting apparatus, we could expect to obtain considerable data on the physical conditions existing on the surface of these planets.

In launching a rocket to Mars or Venus along an optimum trajectory, not only the earth's rotation about its axis may be used, as was shown above, but the earth's velocity around the sun as well. The optimum flight trajectory to Mars from the standpoint of energy expenditure (but not of time) is shown in Fig. 8.20. It represents a curve, approaching an ellipse,

with the sun at one of its foci. For free flight along such a trajectory, the rocket, upon leaving the earth, must have a velocity of about 8.7 mi/sec imparted to it.

The problem of mastering interplanetary space represents a very complex but not an insoluable problem. In 1929 C. E. Tsiolkovski wrote on this subject:

"Interplanetary travel cannot even be compared with flying through the air. The last is a toy compared to the first.

Undoubtedly, success will come, but the time necessary for this achievement cannot be predicted by me.

The notion of the ease of its solution is only a temporary fallacy. Of course, it is beneficial since it adds courage and strength.

If the difficulties of the problem were known, many who are now working with enthusiasm would be dismayed.

But, how beautiful the achieved goal will be."

The time that has passed since these lines were written shows that C. E. Tsiolkovski was very right in his judgements. This problem is very complex indeed. At the present time, rocket technology is on the threshold of solving the first problem of interplanetary flight—the creation of the artificial earth satellite.

IX. The Basic Principles of Stabilization and Steering

A. Methods of Stabilizing and Steering a Rocket

1. Stability and Stabilization

The motion of a rocket under the influence of regular, systematically acting forces (regardless of whether or not these forces are considered in the equations of motions) is customarily called the undisturbed motion of a rocket. For instance, the motion of a long range ballistic rocket, subjected to regular thrust and aerodynamic forces, will be considered undisturbed. The motion of an anti-aircraft rocket seeking a maneuvering airplane, in spite of the indeterminate nature of this maneuvering, must also be considered as undisturbed. Along the pursuit trajectory the rocket is acted upon by steering forces, which are subject to the prescribed conditions, depending upon the method of guidance.

Aside from the regular forces, random, short-duration, external influences may act on the rocket in flight, such as wind gusts, short-duration small variation in the motor thust, etc. Under the action of these forces the rocket will take on a motion which is called the disturbed motion.

Let us assume that the action of random forces is small, and that the disturbed motion differs only slightly from the undisturbed. If, after the discontinuance of the action of the disturbing forces, the oscillatory motion of the rocket again becomes nonoscillatory, this nonoscillatory motion of the rocket should be called stable. If, however, after the discontinuance of the action of the external disturbance the motion of the rocket does not approach nonoscillatory motion, then such motion will be considered unstable (Fig. 9.1).* The concept of stability and instability of the motion is applicable, naturally, not only to rockets, but, in general, to the motion of any body.

It is understood that in all cases, if we wish to control motion, it is necessary, first of all, to eliminate undesirable influences of random external disturbances and thus assure the stability of the motion.

In actual cases, the rocket in flight is continually subjected to the action of various random disturbing factors. In the time interval between two consecutive external disturbances, the oscillatory motion of a rocket does

* In practical investigations of rocket stability, the motion is considered to be stable even in the case where the rocket, after discontinuance of action of the disturbance, returns not to the nonoscillatory, but to some other, slightly different motion.

not have time to return to the parameters of the initial nonoscillatory motion. Therefore, under actual conditions of flight, the motion of the rocket only approximately corresponds to that required. The rocket continually follows an oscillatory motion whose parameters fluctuate about the parameters of the stable nonoscillatory motion. If the deviations from the desired motion are not large, then, for practical purposes, the motion of the rocket may be considered as being stable.

In connection with what has been said, the above definition of stability of motion may be changed. If, in the final analysis, the effect of the disturbing factors on the rocket varies, within allowable limits, the motion may be considered stable. Or, briefly put, in stable motion the external random disturbances do not cause large variations in motion parameters.

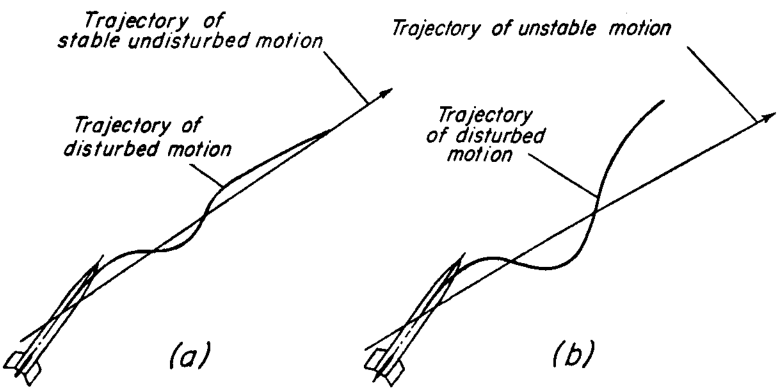


FIG. 9.1. *a*, Stable; and *b*, unstable rocket motions.

The boundaries of allowable variations in the prescribed motion of a rocket are determined by a whole series of considerations. Major among them is the requirement for accuracy of aiming at a target and the structural integrity of the rocket since, at large angles of attack and transverse accelerations, the rocket is subjected to considerable side forces.

It must be said that motion stable relative to one set of parameters may be unstable relative to another set. Therefore, when the stability of motion is discussed, especially stability along allowable deviations, it is necessary to specify the type of motion which is being considered. The motion of a rocket is defined by the motion of its center of gravity and the angular displacement of the whole rocket as a rigid body about the center of gravity.

The concept of stability and instability, in the sense of angular displacement, is usually replaced by the concept of angular stabilization or unstabilization of a rocket. If a rocket to which angular deviation has been imparted

returns to a nonoscillatory motion after the disturbing influence ceases to act, the rocket is called stabilized. If this does not happen, then the rocket is called unstabilized.

In many cases the questions of angular stabilization may be examined independently of the stability of rocket motion as a mass point, even though, in the example of an unguided powder rocket (see page 258), we saw that the angular disturbance is related to the disturbances in the motion of the center of gravity. The rocket may be found to be unstabilized even when deviation from the prescribed motion of its center of gravity may not fall outside the allowable limits.

2. Properties of Oscillating Motion and Stabilization of Unguided Rockets

Stabilizing aerodynamic moment and aerodynamic stabilization have already been mentioned in Chapter VII. If, in referring all aerodynamic forces to the rocket center of gravity, the stabilizing moment acts to decrease the angle of attack, the rocket is called statically stable. If the moment is directed in an opposite direction, the rocket will be statically unstable.*

In Chapter VII the concept of the center of pressure as an intersection point of the resultant of aerodynamic forces with the axis of the rocket was also developed. In the case in which the center or pressure is located aft of the center of gravity the rocket is statically stable. If the center of pressure is located forward of the center of gravity the rocket is statically unstable.

The ratio of the distance between the center of pressure and the center of gravity to the length of the rocket is called the margin of stability.

With a positive margin of stability the rocket having an angular deviation is turned by aerodynamic forces to its initial position. When the angle of attack becomes zero the restoring force will become zero. But, since the rocket maintains angular velocity, the motion will continue in an opposite direction. An angle of attack with an opposite sign will develop and angular oscillations of the rocket will arise. Due to the presence of damping forces the oscillations will decay (Fig. 9.2).

Oscillations of this type are called short period oscillations. Their frequency depends on the margin of stability and the moment of inertia of a rocket. The higher the margin of stability and the lower the moment of inertia, the higher the frequency of these oscillations, the greater the damp-

* In the aircraft industry the frequently used expression "static stability" is equivalent to the concept of "aerodynamic stability." Static stability is ordinarily considered to be stability without interference of steering devices; as opposed to dynamic stability, stability of motion insured by the introduction of a steering system, it being immaterial whether the steering system is automatic or nonautomatic.

ing, the quicker the oscillations will decay, and the faster the rocket is restored to the steady state motion.

However, excessive increase in the margin of static stability may yield negative results. A guided rocket with a high margin of stability becomes unresponsive. The unguided rocket with a high margin of stability will give large values of dispersion (see page 318).

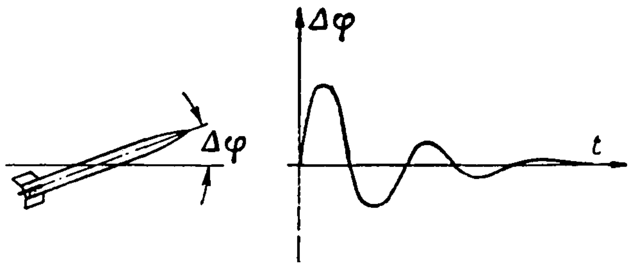


FIG. 9.2. Decay of a short period oscillation.

The short period angular oscillations described above may occur, as is easily understood, about the three central axes.

The presence of the so-called long period, or phugoid oscillations, is peculiar to a rocket, especially a winged rocket, of the pilotless aircraft type. Their origin is most conveniently explained by examining a uniform horizontal flight of a pilotless aircraft whose thrust, on the average, is balanced by a drag force at a constant angle of attack.

FIG. 9.3. Phugoid, or long period, oscillations of a pilotless aircraft.

Let us assume that, for some reason, the thrust has increased for a short period of time. In this case the flight velocity increases, and lift force also increases, becoming greater than the weight force. The pilotless aircraft starts to climb and because of this loses speed. There comes a time when the lift force becomes equal to the weight force, but the aircraft, having vertical velocity, continued to climb because of momentum. Continuing climb will result in the reduction of the lift force. The aircraft will begin to descend. The phugoid oscillations develop in this manner. The aircraft trajectory for this case is shown in Fig. 9.3.

The phugoid oscillations have a long period, frequently considerably exceeding the operating time of the rocket motor, i.e., exceeding the time of flight in the powered phase. Therefore, phugoid oscillations need not be considered for many types of rockets.

The various kinds of rocket oscillations are not independent. During

transverse angular oscillations, due to periodic development of lift force, transverse oscillation of the center of gravity appears about the steady state trajectory.

If the rocket moves with an angle of attack, then, with the appearance of the yaw angle, the rocket will roll. This is clearly evident in Fig. 9.4, which shows a rocket with a yaw angle ψ. The right stabilizer has additional velocity, the lift force on it increasing, whereas on the left it decreases. A moment develops, rotating the rocket about its longitudinal axis and giving rise to rocket roll.

The problem of stabilization consists of limiting and damping out as quickly as possible all oscillations of the types described.

With relatively simple static stabilization, the effects of short duration, random disturbing influences acting on the rocket during flight are satisfactorily excluded, but the influence of systematic, constantly acting factors which are not considered beforehand remains. To their number belong, first of all, technological errors.

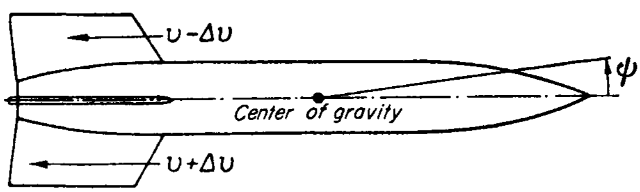


FIG. 9.4. Development of roll during rocket yaw.

The rocket cannot be fabricated exactly. The rocket motor always has a certain eccentricity, and the line of thrust does not pass through the center of gravity of the rocket. This creates a moment, turning the rocket sideways. The external contours of the rocket cannot be fabricated ideally. Therefore, aerodynamic forces also have a certain eccentricity. Stabilizers themselves, whose purpose it is to insure stability of motion, always have inaccuracies due to fabrication which lead to the deviation of the rocket from the required trajectory. The aerodynamic stabilization cannot be effective during flight beyond the limits of the atmosphere.

There is another possible way of assuring stability of motion of an unguided rocket. It consists of imparting to the rocket rapid rotation about its longitudinal axis. This method, as is known, is widely used for stabilizing artillery shells. In this case, the rocket maintains the direction of its axis due to the gyroscopic effect.

The method of spin stabilization is used primarily in small powder field rockets. These rockets, as we already know, are called turbo-reactive shells. Large rockets cannot be stabilized by these means. With rapid rota-

tion it is difficult to insure the necessary strength of a large rocket, let alone the difficulties in making such a rocket guided.

A more refined, but immeasurably more complex, method than the two described is the installation in the rocket of an automatic stabilizer, which will at the same time guide the motion of the rocket according to a predetermined program.

3. Stabilization by Means of an Automatic Pilot

Automatic stabilizers were first produced to insure the constant direction of motion of marine torpedoes. Later, with the development of aviation and a need for automatic steering of aircraft, autopilots were produced whose basic principles were applied to automatic stabilizing apparatus for rockets and pilotless aircraft during the Second World War.

The basic operating principle of such robots is most easily understood from an example of the simplest steering device (Fig. 9.5).

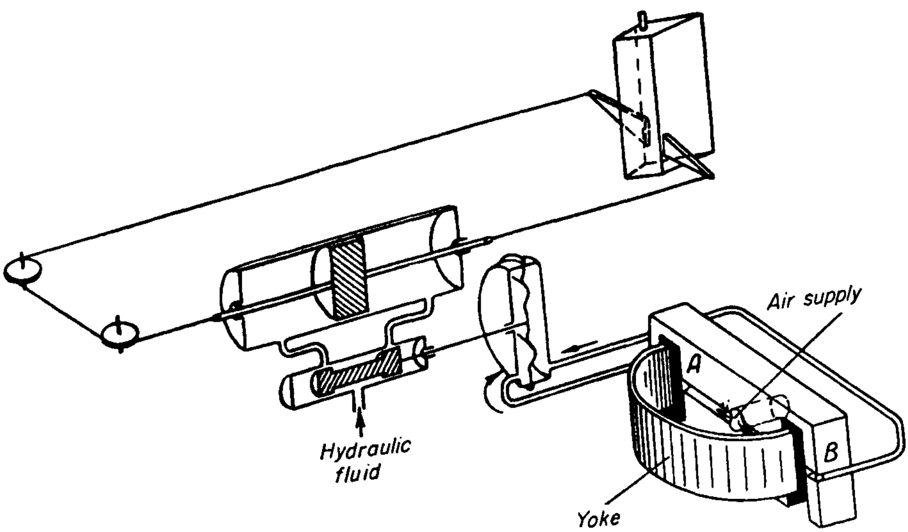


Fig. 9.5. Schematic of the simplest automatic pilot.

The actuating member of the simplest steering device is a cylinder with a movable piston which actuates the vane of the aircraft. The pressure on both sides of the piston in the cylinder is regulated by a distributing valve which is actuated by a sensitive diaphragm. In order to supply oil to the right or left cylinder cavities the piston valve must be moved correspondingly to the right or to the left. The displacement of the regulating valve does not require appreciable force and, therefore, the thin sensitive diaphragm is able to control the motion of the rudder.

The pressure is admitted into both chambers of the diaphragm housing from the pneumatic system of the aircraft through ports half closed by slides. A yoke, rigidly attached to the gyroscope, and therefore remaining motionless during the turning of the aircraft, serves as the slides. While following the course, both ports are closed to the same degree and the pressure in both chambers of the diaphragm housing is equal. The rudder remains in neutral position.

Let us suppose now that the aircraft, for some reason, begins to deviate from the established course, for instance, to the left. Port A will be partially closed and port B opened. The pressure in the right chamber of the diaphragm housing will rise and the piston of the valve will be displaced to the left. This will result in admission of oil into the forward chamber of the actuating mechanism (steering device). This will turn the vane counterclockwise and the aircraft will begin a turn to the right, i.e., resume its course.

It would seem that the described robot fully solves the problem of steering. However, this system has a major deficiency.

Let us see how the aircraft will move later on.

After the aircraft has assumed the desired direction the slides, diaphragm, and valve will return to neutral position, but the piston of the actuating mechanism and the vane will remain in the same position and the

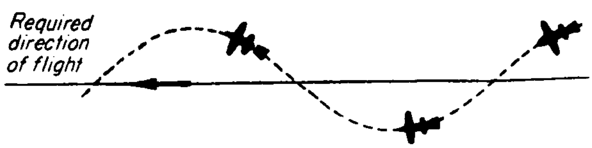


FIG. 9.6. Motion of an aircraft while controlled by the simplest automatic pilot.

aircraft will consequently continue in a right turn. Only when the aircraft overrides to the right, the slides close port B and open port A, and the valve moves in the opposite direction, will the vane force the aircraft to turn left. This gives rise to oscillations about a course—yawing. The aircraft will travel in a "snake" path (Fig. 9.6).

A similar pattern could be observed if the driver of a car in turning to the right began to straighten out his steering wheel only after the car had turned fully to the right. If the driver continued to repeat this error, i.e., was late in straightening out the wheel to a neutral position, the car would also travel like a snake. In order to eliminate this motion the driver must begin straightening out the wheel prior to the time when the car has assumed the proper direction.

Obviously, it is necessary that the steering device, in controlling the motion of the aircraft, return the vane from its deflected position at the

time the aircraft is back on course. This is the purpose of the feedback. In the broad sense of the word, the feedback is an element of a regulating system tying in the following link with one of the preceding ones in such a way that the signals transmitted to the regulating system conform with the results performed by the system.

Figure 9.7 shows the same steering mechanism, but with the vane feedback. Here the air distributor AB is no longer rigidly connected with the aircraft, and is capable of rotation about a vertical axis. This rotation is linked to vane movement.

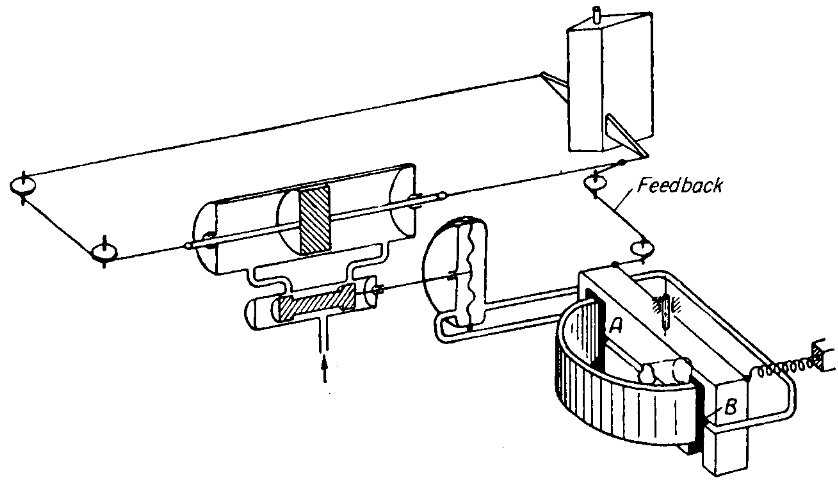


FIG. 9.7. Schematic of an automatic pilot with vane feedback.

Let us now see how the aircraft will get back on course.

Again, let us assume that for some reason the aircraft deviated from the course to the left. The elements of the mechanism will then operate in the above-described sequence and the vane will turn counterclockwise. Simultaneously, by means of feedback, the vane will turn the air distributor and will return it to the neutral position. While the aircraft is turning to the right, port A will begin to open and port B to close, with the result that the vane will turn in the opposite direction. When the aircraft resumes its course the vane and, with it, the slide of the pressure distributor, will assume a neutral position relative to ports A and B. The resumption of course by the aircraft will take place without yawing (Fig. 9.8).

The above-described oscillating instability (see Fig. 9.6) may be avoided by methods other than the introduction of a rigid feedback from the rudder. It is possible, for instance, to give the actuating device—the steering mechanism—signals not only proportional to the angle of rocket deviation

from the true direction, but also proportional to the velocity and acceleration of this deviation, i.e., by introducing the so-called proportional feedback. In this case, during rapidly increasing rocket deviation the stabilizing devices react to the deviation more actively and return the rocket on course more rapidly. This type of stabilizing system is used in ballistic rockets.

The operation of the automatic steering in yaw was described above. It is understandable that for a pilotless missile or a rocket this automation must be realized not only in yaw but also in pitch and roll.

The function of automatic stabilization in a rocket is not confined to pure stabilization alone. Inasmuch as the stabilizing mechanism controls the motion of the rocket according to a definite program, it also becomes a steering device.

The rocket equipped with a stabilizing mechanism is called a guided rocket. If its guidance, from the moment of launch, is not tied in to the

Fig. 9.8. Motion of an aircraft while controlled by automatic pilot with feedback.

earth, then such a guidance system is called self-contained. However, if the link with the ground is maintained after launch, for instance, by a radar beam, then the guidance system is called remote.

The self-contained system has one obvious deficiency, namely, after the moment of launch any further interference with the motion of the rocket is precluded. In particular, it is impossible to correct the motion of the rocket if it begins to deviate from its course.

Transmission of a signal along a beam enables introduction of corrections into the motion of a rocket and attainment of a considerably greater accuracy of hits. For anti-aircraft rockets beam guidance allows the aiming of the rocket directly at the enemy aircraft and sharply increases the percentage of hits as compared to unguided rockets.

However, beam guidance has a major drawback in that the enemy may, by means of creating sufficiently powerful jamming, nullify the advantage of remote guidance.

The problem of beam guidance is one of the basic problems of contemporary rocket technology. Concurrent with the development of beam guidance, means of jamming as countermeasures are also developed. The development of the last necessitates the development of countermeasure-proof guidance apparatus. All of these questions are problems of a special branch of radio technology (electronics).

B. The Gyroscope and Its Application

1. Properties of the Gyroscope

A massive, precisely balanced flywheel, rotating with high angular velocity, is called a gyroscope.

Everyone is well aware of the peculiar behavior of the gyroscope; the rapidly rotating flywheel has the ability more or less to maintain the direction of its axis stably, and displays an unexpected "disobedience" when an attempt is made to rotate this axis.

The gyroscope is the basic element of contemporary autopilots and the majority of stabilizing devices. Therefore, it is reasonable to begin the examination of the operation of stabilizing devices with the gyroscope.

The theory of the gyroscope is a specific problem in the general theory of a moving body with one stationary point. Equations of motion may be derived for such a body (Euler's equations) which in a general form cannot

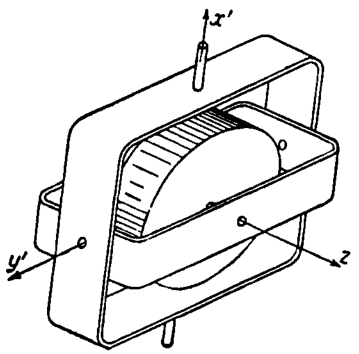


Fig. 9.9. Gyroscope in a gimbal suspension.

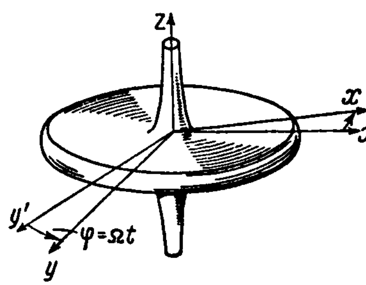


Fig. 9.10. Orientation of fixed gyroscope axes.

be solved, but which enable us to obtain answers to certain specific problems of motion, including the problem of motion of a symmetrical, rapidly rotating disk. The last problem constitutes the substance of the so-called theory of the gyroscope. We will not delve into this theory and will restrict ourselves primarily to the examination of its qualitative side.

First of all, we will assume that the gyroscope consists of a symmetrical body with one stationary point, and that this point is its center of gravity. A suitable mount may be made using a Cardan suspension (Fig. 9.9). Under these conditions, the motion of the gyroscope may be represented as the rotation, with angular velocity Ω, about some instant axis passing through the center of gravity.

Let us take three mutually perpendicular axes, x, y, z, rigidly connected to the gyroscope (Fig. 9.10). Let us direct axis z along the main spin axis

of the gyroscope. The other two axes will pass through the center of gravity of the gyroscope perpendicular to axis z.

The vector of the instant angular velocity $\mathbf{\Omega}$ may be resolved along these axes into components ω_x, ω_y, ω_z. For a rapidly rotating gyroscope the absolute value ω_z is immeasurably greater than the values of ω_x and ω_y. These last for a balanced gyroscope are generally equal to zero and appear, as we shall see later, only when external forces begin to act on the gyroscope. Inasmuch as ω_x and ω_y are small, $\omega_z \approx \Omega$.

The components of the main angular momentum vector $\mathbf{N}$ will be $A\omega_x$, $B\omega_y$, $C\omega_z$, where A, B, C are the moments of inertia of the gyroscope about axes x, y, z. From the conditions of symmetry $A = B$.

An obvious fact must be noted: vector $\mathbf{N}$, generally speaking, does not coincide in direction with vector $\mathbf{\Omega}$. Only because, for a gyroscope, ω_x and ω_y are very small compared to ω_z, is it possible to consider that vectors $\mathbf{N}$ and $\mathbf{\Omega}$ are directionally quite close to each other. This considerably facilitates the problem and enables the immediate determination of the direction of the vector $\mathbf{\Omega}$ by observing the behavior of vector $\mathbf{N}$ (being coincident with it).

The basic relationship determining the rotational motion of a body is the law of the change of the angular momentum

$$d\mathbf{N}/dt = \mathbf{M} \tag{9.1}$$

where $\mathbf{M}$ is the vector of the moment acting on the body; and $\mathbf{N}$ is the vector of the angular momentum.

Let us introduce a system of semi-fixed central axis $x'y'$ (see Fig. 9.10). These axes are rigidly fixed to the axis z and turn with it, since they are not fixed to the rotor of the gyroscope, i.e., they do not rotate with the spin velocity Ω. For the gyroscope shown in Fig. 9.9 axes x' and y' coincide with the gimbal axes.

Let us now see what motion will be performed by the gyroscope axis z if we try to turn the gyroscope by applying to it, through the gimbal ring, a moment $\mathbf{M}$ about axis x' (Fig. 9.11).

Up to the time of the application of moment $\mathbf{M}$, the gyroscope has an angular momentum

$$\mathbf{N} = C\mathbf{\Omega}.$$

According to expression (9.1) the change in the angular momentum in time Δt is

$$\Delta\mathbf{N} = \mathbf{M}\Delta t.$$

Vector $\Delta\mathbf{N}$ coincides in direction with vector $\mathbf{M}$. Adding $\mathbf{N}$ and $\Delta\mathbf{N}$ we obtain a new angular momentum vector $\mathbf{N}'$. Due to the smallness of $\Delta\mathbf{N}$ vectors $\mathbf{N}$ and $\mathbf{N}'$ differ from each other only in direction. Therefore, we

see that as a result of the action of moment **M**, the angular momentum vector **N**, without changing its magnitude, rotates through an angle $\Delta N/N$ about axis y'.

It has already been mentioned above that, for a gyroscope, the angular momentum vector almost coincides with the instant axis of rotation. Con-

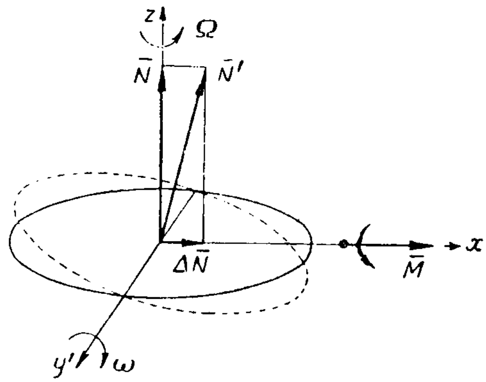


FIG. 9.11. Derivation of an expression for angular precession velocity.

sequently, we can say that under the influence of moment M the axis of gyroscope rotation turns through the same angle $\Delta N/N = M\Delta t/N$ about axis y' during time Δt. Therefore, the angular velocity of the gyroscope axis

$$\omega = M/N = M/C\Omega. \tag{9.2}$$

The described motion of the gyroscope about axis y' is called precession, and angular velocity ω is called the precessional rate.

The principle obeyed by the motion of the gyroscope during the application of an external moment may be formulated in the following manner: If a gyroscope, having three degrees of freedom, has a moment applied to it about an axis perpendicular to the axis of rotation, the gyroscope will turn in such a manner as to bring the vector of the main rotation Ω into coincidence with the moment vector M along the shortest path (see Fig. 9.11).

Strictly speaking, this law contains all of the "unusual" behavior of the gyroscope. The gyroscope axis turns not in the plane of the applied moment but in a plane perpendicular to it. Besides, the precessional motion is not accelerated. Precessional rate ω increases only while the applied moment increases. With a constant moment the angular velocity of precession remains unchanged and, after the action of the moment is stopped, precession ceases.

If initial rotation with velocity Ω were not imparted to the gyroscope, it would behave as an ordinary body with the application of an external

moment. From Fig. 9.11 it is seen that in that case (when $N = 0$), vector ΔN would entirely determine the motion of the flywheel, and the last, due to the action of the moment, would be rotating with an acceleration about axis x', i.e., in a plane of the action of the moment.

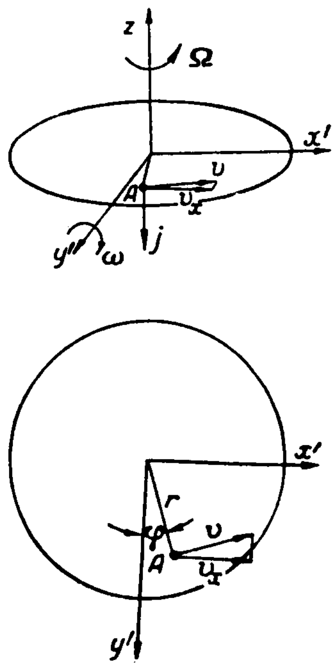


FIG. 9.12. Explanation of precession by means of Coriolis acceleration.

The behavior of the gyroscope may also be quite simply explained by examining accelerations which develop during the precession.

For simplicity's sake let us examine a flat disk rotating with velocity Ω (Fig. 9.12). Let us suppose that by some means precessional motion about axis y' has already been imparted to the disk.

Due to the smallness of ω, the velocity of an arbitrarily chosen point A

$$v = \Omega r.$$

The component of this velocity perpendicular to axis y'

$$v_x = \Omega r \cos \varphi.$$

The turning of the disk about axis y' gives rise to Coriolis acceleration at point A, whose absolute value is

$$j = 2\omega v_x = 2\omega\Omega r \cos \varphi. \tag{9.3}$$

The vector of Coriolis acceleration is perpendicularly directed to the plane of the rotating disk. By varying r and φ it is possible to find accelerations j at each point in the disk. The distribution of these accelerations on the surface of the disk (acceleration diagram) is presented in Fig. 9.13.

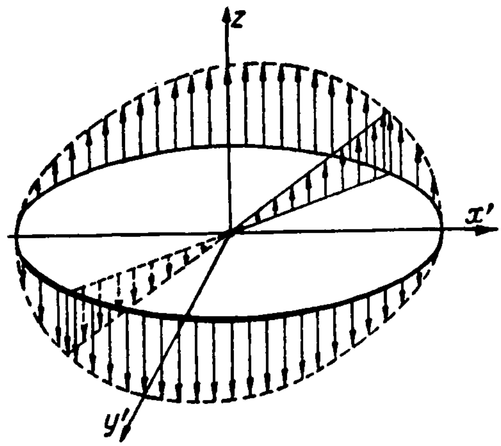


FIG. 9.13. Distribution of Coriolis acceleration in the plane of a disk.

In order to impart this system of accelerations to the disk it is necessary, as we see, to apply to it a moment about axis x' perpendicular to the precession axis y'.

The magnitude of the moment

$$M = \int_V jy'dm$$

where $y' = r \cos \varphi$ is the moment arm of the element, and dm is the element of the mass.

Integration is done for the entire volume of the disk V. Considering Eq. (9.3) we obtain

$$M = 2\omega\Omega \int_V r^2 \cos^2 \varphi dm = 2\omega\Omega \int_V y'^2 dm.$$

But the last integral is the moment of inertia of the disk about axis x'. For a flat disk it is equal to half of the moment of inertia about axis z.

$$\int_V y'^2 dm = A = C/2.$$

In this way we arrive at expression (9.2)

$$M = C\Omega\omega.$$

Knowing the law of precession it is possible to explain and foresee many manifestations connected with gyroscopic effect.

For instance, let us trace the behavior of a spin stabilized missile in flight.

The missile leaves the launching rails with zero, or almost zero, angle of attack. Having high angular velocity during its further motion, it behaves like a gyroscope and strives to maintain, unchanged, the direction of the longitudinal axis. Due to the curvature of the trajectory an angle of attack appears, and with it an aerodynamic moment in the vertical plane (Fig. 9.14).

If the missile is statically unstable (which is usually the case with the nonfinned missiles), the aerodynamic moment will be directed so as to increase the angle of attack, and the vector of the moment will be perpendicular to the plane of the drawing and directed toward the nose. With axial rotation of the missile to the right, vector Ω is directed forward. Therefore, according to the principle of precession, the missile begins to precess to the right. During further motion, the axis of the missile will be describing a cone about the trajectory of the center of gravity with the motion to the right (Fig. 9.15). For a statically stable missile, the precession will be to the left.

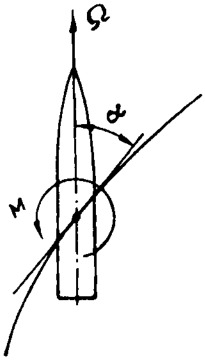


FIG. 9.14. Spin stabilized missile in a trajectory.

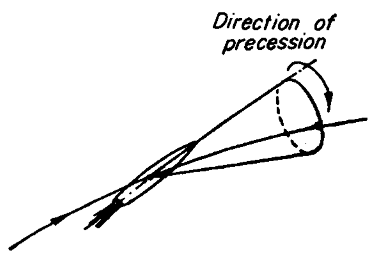


FIG. 9.15. Precession of a spin stabilized missile.

2. Application of the Gyroscope

The property of the gyroscope to maintain the direction of the spin axis is widely used in the field of navigation and stabilizing devices.

This widely accepted "property," however, is conditional. We already know that with the application of an external moment the gyroscope precesses and does not maintain unchanged direction of the spin axis.

A gyroscope installed in a gimbal mount is subject to the moments of friction forces in the bearings of the gimbal. These moments develop from the rotation of the gyroscopic rotor itself, and also from the rotation of the

external gimbals of the gyroscope relative to the rotor during oscillating motion of the flying apparatus.

After the passage of more or less considerable time, the axis of a free gyroscope deviates noticeably from the original direction. Even with well-designed gimbal bearings, unpermissible deviation of the axis takes place within a few minutes of the unrestrained operation of the rotor. Therefore, it is clear that in the operating cycle of a gyroscopic device the position of the gyroscopic axis must be continuously corrected.

As a simpler, and at the same time quite ingenious, method of gyroscope correction, the operational scheme of the simplest artificial horizon should be examined.

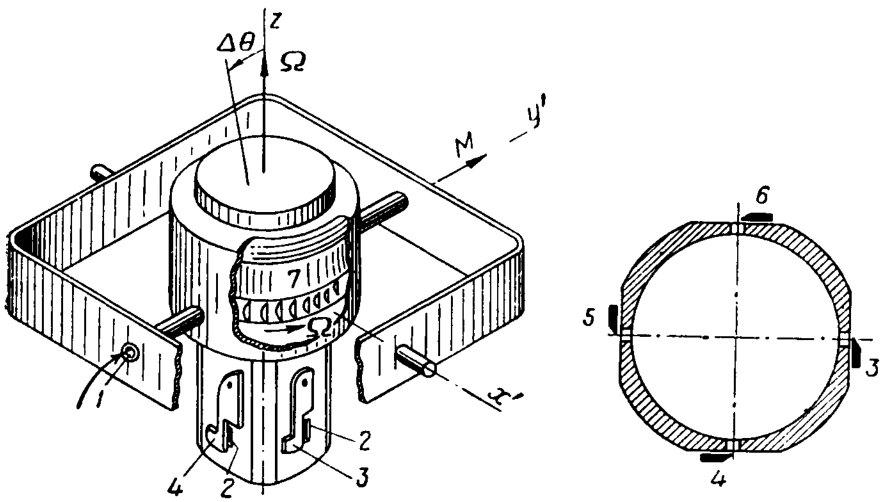


FIG. 9.16. Schematic of the artificial horizon device: *1*, air supply tube; *2*, air exit ports; *3*, *4*, *5*, *6*, dampers; and *7*, rotor.

The artificial horizon instrument is used to show the pilot, while flying blind, the position of the horizon. The sensitive element of this device, in this case the gyroscope, must constantly maintain its orientation in space.

Figure 9.16 shows schematically the construction of an artificial horizon.

The device consists of a gyroscope with a vertical axis mounted in a gimbal.

The rotor of the gyroscope is set in motion by a stream of air entering through the axis tube *1*, and impinging on the ribbed surface of the rotor. From the rotor box the air escapes through four openings *2*, halfway closed by pendulum dampers *3*, *4*, *5*, and *6*.

Let us see how the correction is performed. Let us assume that, due to the action of friction forces in the bearings, the gyroscope axis turns about axis x' through an angle $\Delta\theta$ (Fig. 9.16). During this motion damper *3* will open up and damper *5* will close. The escaping stream of air will create a

reacting moment M about axis y'. According to the law of precession, the gyroscope will start to turn in order to bring vector Ω closer to vector M along the shortest path, and the vertical position of the gyroscope axis will be restored. An analogous sequence will take place if the gyroscope, due to the action of friction forces, turns about axis y'.

There are other schemes of correction besides the one described. In particular, the magnetic needle of an ordinary compass is usually used for the correction of a directional gyroscope, and the gyroscope is oriented along its direction.

In examining all of these schemes a natural question comes up. Is the gyroscope so necessary if it has to be corrected anyway with the aid of an ordinary pendulum or an ordinary magnetic needle? Is it not simpler to use the pendulum directly as an indicator of the vertical, and the magnetic needle as an indicator of direction?

The fact is, however, that the pendulum and the needle of a compass have very low masses and low inertias, and are very much subject to various spurious effects. For instance, should an aircraft go into a turn the pendulum would orient itself in the direction of total acceleration and the device, without a gyroscope, would not indicate the true horizon. The gyroscope has considerable inertia and a prolonged systematic disturbance is required, for instance, a prolonged bank, for the artificial horizon equipped with a gyroscope to show a noticeable error. The greater the kinetic moment of the gyroscope $C\Omega$, the more inert is the gyroscope, and the more effective is its operation.

Therefore, the gyroscope finds application in apparatus due to its high inertia, the low frequency of its own oscillations, and low susceptibility to the action of spurious disturbing forces. As opposed to the magnetic needle and the pendulum, the gyroscope may be used in instruments as a power source setting in motion certain mechanisms.

C. Gyroscopic Devices and Actuating Components of the Automatic Stabilizer of a Long Range Rocket

1. Arrangement and Function of the Rudders of a Long Range Rocket

We will examine the construction and operation of an automatic stabilizer of a long range rocket, using the self-contained steering apparatus of the V-2 ballistic rocket as an example.

The rocket is equipped with four pairs of rudders. The arrangement of rudders is shown in Fig. 9.17. Rudders 1, II, III, and IV are jet vanes, while I′, II′, III′, and IV′ are fins.

Fins and vanes are numbered clockwise, looking forward. The forward vane, which is in the plane of the trajectory, is designated by the number I.

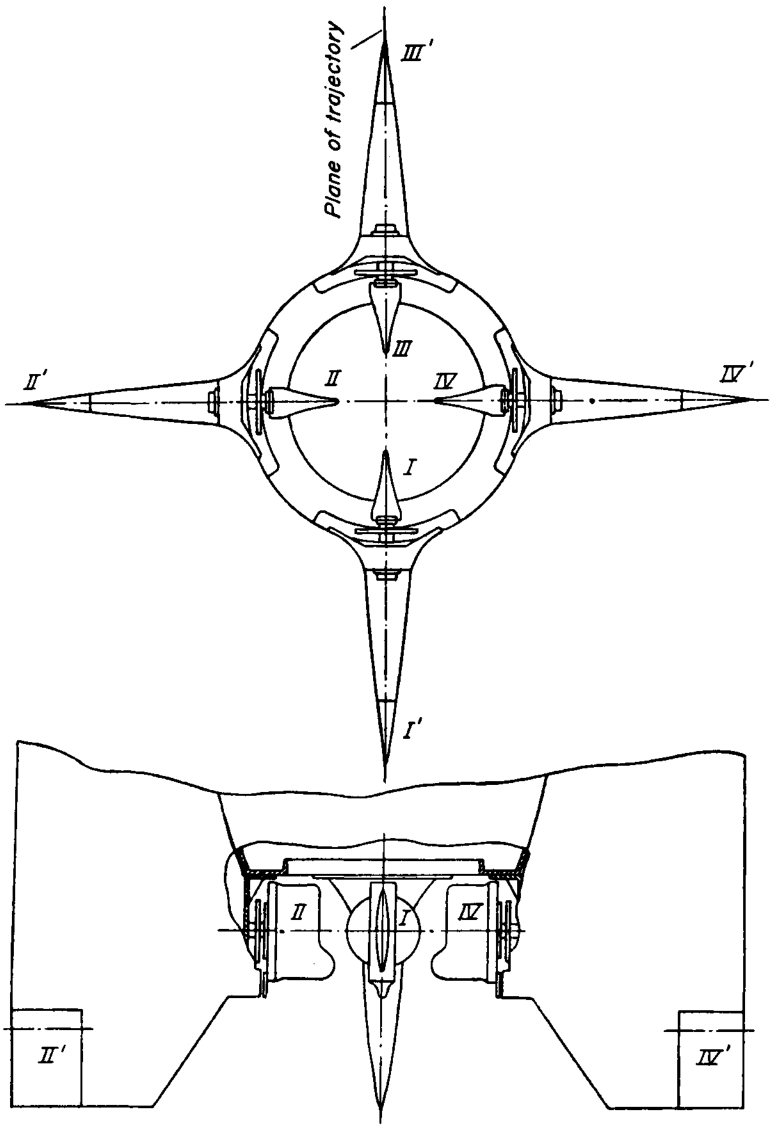


FIG. 9.17. Schematic arrangement of jet vanes and fins on a V-2 long range ballistic rocket.

Since in guided flight the rocket does not rotate about its longitudinal axis, this orientation of vanes is maintained during the entire powered phase of the trajectory.

Vanes II and IV stabilize the rocket in pitch. By turning vanes II and IV in synchronism, according to a definite plan, we are able to program a rocket in any manner.

The function of vanes I and III is to maintain the rocket on a given course, i.e., not to permit it to deviate from the plane of the trajectory. This is accomplished by synchronous turning of vanes I and III. Roll stabilization of the rocket is performed by the same vanes, I and III. If the rocket has a small angle of roll, vanes I and III turn in opposite directions, thereby establishing a restoring moment about the longitudinal axis.

Fins I′, II′, III′, and IV′ have an auxiliary purpose and perform the same functions as the jet vanes.

Their installation on the rocket is not essential in principle. Therefore, we will not discuss the operation of the fins.

In examining the operation of the automatic stabilizer we will begin with a description of the construction and operation of the gyroscopic devices.

2. The Horizontal Free Gyroscope

The first device, called the horizontal free gyroscope, is designed for stabilizing the rocket in the plane of the trajectory. This same device also programs the rocket. Therefore, the signals from this gyroscope must be transmitted to vanes II and IV.

The construction of the horizontal free gyroscope is shown in Fig. 9.18.

The gyroscope is mounted on a gimbal in such a way that the spin axis of the rotor is horizontal and lies in the plane of the trajectory.

Rotor *1* also serves as the armature of a nonsynchronous motor, whose stator winding is supplied by a 500-cycle alternating current. The rotor is brought up to speed a few minutes before rocket launching and automatically assumes the necessary orientation in space with the aid of a pendulum-actuated correcting device.

If the axis x' (see Fig. 9.18) inclines to the vertical, the pendulum, *3*, will close a contact, and the electromagnet, *4*, will receive a properly directed signal. The electromagnet will create a moment about the vertical axis x' which, as we already know, will cause precession of the gyroscope about the horizontal axis y'. This precession will continue until the moment about axis x' becomes zero, i.e., until the pendulum contact, *3*, is opened.

With the deflection of the axis x' in the other direction pendulum *3* will close a contact on the other side. In this case the electromagnet will create a moment of an opposite sign.

Orientation of the rotor relative to the gimbal frame, *5*, is accomplished with the aid of an analogous device. If the rotor turns about axis x', then one of the contacts, *6*, will be closed and a signal will be transmitted to the electromagnet, *7*. Due to the action of this electromagnet, precession will start about axis x' and the necessary orientation will be restored.

The system of gyroscopic correction operates up to the moment of launch. After launching the correction system is disengaged and the gyro-

scope becomes free. During the time of the guided flight (64 sec), the axis of the uncorrected gyroscope does not have a change to deviate noticeably from the given direction.

Should the rocket in flight deviate from the required direction in the plane of firing in a given trajectory plane, the potentiometer, *8*, connected

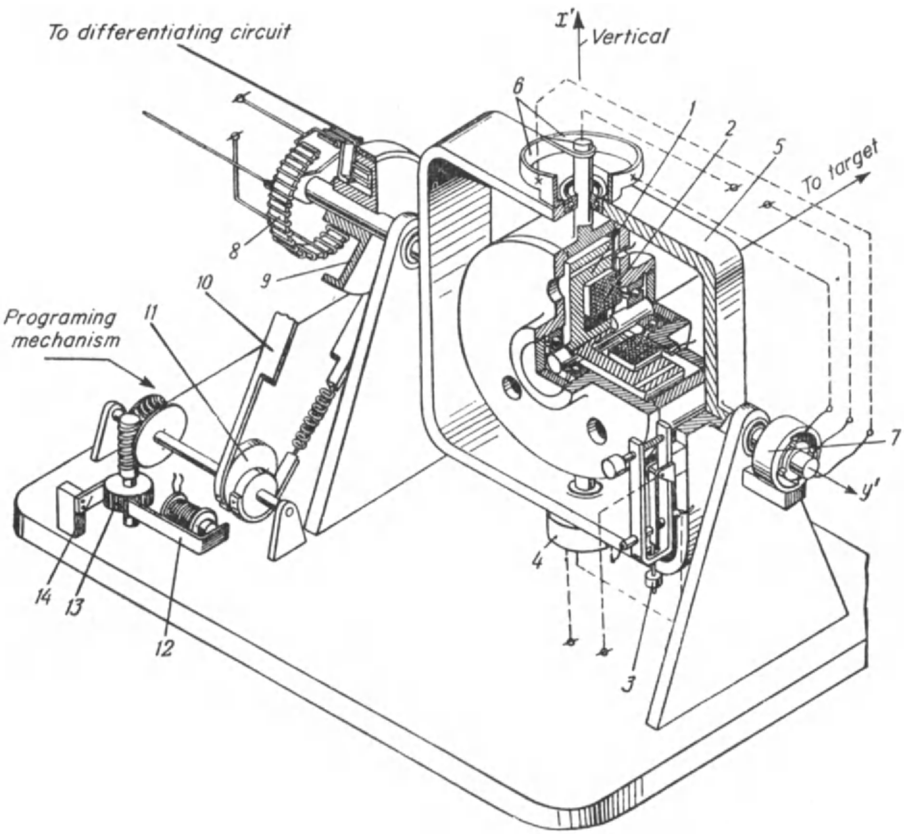


FIG. 9.18. Construction of a horizontal free gyroscope: *1*, gyroscope rotor; *2*, stator winding; *3*, pendulum; *4*, electromagnet; *5*, gyroscope gimbal; *6*, contact arrangement; *7*, electromagnet; *8*, potentiometer; *9*, potentiometer wheel; *10*, metal band of the programming mechanism; *11*, eccentric of the programming mechanism; *12*, pawl of the step relay; *13*, ratchet wheel; and *14*, latch.

to wheel *9*, and through it to the frame of the rocket, will rotate together with the rocket about a stationary gyroscope, thereby generating an electrical signal which, having passed through the amplifier-transformer system, will be applied to the actuating components—the steering mechanisms. Vanes II and IV will turn, and the required rocket direction will be reestablished.

If, during flight, the potentiometer is turned through some angle $\Delta\varphi$ relative to the frame of the rocket, then obviously the vanes will respond in the same manner as if the rocket itself deviated by that angle and will turn the rocket through an angle $\Delta\varphi$. Therefore, by turning the potentiometer, *8*, according to a given plan, we will cause the turning of a rocket according to the same plan in the plane of the trajectory, i.e., we will impart to the rocket its program. The turning of the potentiometer is performed by the so-called programming mechanism (see Fig. 9.18).

Potentiometer *8*, and the wheel, *9*, constitute a rigid unit. A thin metal band, *10*, is passed over the wheel, *9*. This metal band is connected on the opposite side to the eccentric, *11*, which is profiled to correspond to a given program. Eccentric *11* is rotated through a worm gear transmission by a step relay, *12*. The last consists of an electromagnet with an armature. When the electromagnet receives an impulse, the armature is attracted to the magnet and by its pawl moves the ratchet wheel, *13*, by one tooth. Therefore, the speed of rotation of the wheel, *13*, depends on the impulse frequency received by the electromagnet.

The leaf spring, *14*, shown in Fig. 9.18, is a latch for the ratchet wheel which prevents its rotation in the reverse direction.

. 3. The Vertical Free Gyroscope

The second gyroscopic device of the automatic stabilizer is the vertical free gyroscope. This apparatus is installed in the rocket to insure stabilization in yaw and roll and controls vanes I and III.

The construction of the device is shown in Fig. 9.19.

The axis of the rotor is perpendicular to the plane of the trajectory. Therefore, the gyroscope is insensitive to the rocket maneuvers in the plane of the trajectory (in pitch), but reacts to roll and yaw.

As in the case of the horizontal free gyroscope, alternating current is supplied to the coil, *2*, for the rotation of the rotor, *1*.

The correction of the device is the same as for the horizontal free gyroscope and is also performed only up to the time of launching.

With the deviation of axis x' from the vertical, the pendulum, *3*, closes one of the contacts, resulting in a signal being supplied to the electromagnet, *4*, and axis x' returns to the vertical position. If the gyroscope turns about axis x', a signal is supplied by potentiometer *5* to electromagnet *6*, and the normal position of the gyroscope is again restored.

After launching, potentiometer *5* transmits a signal, not to the correcting magnet *6*, but to vanes I and III. Since the axis x' coincides with the longitudinal axis of the rocket, it is obvious that, in flight, potentiometer *5* will respond to the roll of the rocket and will transmit signals to move vanes I and III in opposite directions.

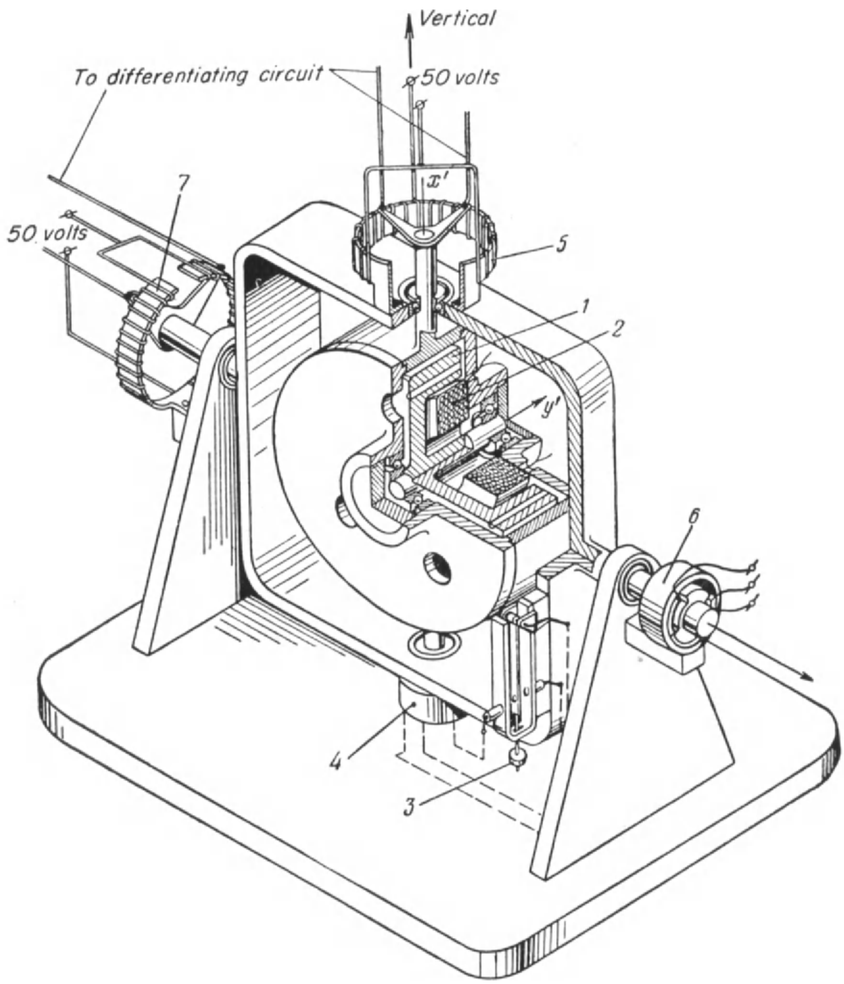


FIG. 9.19. Construction of a verticle free gyroscope: *1*, gyroscope rotor; *2*, stator winding; *3*, pendulum; *4*, electromagnet; *5*, potentiometer; *6*, electromagnet; and *7*, potentiometer.

Potentiometer *7* will transmit signals for correcting direction. These signals must be transmitted to vanes I and III in a manner that will cause them to turn synchronously.

4. Steering Mechanisms

The steering mechanisms are the actuating components which turn the vanes in the necessary direction and by a necessary amount depending on the received signal.

Figure 9.20 shows a schematic, and Fig. 9.21 the construction of a hydraulic steering mechanism.

The signal obtained from the gyroscope potentiometer, transformed and amplified in the intermediate devices of the automatic stabilizer, is received by the polarized relay *1* of the steering mechanism. Depending on

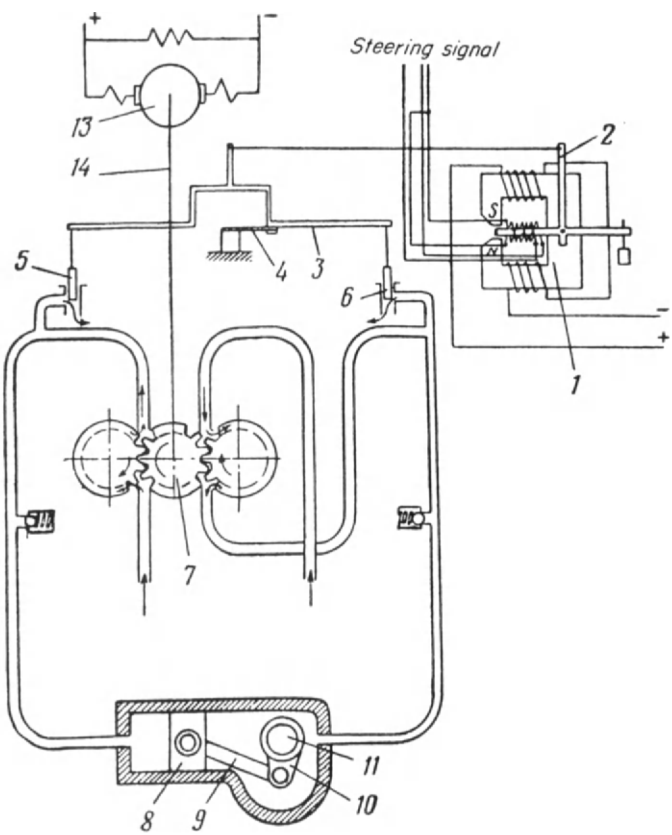


FIG. 9.20. Schematic of a rocket steering mechanism (see also Fig. 9.21).

the sign of the signal, the polarized relay moves yoke *2* in the proper direction and piston valve distributor *3*, connected to it and mounted on a flat spring *4*, which replaces the mechanical hinge.

With the rotation of the distributor, one of the pistons (*5* or *6*) will close the bypass, and the oil, supplied by the gear pump, *7*, will enter the chamber of the power cylinder, displacing the power piston, *8*. The force will be transmitted to the vane shaft, *11*, by the piston rod, *9*, and the crank, *10*. The vane, *12*, attached to the shaft will turn in the required direction. The gear pump is rotated by motor *13* by means of shaft *14*.

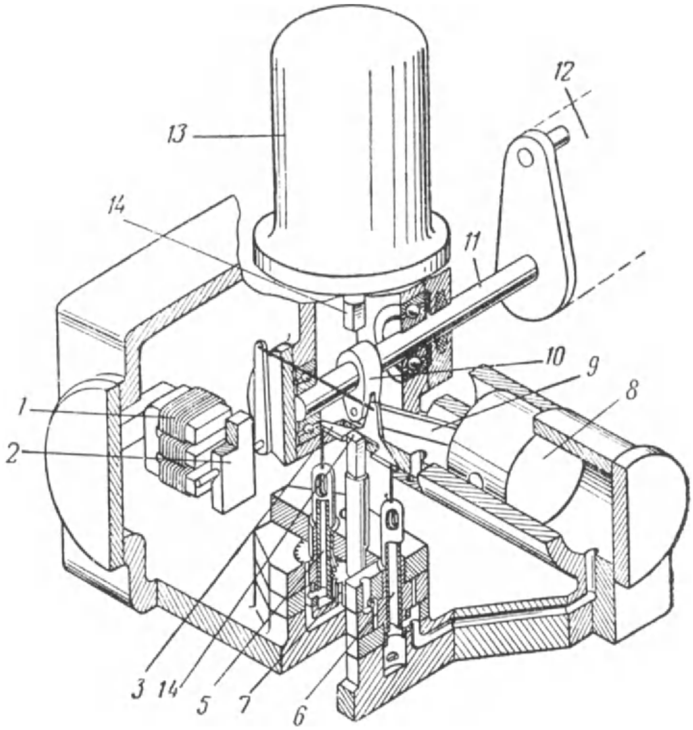


FIG. 9.21. Construction of a steering mechanism: *1*, polarized relay; *2*, yoke; *3*, slide valve; *4*, spring; *5*, *6*, slide valve pistons; *7*, gear pump; *8*, power piston; *9*, rocker arm; *10*, crank; *11*, vane shaft; *12*, vane; *13*, gear pump motor; and *14*, motor shaft.

The described steering mechanism operated only one vane. There must be four such mechanisms installed in the rocket.

5. Conditions for Stable Flight

In the example of the simplest automatic steering (see page 279), we have already seen that with the deviation of the flying apparatus off course the simple tilting of the rudder in the required direction is insufficient for the attainment of stable equilibrium. If the rudder tilt is in the proper direction but not in the required degree, or is lagging, we always risk having an unstable regime of rocket steering—divergent steering or steering with full departure from the given course.

Let us determine the conditions which must be satisfied by a stabilizing system in order to insure stable motion. With this in mind, let us turn to the equation of rocket motion, Eq. (7.3).

$$J\ddot{\varphi} + M + L_g D_g + M_h = 0$$

and solve for the type of motion that the rocket will have after it has received some disturbance, even in the guise of an instantaneous angular impulse.

We will simplify the problem by eliminating from the equation the hinge moment M_h as a quantity small compared to L_vD_v. Besides, we will consider that the remaining parameters of rocket motion, after imparting to it the disturbance corresponding to angle φ, remain unchanged. So, for example, the rocket velocity v and aerodynamic coefficients will be considered as constant, retaining the value which they had prior to the time of disturbance.

In the first approximation of stability evaluation this assumption is quite acceptable, inasmuch as we are interested only in the deviation of the rocket from the steady state motion and not in the motion of the rocket itself.

The initial equation of motion, Eq. (7.3), being analyzed, takes the form

$$J\ddot{\varphi} + M + L_vD_v = 0. \tag{9.4}$$

With a disturbance, the quantities entering into this equation change, and we get

$$J(\varphi + \Delta\varphi)^{\cdot\cdot} + M + \Delta M + (L_v + \Delta L_v)D_v = 0 \tag{9.5}$$

where $\Delta\varphi$ is the angle of deviation from the programmed angle resulting from disturbance; ΔM, the corresponding change in aerodynamic and damping moments; and ΔL_v, the change in vane lift force.

Subtracting Eq. (9.5) from Eq. (9.4) we find

$$J\Delta\ddot{\varphi} + \Delta M + \Delta L_vD_v = 0. \tag{9.6}$$

Moment M is composed, as we know, of stabilizing and damping moments. The first of these is proportional in the first approximation to the angle of attack, and the second to the angular velocity of the rocket rotation. Consequently, the change in the moment ΔM will be composed of two addends, one of which is proportional to the angle of rotation $\Delta\varphi$, and the second to the velocity of rotation $\Delta\dot{\varphi}$

$$\Delta M = a\Delta\varphi + b\Delta\dot{\varphi} \tag{9.7}$$

where a and b, due to our assumption of the constancy of flight parameters of disturbed flight, are constant values.

As to the lift force L_v, we already know that, in the first approximation, it is proportional to the vane rotation angle β. Consequently

$$\Delta L_v = c\Delta\beta$$

where $\Delta\beta$ is the additional angle to vane rotation due to the reaction of the automatic stabilizer to the disturbed motion of the rocket; the quantity $\Delta\beta$ is a function of time.

Therefore, Eq. (9.6) takes the form

$$J\Delta\ddot{\varphi} + b\Delta\dot{\varphi} + a\Delta\varphi + cD_v\Delta\beta = 0. \tag{9.8}$$

Now the problem consists in finding out in what way the angles of vane rotation $\Delta\beta$ and rocket disturbance $\Delta\varphi$ are interrelated. If they are so related that the solution of Eq. (9.8) will yield decaying or, even better, the rapidly decaying solution for $\Delta\varphi$, the rocket steering will be stable. If in solving Eq. (9.8) we discover that function $\Delta\varphi$ does not decay with time, it will indicate that automatic stabilization does not insure stable motion.

The angle of vane rotation and the hinge moment M_h are related by the following equation

$$M_h = a_0\beta + a_1\dot{\beta} + a_2\ddot{\beta}$$

where a_0, a_1, a_2 are constant quantities.

Consequently

$$\Delta M_h = a_0\Delta\beta + a_1\Delta\dot{\beta} + a_2\Delta\ddot{\beta}. \tag{9.9}$$

However, the variation in the hinge moment ΔM_h depends on the signals which are transmitted to the steering mechanism from guidance devices. Depending on the method of reception and transmission of signals from the guidance gyroscope, the addition of the hinge moment ΔM_h may be, for example, proportional to the angle of deviation of the rocket axis $\Delta\varphi$ from the program. It is possible, for instance, to arrange for the value ΔM_h to be proportional to the velocity or the acceleration of the angular disturbance of the rocket motion ($\Delta\dot{\varphi}$ or $\Delta\ddot{\varphi}$).

Let us assume, for instance, that

$$\Delta M_h = b_0\Delta\varphi + b_1\Delta\dot{\varphi} + b_2\Delta\ddot{\varphi}. \tag{9.10}$$

Coefficients b_0, b_1, and b_2 are constant quantities and, for the time being, undetermined, but it is they which characterize the behavior of the steering system. The right hand side of Eq. (9.10) is called the time constant. Introducing design changes into the steering system, we can change values b_0, b_1, and b_2 and achieve stable motion of a rocket by the stabilizing system.

Comparing expressions (9.9) and (9.10) we find

$$a_0\Delta\beta + a_1\Delta\dot{\beta} + a_2\Delta\ddot{\beta} = b_0\Delta\varphi + b_1\Delta\dot{\varphi} + b_2\Delta\ddot{\varphi}.$$

Let us substitute into this the expression for $\Delta\beta$ from Eq. (9.8). As a result we will get one equation in terms of $\Delta\varphi$.

$$\Delta\ddot{\varphi}^{\cdot\cdot} + d_3\Delta\ddot{\varphi}^{\cdot} + d_2\Delta\ddot{\varphi} + d_1\Delta\dot{\varphi} + d_0\Delta\varphi = 0 \tag{9.11}$$

where

$$\left.\begin{aligned} d_0 &= \frac{a_0 a}{a_2 J} + \frac{b_0 c D_v}{a_2 J} \\ d_1 &= \frac{b a_0 + a_1 a}{a_2 J} + \frac{b_1 c D_v}{a_2 J} \\ d_2 &= \frac{a_0 J + a_1 b + a_2 a}{a_2 J} + \frac{b_2 c D_v}{a_2 J} \\ d_3 &= \frac{a_1 J + a_2 b}{a_2 J}. \end{aligned}\right\} \tag{9.12}$$

Thus, the principle of the disturbed rocket motion will be determined by the solution of a linear differential equation, Eq. (9.11).

Solution of Eq. (9.11) we will seek as usual in the form

$$\Delta\varphi = Ae^{kt}.$$

Substituting this expression into Eq. (9.11), we will get the typical equation

$$k^4 + d_3k^3 + d_2k^2 + d_1k + d_0 = 0. \tag{9.13}$$

Equation (9.13) has four roots: k_1, k_2, k_3, and k_4. As a result, the solution of Eq. (9.11) will take the form

$$\Delta\varphi = A_1e^{k_1t} + A_2e^{k_2t} + A_3e^{k_3t} + A_4e^{k_4t}. \tag{9.14}$$

Let us see now under what conditions the motion will be decaying.

The roots of Eq. (9.13) may be either real or complex.

For real roots of k, the function e^{kt} will be divergent for positive values of k and convergent for negative values of k.

If the root k is complex

$$k = \mu + \nu i$$

where μ and ν are real numbers. Then, according to Euler's identity

$$e^{kt} = e^{(\mu+\nu i)t} = e^{\mu t}(\cos \nu t + i \sin \nu t).$$

Each algebraic equation with real coefficients yields only an even number of complex roots, whereupon these roots, as is known, are conjugate pairs.

Suppose, for instance, that the roots k_1 and k_2 are conjugate

$$\begin{aligned} k_1 &= \mu + \nu i \\ k_2 &= \mu - \nu i. \end{aligned}$$

Then, for the first two addends in the right hand side of Eq. (9.14) we will find

$$A_1e^{k_1t} + A_2e^{k_2t} = A_1e^{\mu t}(\cos \nu t + i \sin \nu t) + A_2e^{\mu t}(\cos \nu t - i \sin \nu t)$$

or

$$A_1e^{k_1t} + A_2e^{k_2t} = (A_1 + A_2)e^{\mu t} \cos \nu t + i(A_1 - A_2)e^{\mu t} \sin \nu t.$$

Denoting

$$A_1 + A_2 = C_1 \qquad i(A_1 - A_2) = C_2$$

we get

$$A_1e^{k_1t} + A_2e^{k_2t} = C_1e^{\mu t} \cos \nu t + C_2e^{\mu t} \sin \nu t. \tag{9.15}$$

If the other two roots, k_3 and k_4, in Eq. (9.13) are also complex, then the sum $A_3e^{k_3t} + A_4e^{k_4t}$ on the right side of Eq. (9.14) will be transformed analogously.

The obtained function (9.15) will be damped if μ is negative. The case of $\mu = 0$ indicates oscillating undamped motion; the case of $\mu > 0$ indicates oscillations with increasing amplitude.

Therefore, the conditions for stable steering may be formulated in the following form: For stable steering it is necessary for the real parts of all roots of Eq. (9.13) to be negative. This condition may be fulfilled by choosing, in a proper manner, Eq. (9.13) and coefficients d_0, d_1, d_2, and d_3, which depend on the parameters of the steering system.

The answer to the question of whether the steering system is stable or unstable may be obtained without solving Eq. (9.13). Using methods of higher algebra, it is possible to determine relationships between coefficients d_0, d_1, d_2, and d_3 for which the real parts of all roots of the algebraic equation will be negative. For the equation of the fourth degree, Eq. (9.13), it is necessary and sufficient to satisfy the condition of Routh according to which the real parts of all roots will be negative if simultaneously

$$d_0 > 0,\ d_1 > 0,\ d_2 > 0,\ d_3 > 0$$

and

$$d_1d_2d_3 - d_0d_3^2 - d_1^2 > 0.$$

It is not difficult to be convinced of the fact that the motion of a guided rocket may be either stable or unstable.

For instance, let a signal be transmitted to a steering mechanism which is proportional to the deviation of the rocket, and which is independent of the angular velocity or acceleration of the rocket. Then, as it follows from expression (9.10), $b_1 = b_2 = 0$. Further, let us suppose for the sake of simplicity that the vane masses, and moments of inertia during their rotation, are small, that the vanes themselves are totally compensated in the flow,

and that the hinge moment is independent of the angle of rotation. This means that, in Eq. (9.9), $a_0 = a_2 = 0$. Besides, if it is assumed that the rocket has left the region of the atmosphere and the role of aerodynamic forces is negligible, then, in expression (9.7), $a = b = 0$.

Multiplying all terms of Eq. (9.11) by a_2J, and keeping in mind the notation of Eq. (9.12), we get

$$a_1J\Delta\dddot{\varphi} + b_0cD_v\Delta\varphi = 0$$

or

$$\Delta\dddot{\varphi} + d^3\Delta\varphi = 0$$

where

$$d^3 = b_0cD_v/a_1J.$$

The typical equation will have the form

$$k^3 + d^3 = 0$$

whence

$$k_1 = -d, \qquad k_{2,3} = \tfrac{1}{2}d \pm \tfrac{1}{2}id\sqrt{3}.$$

The real part of roots k_2 and k_3 turns out to be positive. Consequently, the motion of the rocket will be unstable. This came about not because we have neglected vane inertias and aerodynamic forces, but primarily because the chosen principle of regulation, with the transmittal of signals to the vanes proportional only to the angle $\Delta\varphi$, was incorrect.

In order to obtain stable motion, it is sufficient to transmit to the steering mechanisms signals which are proportional to $\Delta\dot{\varphi}$ and, for the improvement of the control system, also proportional to $\Delta\ddot{\varphi}$. Then, in expression (9.11), b_1 and b_2 will not be equal to zero. By the proper selection of these coefficients it is easy to obtain a stable system of stabilization.

A stabilizing system of the V-2 long range ballistic missile is based precisely on this principle. Signals obtained from the potentiometers of the horizontal and vertical free gyroscopes are differentiated by special devices and are transmitted to the vanes in the form of functions of deviation angle $\Delta\varphi$, Eq. (9.10).

Let us turn to the examination of intermediate devices which perform the transmission of signals from the gyroscopes to the steering mechanisms.

D. Intermediate Devices of the Long Range Rocket Automatic Stabilizer

1. The General Schematic of the Intermediate Devices

The intermediate devices of the automatic stabilizer are intended for transformation and amplification of signals obtained from gyroscopic potentiometers, and their transmission to the steering mechanisms.

The basic schematic of the intermediate devices of the automatic stabilizer is shown in Fig. 9.22.

Stabilization in the plane of the trajectory with the aid of the horizontal free gyroscope is performed independently of stabilization in yaw and roll. Therefore, signals transmitted by the horizontal and the vertical free gyroscopes have independent systems of transformation and transmission to the vanes.

The signal in the form of dc voltage obtained by the horizontal free gyroscopic potentiometer is transmitted, first of all, to a differentiating circuit where it is transformed into the form

$$v \approx c_0\Delta\varphi + c_1\Delta\dot{\varphi} + c_2\Delta\ddot{\varphi}. \tag{9.16}$$

The function of the differentiating circuit consists, therefore, in adding to the voltage, which is proportional to the angle of rocket deviation, components which are proportional to the angular velocity and angular acceleration of the rocket. These additional signals, as we already know, are necessary in order to insure steering stability.

The potentiometer is supplied by a source of direct current, since it is very difficult to design a differentiating circuit which would operate on alternating current.

After the differentiating circuit, the signal is transmitted to a converter, where it is converted into an amplitude-modulated alternating current signal with a frequency of 500 cycles. This conversion is performed in order to simplify the subsequent amplification, since amplification of a direct current signal poses great difficulties.

The converted signal is transmitted through a transformer to the amplifiers, which control the steering mechanisms of vanes II and IV.

Since vanes II and IV are not mechanically interconnected, a synchronizing device is introduced into the pitch system of stabilization in order to insure synchronous operation of vanes II and IV, excluding, in this manner, the occurrence of undesirable perturbations in roll. When vanes II and IV, together with the synchronizing potentiometers, are turned to the same angle, no signal is transmitted to the primary coil of the transformer Tp_1. When the vanes are turned to a different angle, however, the primary coil of the transformer Tp_1 will receive a signal proportional to the difference of the vane angles. This signal will then be transmitted to the amplifiers and, through them, to the relay of the steering mechanisms. As a result, the coordination of the vanes will be reestablished.

The system of control for vanes I and III is analogous to the control system of vanes II and IV. Signals from two potentiometers are separately differentiated, converted, and transmitted through transformers to the amplifiers of the steering mechanisms of vanes I and III. The signal from

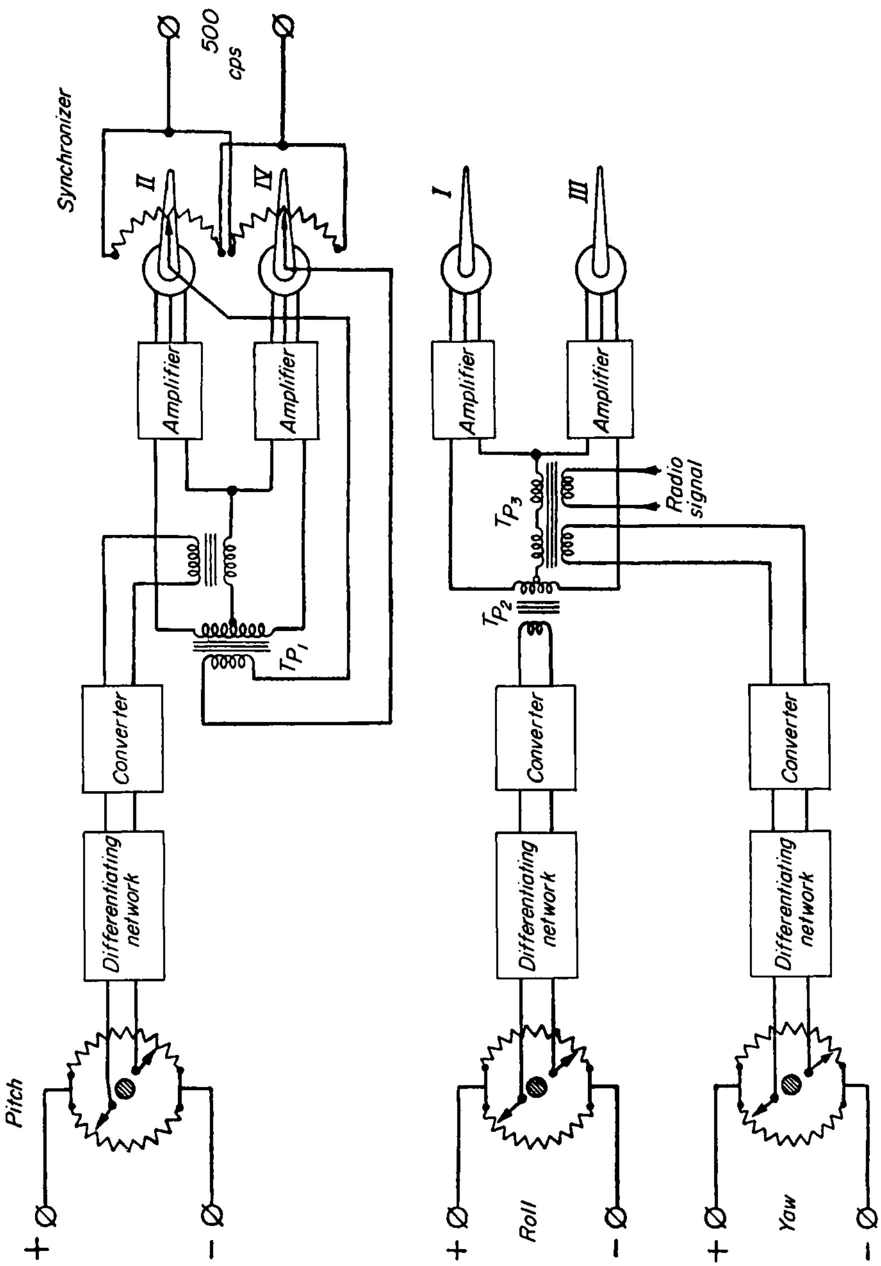


FIG. 9.22. Block diagram of intermediate devices of an automatic stabilizer.

the roll potentiometer forces the vanes to deflect in opposite directions, and a signal from the yaw potentiometer is transmitted in such a way that vanes turn in the same direction simultaneously.

In the schematic of Fig. 9.22 an extra spare coil is shown in transformer Tp_3. Through it may be impressed a signal from radio control. This signal, obviously, as the signal from the yaw potentiometer, will correct direction of rocket flight.

Let us examine separate elements of the intermediate devices.

2. The Differentiating Circuit

Voltage v from the gyroscopic potentiometer is proportional to the angle of rocket deviation $\Delta\varphi$. Since $\Delta\varphi$ is a function of time, v is also a function of time.

Differentiation of the signal v may be performed with the aid of the simplest RC circuit, i.e., by introducing in sequence capacitance and ohmic resistance (Fig. 9.23a).

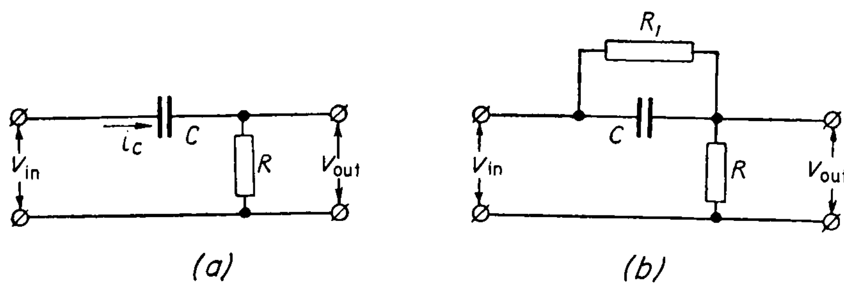


Fig. 9.23. Schematic of the simplest differentiating circuit.

The signal being differentiated is applied across input terminals and its derivative is picked off at output terminals. Obviously,

$$v_{\text{in}} - v_c = v_{\text{out}}$$

where v_c is the voltage drop across the condenser.

The current which passes through the capacitance

$$i_c = C(dv_c/dt).$$

If no load is connected to the terminals, i.e., if the circuit operates with an amplifier, then

$$v_{\text{out}} = i_c R.$$

Eliminating from the three obtained equations v_c and i_c we get

$$v_{\text{out}} + RC(dv_{\text{out}}/dt) = RC(dv_{\text{in}}/dt).$$

If the selected value of RC is sufficiently small, it is possible to neglect the second addend in the left part of this equation, and we will then have almost pure differentiation

$$v_{\text{out}} \approx RC(dv_{\text{in}}/dt) = RC\dot{v}_{\text{in}}$$

i.e., the output will be a voltage almost proportional to the derivative of the input signal.

It is essential to note that, inasmuch as RC must be very small, the converted signal, as it follows from the last expression, is sharply reduced in strength by the differentiating circuit. Therefore, the introduction of amplifiers further on becomes necessary.

If we wish to obtain an output signal which depends not only on the derivative dv_{in}/dt, but also on the function v_{in} itself, then it is necessary to add to the simplest differentiating circuit the resistance R_1 in parallel with capacitance C (Fig. 9.23b). The output signal will then have the form

$$v_{\text{out}} \approx m_1 v_{\text{in}} + m_2 \dot{v}_{\text{in}}$$

where m_1 and m_2 are constants depending on parameters R, R_1, and C.

Therefore, the output voltage in a circuit becomes proportional to voltage v_{in} and its first derivative, and the loop shown in Fig. 9.23b performs a

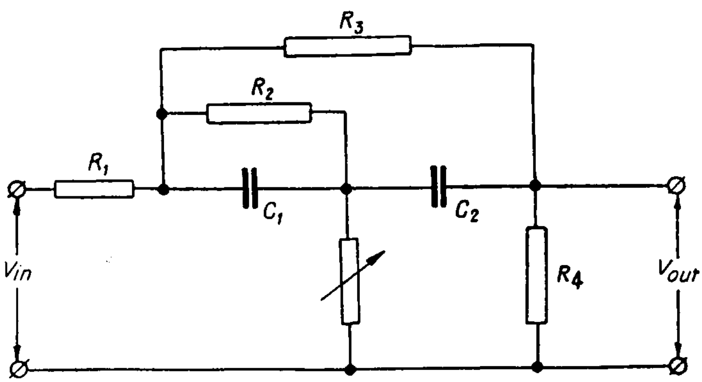


FIG. 9.24. Differentiating circuit schematic of the automatic stabilizer.

single differential transformation of the signal. A scheme shown in Fig. 9.24 is used for double differentiation of a signal. The output voltage of this circuit is

$$v_{\text{out}} \approx n_0 v_{\text{in}} + n_1 \dot{v}_{\text{in}} + n_2 \ddot{v}_{\text{in}}$$

where n_0, n_1, and n_2 depend on the parameters of the circuit.

This signal, converted and amplified, finally reaches the steering apparatus in the form of a demand to vary the hinge moment, Eq. (9.10)

$$\Delta M_h = b_0 \Delta\varphi + b_1 \Delta\dot{\varphi} + b_2 \Delta\ddot{\varphi}.$$

The relationships among coefficients b_0, b_1, and b_2 are determined by the relationships among coefficients n_0, n_1, and n_2. Therefore, the relationships among coefficients b_0, b_1, and b_2 necessary for stable steering are obtained by suitable selection of resistances and capacitances of the differentiating circuit.

3. Signal Converter (Modulator)

In passing through the differentiating circuit the signal is drastically weakened and must be amplified. Since amplification of a direct current signal poses considerable difficulties, in the system of the automatic stabilizer under discussion the signal is converted to an alternating current modulated in amplitude. A scheme shown in Fig. 9.25 is used for this.

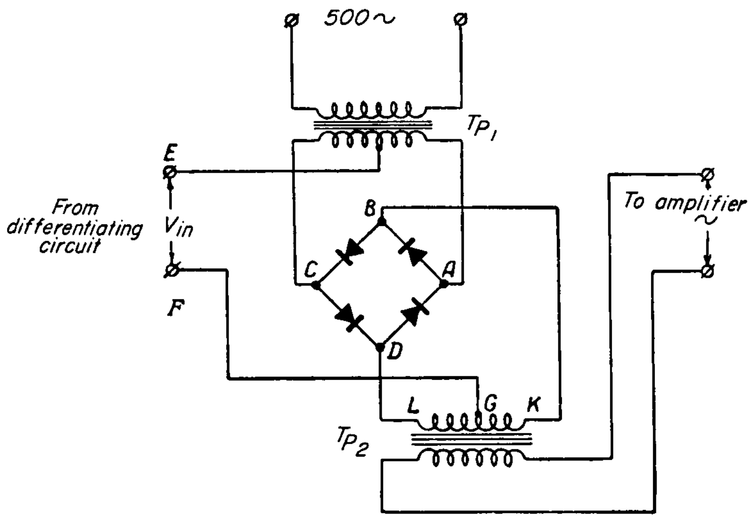


Fig. 9.25. Converter schematic.

The voltage from the differentiating circuit is applied to the center taps of the coils of the two transformers: the first, Tp_1, input, excited by 500-cycle alternating voltage, and the second, Tp_2, output, from which the converted signal is picked off.

The basic element of this scheme is a bridge of four selenium rectifiers.

A selenium rectifier consists of a stack of iron or aluminum disks covered on one side with a layer of selenium. In its turn a layer of some fusible metal is deposited on the selenium. A thin layer at the boundary between the selenium and the fusible metal has the property of being a good conductor in one direction and a very poor conductor in the opposite direction. The graph in Fig. 9.26 shows the amount of current passing through a selenium rectifier with respect to the applied voltage.

The selenium rectifier bridge $ABCD$ (see Fig. 9.25) is made strictly symmetrical. In the absence of an input signal at terminals EF it is balanced. Notwithstanding the fact that the transformer Tp_1 impresses a voltage across the diagonal AC, the voltage in the second diagonal BD connected to the primary coil of the output transformer Tp_2 is equal to zero.

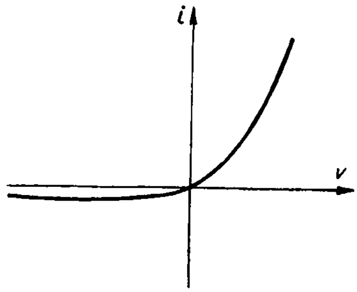


FIG. 9.26. Characteristic of a selenium rectifier.

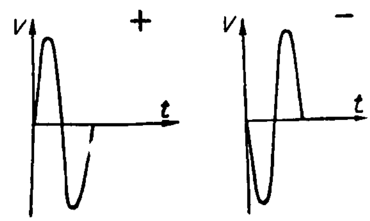


FIG. 9.27. Change in the output signal during one cycle of the alternating current carrier depending on the polarity of the signal.

Upon transmittal of a signal the symmetry of the bridge, because of the nonlinear characteristic of selenium rectifiers, is disturbed and there is a voltage across the diagonal BD proportional to the magnitude of the input signal. Voltage also appears across the secondary coil of the transformer Tp_2. This will be an alternating voltage whose amplitude is proportional

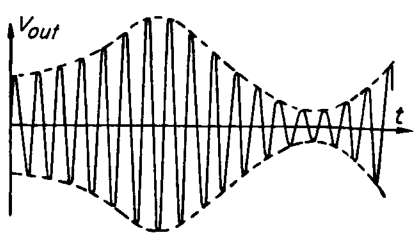


FIG. 9.28. Modulated signal.

to the magnitude of the direct current input signal. If the direction of the input signal current changes then the phase of the output alternating voltage will change by 180° (Fig. 9.27).

As a result we get in the secondary coil of the output transformer an alternating voltage with a frequency of 500 cycles, and an amplitude variable with the signal arriving from the differentiating circuit (Fig. 9.28). The phase of the output alternating voltage depends on the polarity of the input signal.

4. Amplifier and Demodulator

A schematic of the amplifier and demodulator is shown in Fig. 9.29.

The basic element of the amplifier is an electronic tube (pentode), widely used in the most diversified devices and systems. The tube is supplied from a voltage source with 200–250 volts.

Voltage from the secondary coil of the output converter transformer is impressed across the grid of the tube. This voltage, amplified by the tube, is supplied to the coil, A, of a three-cored transformer (coils of the transformer are wound on three cores).

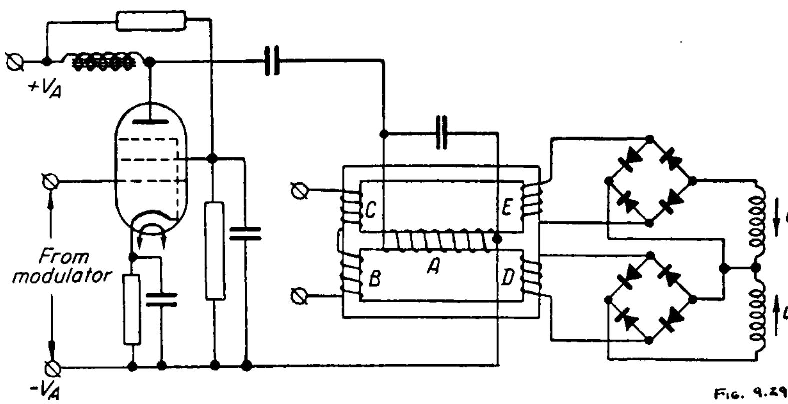


Fig. 9.29. Amplifier schematic.

The amplified voltage is the voltage of the alternating current with a frequency of 500 cycles, whose amplitude is proportional to the output signal of the differentiating circuit and whose phase depends on the polarity of the signal.

Since, in the final analysis, the signal must control a polarized relay of the steering mechanism which reacts only to the direct current signals, it is necessary for the amplified signal to be reconverted into direct current signal. This reverse, second, conversion of the signal (or demodulation) is performed by the three-cored transformer and selenium rectifiers connected as shown in Fig. 9.29.

The three-cored transformer, besides the amplified signal voltage (coil A), is supplied by the alternating voltage with a frequency of 500 cycles from the power source. The last is called a carrier, since the phase of the amplified signal is determined by comparison with it, i.e., the sign of the original signal obtained from the gyroscopic potentiometer.

If the steering signal is absent in coil A, the voltages across coils E and D are equal to each other.

Voltages across coils E and D are rectified by selenium rectifiers and are supplied to the two coils of a polarized relay. The rectifiers are so connected that the currents in each coil oppose each other. For a zero signal, both currents i_1 and i_2 (see Fig. 9.29) are equal in magnitude and the total net effect is equal to zero. Consequently, the armature of the relay remains at rest.

The three-cored transformer operates in such a manner that, depending on whether the currents in coils BC and A correspond or are opposite in phase, the voltage in coil E correspondingly increases or decreases (while the voltage across coil D correspondingly decreases or increases).

As the signal is transmitted, one of the currents (i_1 or i_2) will increase and the other one will decrease, depending on the sign of the signal. The polarized relay will then operate either in one direction or the other. The net result is the turning of the vane, depending upon the angle, angular velocity, and angular acceleration of the rocket.

E. Range Control

1. Methods of Range Control

Range is one of the elements of flight trajectory and is determined, generally speaking, by the parameters of rocket motion.

If it is possible to interfere with the motion of the rocket along the trajectory, the reaching of a given target and the necessary range may be obtained with various relationships of flight parameters. To put it another way, the rocket may be guided to a given target by various trajectories.

However, starting with the time when the rocket ceases to be guided and continues its motion subject to forces which cannot be changed by us, the flight trajectory no longer depends on our wishes and is almost entirely determined by those initial conditions of the coasting flight which were imparted to the rocket at the end of the powered phase. Therefore, for a rocket to reach a given target it is necessary that the parameters at the beginning of coasting flight have definite relationships.

The initial parameters determining the motion of a ballistic body are, obviously, velocity v_0, the direction of velocity vector (for a plane motion, angle θ_0), and the coordinates at the end of the powered phase x_0 and y_0 (Fig. 9.30). Besides, the rocket flight during the coasting phase may be affected by such factors, variable with time, as wind direction and meteorological environment for the entire coasting phase or during that part of the trajectory which takes place in sufficiently dense regions of the atmosphere. The trajectory of flight is also subjected to systematic secondary factors, for instance, the velocity of earth's rotation and the variation in the magnitude of gravity acceleration vector, depending on the latitude of the locale.

Therefore, the rocket range is a function of several variables:

$$L = f(x_0, y_0, v_0, \theta_0, c) \tag{9.17}$$

where parameter c is a correction which takes into account the effect of the enumerated secondary factors.

The range control of powder rockets is performed simply by means of changing the elevation angle of the launcher during launch (Fig. 9.31).

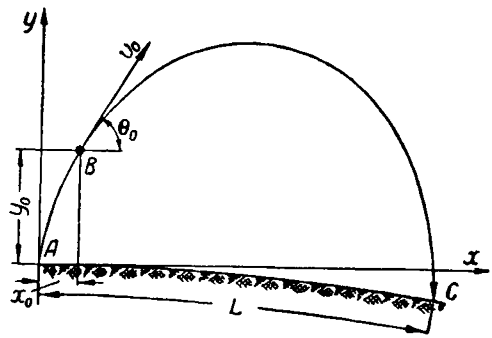


FIG. 9.30. Rocket trajectory and range.

With this method of range control it is obviously necessary that the thrust and operating time of the motor do not vary from rocket to rocket. Otherwise, for a constant angle θ_0, there will be a large variation in range. In order to avoid this dispersion, increased requirements are imposed upon powder grains in dimensional stability and weight and also in accuracy of fabricating powder chambers and nozzles.

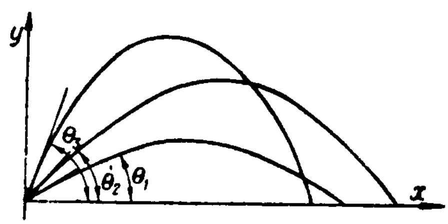


FIG. 9.31. Variation in range with the projection angle θ_0.

However, even with full conformity to the indicated conditions there is always range dispersion, inasmuch as for individual rockets, variations in ignition, combustion regime, and afterburning of a powder charge are always possible. With small rockets, this dispersion may be compensated by the number of rockets released to cover a given area. For long range rockets, because of their high cost, such compensation cannot be allowed. For these rockets it is necessary to introduce methods of range control which would insure substantially greater accuracy.

In examining expression (9.17), it is not difficult to note that the range control of a rocket may be attained only by variation in quantities θ_0 and v_0, since the parameter c is a quantity not subject to arbitrary change and variation in coordinates x_0 and y_0 may influence the range only within restricted limits.

If we are to control the range of a ballistic rocket by changing angle θ_0, assuming that the constancy of velocity v_0 is insured by the operating time of the motor with a given thrust, then obviously we will get a range dispersion of the same order as for nonguided powder rockets. At the same time, the necessity for changing the program with range will be an inescapable inconvenience.

It is more rational to control range by means of changing velocity v_0 at the end of the powered phase.

The first, somewhat coarse, method of control consists of switching off the motor (cut-off of fuel components) at an appropriate time, depending on the required range. This method of control, based on the assumption of invariable parameters for a group of rockets, does not, however, yield satisfactory accuracy. The thrust variation of actual samples of the same motor is noticeably greater than the allowable dispersion which is required here. The thrust regulation, in order to maintain strictly its constant value for a group of rockets, involves considerable technical difficulties.

Another, more acceptable, method of range control is the fuel component cut-off according to actual attained velocity (and not according to calculated time function). Thus, the second quantity (v_0) of the two (θ_0 and v_0), which more strongly affects range, is under direct control. As to the secondary factors x_0 and y_0, the influence of the change of these quantities on range need not be taken into account in the first approximation. This method of control yields considerably greater range accuracy compared to the motor cut-off with time.

The velocity of the rocket may be measured with the aid of radar, utilizing the Doppler effect as its operating principle. The device measures the difference in frequency of radio waves transmitted to, and reflected from, the rocket. This frequency difference, as is known, depends on the velocity of the reflecting object, in this case a rocket. Motor cut-off is performed at the required time by radio beam.

This method of range control yields high accuracy. However, it requires a complicated apparatus, and like any other method of radio control is subject to the action of countermeasures (interference).

A simpler method and one free of the indicated drawback is the method of mechanical integration of axial rocket accelerations during the powered phase. The device which performs the acceleration integration and thus determines the velocity of a rocket is the so-called integrator.

2. Construction and Operation of a Gyroscopic Integrator

The most feasible, for application, gyroscopic integrator is shown in Fig. 9.32.

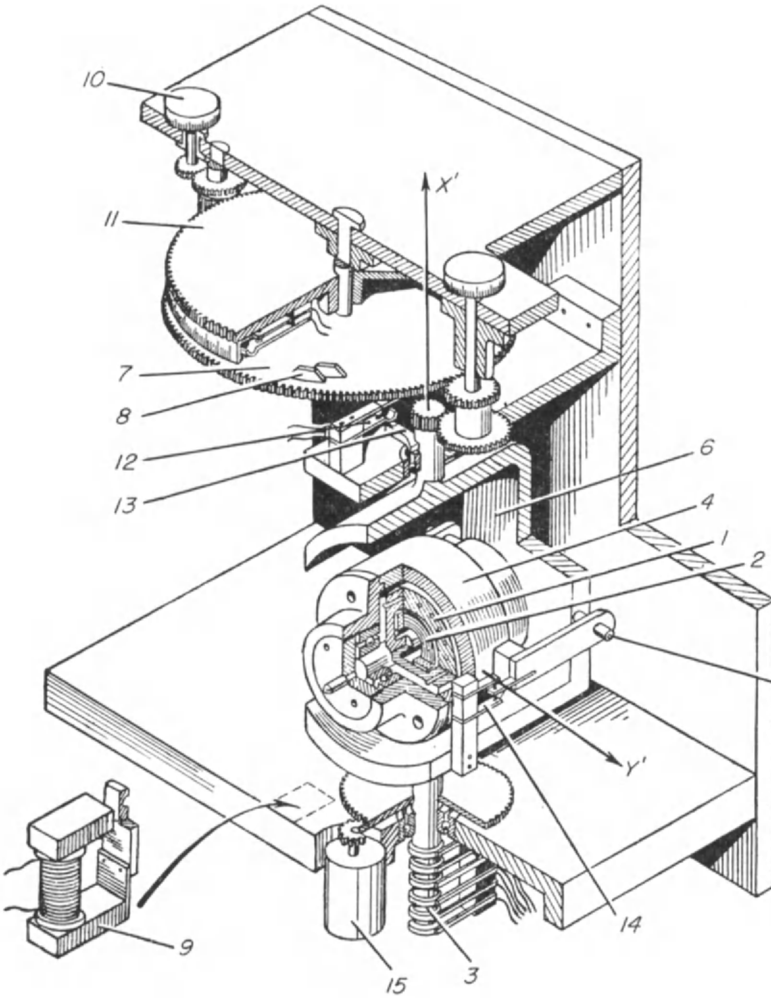


Fig. 9.32. Construction of an integrator: *1*, gyroscope rotor; *2*, stator winding; *3*, slip rings and slide contacts; *4*, rotor housing; *5*, gyroscope suspension axis; *6*, external yoke; *7*, gear; *8*, motor cut-off cogs; *9*, latch; *10*, range setting; *11*, gear; *12*, *13*, zeroing contacts; *14*, erection contacts; and *15*, erection motor.

The integrator has a gyroscopic rotor *1*, which is actuated by alternating current supplied to the stator winding *2*. The current and signals from the integrator are conducted through rings *3* with sliding contacts.

The rotor is located in housing *4*, which is free to turn about axis *5*. The weight of the rotor and housing creates a moment about this axis.

The axis *5* is connected with the external gyroscopic gimbal *6*, which may turn about axis x'. Through a train of gears the rotation is transmitted to disk *7*, with cogs *8*.

A few minutes before launching, the gyroscope is brought up to speed. During this time the gyroscope axis is immovably locked by means of a detent *9*. In Fig. 9.32 the detent is brought out to the side. It consists of an electromagnet and two plates. When a signal is transmitted to the electromagnet (this takes place at the instant of rocket take-off from the pad), the plate of the latch frees the axis of the gyroscope and the last, together with the housing, becomes supported by axis *5*.

Precession about axis x' begins due to the action of weight forces.

For a stationary rocket, according to Eq. (9.2), the angular velocity of precession is

$$\omega = Pa/C\Omega$$

where P is the weight of the rotor and housing; and a is the moment arm of force P about axis *5*.

For a rocket moving straight up (as we will assume for the time being for the sake of simplicity), with acceleration j, the apparent force of the weight will increase by comparison with P by an amount proportional to j/g, and then

$$\omega = (Pa/C\Omega)(1 + j/g).$$

The angle of gyroscopic rotation about axis x' in time t

$$\vartheta = \int_0^t \omega dt.$$

If the kinetic moment of the gyroscope remains constant, then

$$\vartheta = \frac{Pa}{C\Omega}\int_0^t \left(1 + \frac{j}{g}\right) dt = \frac{Pa}{C\Omega}\left(t + \frac{v}{g}\right) \tag{9.18}$$

i.e., the angle of gyroscopic rotation about axis x' has a linear relationship dependent on the flight velocity of the rocket at a given instant of time.

If the time t of rocket flight is known, then by eliminating addend $(Pa/C\Omega)t$ in the last expression, it is possible to find the rocket velocity by means of angle ϑ after a period of time t after launching.

The rocket range setting is performed by a knob, *10*, on the integrator. The disk, *11*, with a contact arrangement, rotates through the required angle relative to cogs *8*. This setting is performed, taking into consideration the correction due to the first addend in expression (9.18). This correction is easily introduced, since the rocket program is known in advance.

When the rocket attains the required velocity, one of the cogs, *8*, will initiate a signal to switch the motor into a preliminary stage, and the second cog will initiate total cut-off.

During operation, the spin axis of the gyroscope must always remain perpendicular to axis x'. In order to fulfill this requirement a correcting device is made part of the integrator.

If the spin axis of the gyroscope deviates either up or down, either the upper or lower contacts, *14*, will be closed and the motor, *15*, will receive a signal of the corresponding sign. Thus, a moment will act on the gyroscope

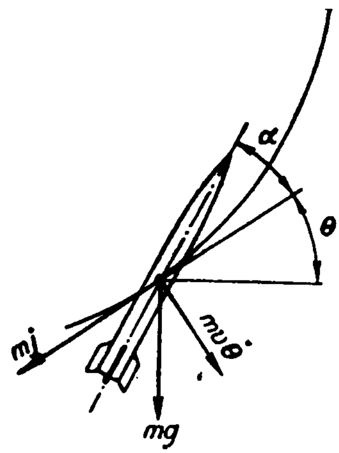


FIG. 9.33. Determination of the apparent axial acceleration.

about axis x' through a gear train. Such a moment, as we already know, will neither accelerate nor decelerate the rotation of the gyroscope relative to axis x', i.e., it will not vary the conditions of integration, but will only force the gyroscope to turn about axis *5* and resume the proper position.

The integration of accelerations in the manner described has its defects. One of these is the necessity of taking into account the rocket program in determining range, which leads to the so-called precedural error of the integrator.

Axis x' is parallel to the rocket axis. Consequently, the integrator will react, not to the accelerations tangential to the trajectory, but to the axial rocket accelerations.

As is seen in Fig. 9.33 the apparent weight force acting on an integrator rotor in the axial position is

$$P' = m[j \cos \alpha + v\dot{\theta} \sin \alpha + g \sin (\theta + \alpha)]$$

where $m = P/g$ is the mass of the gyroscopic rotor and housing.

In this case the precessional velocity of the gyroscope

$$\omega = (Pa/gC\Omega)[j \cos \alpha + v\dot{\theta} \sin \alpha + g \sin (\theta + \alpha)].$$

The angle of rotation of the gyroscope relative to axis x'

$$\vartheta = \int_0^t \omega dt = \frac{Pa}{gC\Omega} \int_0^t [j \cos \alpha + v\dot{\theta} \sin \alpha + g \sin (\theta + \alpha)] dt.$$

Therefore, we see that the integrator does not yield, in a pure form, the rocket flight velocity, and its indications require corrections on the basis of the required program $\theta + \alpha = f(t)$ and the calculated change in the angle of attack α along the trajectory. Since, in flight, the program is executed with certain errors, these errors are reflected, to some degree, in the determination of the rocket velocity v by the integrator.

F. Rocket Dispersion

1. The Measure of Dispersion

The same indices as used in artillery practice may be used to evaluate the degree of accuracy of rocket firing.

Let us assume that we are firing at a certain target with an unlimited number of similar rockets with constant launching conditions (Fig. 9.34).

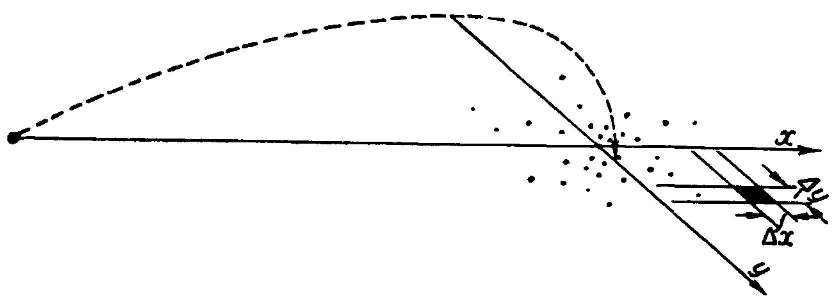


Fig. 9.34. Dispersion of rocket fire.

In this case, when we speak of "similar rockets" and "constant conditions" we understand by this only "similarity" and "constancy" within those limits which can be insured by technological considerations and by practical conditions of conducting the firings.

In actuality, the geometric dimensions of the rocket will vary somewhat from round to round. The conditions of powder grain burning in one rocket will differ from conditions of burning in another, if only slightly. The condition of launching itself will not remain invariant. Accidental, even though completely unnoticeable, displacement of the launcher from shot

to shot and the random wind gusts during rocket flight will change in some degree the conditions of launching.

However, in speaking of similar rockets and constant conditions we at the same time keep in mind that systematic errors and systematic deviations from certain average standards are excluded from the technical rocket equipment, their geometry, and conditions of launch.

For instance, the inaccuracy of the master gage, which might lead to the systematic deviation in the geometric dimension of a rocket or its charge, is eliminated. The constant wind velocity, which affects the range, is taken into account, etc. Therefore, in firing similar rockets with constant launching conditions, the deviation of the rocket from the target will have a random character and the points of impact shown in Fig. 9.34 will be, to a greater or lesser extent, scattered relative to the target. For a large number of shots, the random deviation from the target or, more accurately, the distribution of missiles on a plane, will follow the Gauss random distribution law.

If, in plane x–y (see Fig. 9.34), a rectangle is isolated with sides Δx and Δy, then the number of missiles ΔN hitting this rectangle, with a large number of firings, will be proportional to the area $\Delta x \cdot \Delta y$

$$\Delta N = \Phi \Delta x \Delta y.$$

The proportionality coefficient Φ is a function of coordinates x and y

$$\Phi = \Phi(x, y).$$

This is the distribution function. According to the Gauss law

$$\Phi = Ae^{-k^2x^2-h^2y^2} \tag{9.19}$$

where A, k, and h are certain constants and where the quantities k and h determine the degree of dispersion along the axes x and y, respectively.

The surface defined by Eq. (9.19) is shown in Fig. 9.35.

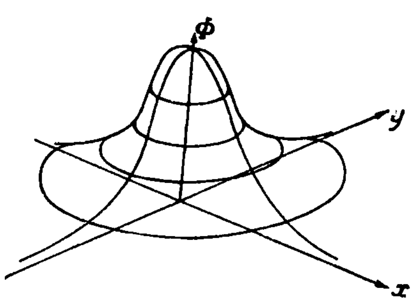


FIG. 9.35. Surface described by the distribution function $\Phi(xy)$.

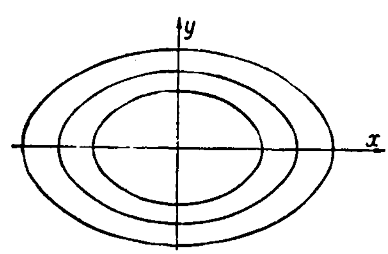


FIG. 9.36. Family of dispersion ellipses.

The function Φ is a maximum at the origin of the coordinates when $x = y = 0$, and becomes zero when $x = \pm\ \infty$ or $y = \pm\ \infty$.

Let us determine the geometric locus of points of equal distribution. Assuming $\Phi = C =$ constant, from Eq. (9.19) we will find

$$k^2x^2 + h^2y^2 = \ln\ (A/C)$$

i.e., the equation of an ellipse with axes coinciding with the axes of the coordinates. The dimensions of the semi-axes of the ellipse depend on the value of C. Therefore, the family of contours of the surface $\Phi(x, y)$ consists of a family of ellipses of dispersion (Fig. 9.36).

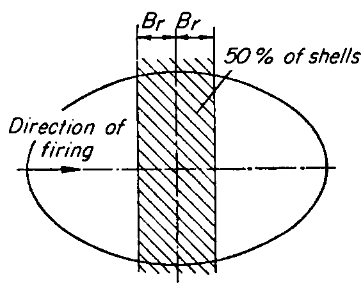


FIG. 9.37. Determination of B_r.

As a measure of missile dispersion during firing, the axis dimensions of one of these ellipses may be used, for instance, an ellipse within whose boundaries half of a large number of missiles impacts, i.e., an ellipse of the best half.

In artillery practice the measure of dispersion is not the dimension of an ellipse, but the width of a strip within whose bounds half of a large number of shells impact. The dimensions of this stip in the direction of firing (down range) are designated by $2B_r$ (Fig. 9.37) and, sideways, by $2B_s$.

2. Dispersion of Unguided Rockets

At the present time, the dispersion of unguided rockets is many times greater than the dispersion of ordinary artillery shells. This is explained, primarily, by the fact that the powered phase of the unguided rocket flight, i.e., that phase during which it accelerates up to speed, is not sufficiently stable, while the artillery shell, during acceleration, i.e., in the gun barrel, has a strictly determined direction of flight. The above is also confirmed by the fact that the unguided rockets, whose burning time ends while still on the launching rails, have dispersion approximately of the same order of magnitude as artillery shells.

Therefore, in discussing the sources causing rocket dispersion it is necessary to differentiate between those which act during the powered and during the coasting phases.

The basic causes of rocket dispersion during the powered phase are the eccentricity of application of the motive and aerodynamic forces and external disturbances which force the rocket to turn in flight about a transverse axis.

With an eccentrically applied thrust force, there develops a constantly acting moment, forcing the rocket to move continuously along a curved

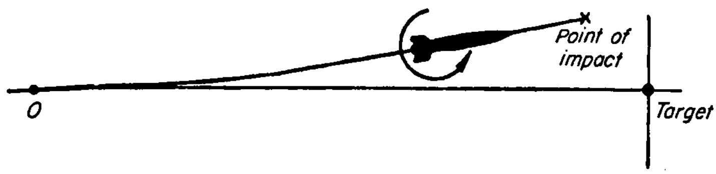


FIG. 9.38. Rocket dispersion due to eccentrically applied thrust (thrust misalignment).

trajectory and deflecting the rocket from the target (Fig. 9.38). In other words, during the powered phase, it is essential that the direction of a motive force constantly follow the direction of the turning rocket axis. If this force deflects the axis of the rocket from a given direction then it itself follows the turn and deflects the rocket from its target even further.

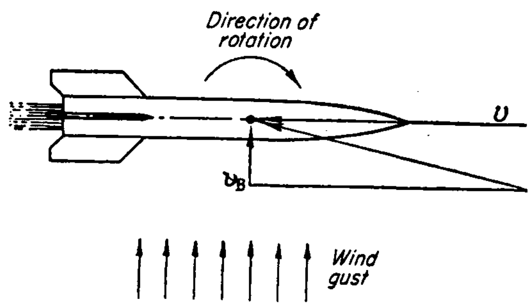


FIG. 9.39. Effect of cross wind on the flight of an unguided rocket.

Aerodynamic eccentricity acts during the powered phase in an analogous manner, inasmuch as it changes the direction of the rocket axis and, at the same time, the direction of the thrust force.

The action of external disturbing factors is quite singular. A cross wind gust, for instance, forces the rocket to deviate from its course, as opposed to an artillery shell, not with the gust but against it. Figure 9.39 shows a rocket in flight with a cross wind gust acting upon it. If the rocket is statically stable (and it is specifically made so), then it will turn into the resultant flow like a weather vane and will continue its flight, due to the action of thrust, in a new direction. In this sense, it is not advantageous to have a rocket with a large margin of static stability. Such a rocket will be overly

sensitive to random disturbances of this type and the dispersion will be large.

In order to reduce the effect of the eccentricity of thrust and aerodynamic forces, the unguided rocket is rotated in flight, i.e., it is made to turn slowly relative to the longitudinal axis. This rotation should not be confused with the rapid rotation which is imparted to spin stabilized missiles. Here, the turning of the rocket is necessary only as a means of canceling the moment of the eccentrically applied forces.

The rotation is imparted to the rocket either by canted rails or by means of inclined fins installed on the rocket. In some cases the rotation is imparted

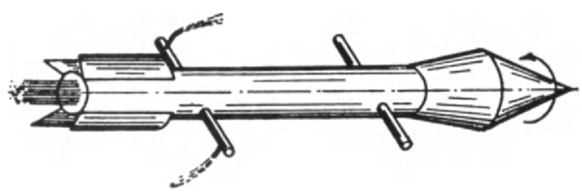


FIG. 9.40. Rocket spin during powered phase.

by the reactive force of jets emerging from small auxiliary nozzles drilled in transverse rods of the rocket (Fig. 9.40). The rocket slides along the rails by means of these rods during launching.

Aside from the enumerated basic causes which affect the dispersion of a rocket there are also other causes which we have touched upon earlier. They are fabricating variation in the dimensions of the rocket, the size of the charge, and variation in the combustion process itself, whose uniformity from rocket to rocket is quite difficult to maintain.

During flight in the coasting phase, the character of rocket dispersion is approximately the same as for ordinary artillery shells. The dispersion is basically conditioned by variation in aerodynamic properties from rocket to rocket and by eccentricity of aerodynamic forces. In the presence of aerodynamic eccentricity the rocket flies with some angle of attack. This leads to the appearance of a transverse force deflecting the rocket to one side.

From thernathematical point of view, the dispersion can be considered as the result of changes introduced into the trajectory parameters by various sources.

For instance, in the simplest case of a rocket, or an artillery shell, projected at an angle to the horizon, in the absence of resistance the range is

$$L = (v_0^2/g) \sin 2\theta_0.$$

In this case, the range dispersion may be considered the result of scatter of Δv_0 and $\Delta\theta_0$ in the values of v_0 and θ_0. Assuming that the changes in the

values of v_0 and θ_0 are small, we will get

$$\Delta L = (\partial L/\partial v_0)\Delta v_0 + (\partial L/\partial \theta_0)\Delta \theta_0$$

or

$$\Delta L = (2v_0/g)(\sin 2\theta_0 \Delta v_0 + v_0 \cos 2\theta_0 \Delta \theta_0).$$

It may be seen from the last expression that the deviation in the values of v_0 and θ_0 have various effects on range dispersion depending on velocity v_0 and projection angle θ_0. Specifically, if the firing is conducted under conditions of maximum range, i.e., when $\theta_0 = 45°$, the range dispersion is determined only by the value Δv_0.

Relative range dispersion for the example under discussion

$$\Delta L/L = (2/v_0)(\Delta v_0 + v_0 \operatorname{ctn} 2\theta_0 \Delta \theta_0).$$

The value $\Delta L/L$ is a minimum when $\theta_0 = 45°$ and becomes infinite when $\theta_0 = 0$ and $\theta_0 = 90°$, since for these values the range L becomes zero.

A similar analysis with respect to changing parameters may be performed for rockets, taking into account the air resistance and certain other factors. While doing this it is possible to determine not only the qualitative but, in certain cases, the quantitative, portion of the problem and approximately evaluate the magnitude of dispersion for proposed rockets.

3. Dispersion of Guided Ballistic Rockets

As has been indicated above, the range dispersion of guided long range ballistic rockets is tied in primarily with the errors in the integrator. Another cause of dispersion of rockets with automatic stabilization is the transverse drift to which the described gyroscopic devices do not react. If the flow around a rocket is asymmetrical, the development of lateral

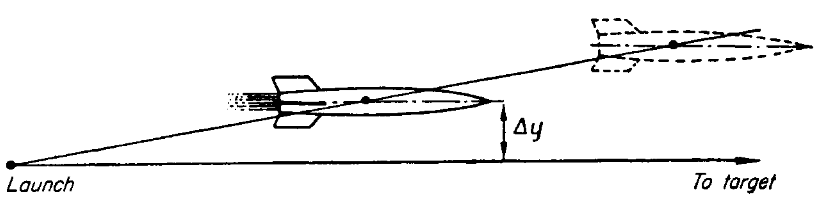


Fig. 9.41. Drift of a rocket equipped with an automatic stabilizer.

aerodynamic forces is possible during its motion in the dense atmospheric region. Lateral force also develops if the line of action of thrust is eccentric and is not directed along the axis of the rocket (along the gyroscopic axis).

Due to the action of transverse force, the rocket is laterally displaced (Fig. 9.41). This displacement or, as it is known, drift, will take place without rotation of the rocket axis (inasmuch as this rotation is not permitted by the automatic stabilizer). The skidding may take place in a

plane of the trajectory as well as in the direction perpendicular to this plane.

Gyroscopic apparatus react only to the angular displacement of the rocket and drift is not corrected by them. As a result, noticeable dispersion develops.

The drift may be eliminated only with the aid of special apparatus.

One of the possible means is the design of an independent integrator of lateral accelerations. By means of second order integration of accelerations it is possible to determine drift and to transmit an appropriate signal to vanes I and III (see Fig. 9.17). Practical implementation of such a measure, however, requires somewhat cumbersome gyroscopic apparatus necessary primarily to eliminate the effect of earth's gravity.

X. Ground Equipment and Launching Devices

A. Launching Devices of the Field, Anti-aircraft, and Aircraft Rockets

The complex of ground equipment is an independent, and quite important, part of rocket technology. The success or failure of launchings, the rate of design development, and the practical application of rockets depend, to a considerable degree, on the arrangement of ground servicing, design philosophy, and simplicity.

The ground complex includes all physical and organizational provisions for rocket launching, subsequent observation of flight, and guidance, if it is done from the ground. The physical provision for launching consists of means of transporting the rocket to the launching area, fuel supply (if fueling is done prior to launching), fueling, and auxiliary facilities.

In practice, the problems of physical and organizational provisions for rocket launching are solved variously, depending on the arrangement, purpose, and design of the rockets. For large, long range rockets, the implementation of launching represents a rather complicated problem. Conversely, for small, field rockets, this problem is extremely simple and does not present any greater complexity than the firing of an artillery shell. For a start, we will dwell on these simpler problems of launching short range anti-aircraft and aircraft rockets.

Transportation of small rockets (weighing up to 1 to 1.5 tons) does not present special difficulties. Special crating and ordinary means of transportation are used to deliver the rocket to the launching area. Larger rockets may require special dollies and hoists for their transportation. Understandably, special hoisting means are necessary for lifting such a rocket.

The launching of a large anti-aircraft rocket is also done in a different manner from the launching of medium-sized rockets. The method of launching large anti-aircraft rockets links them with the long range rockets. In particular, the launching area of one such anti-aircraft rocket will be examined below, in a section dealing with the launching of long range rockets.

Guides of various designs are used as launchers for powder rockets.

In firing powder rockets, the guides are assembled in blocks and, for mobility, are installed on some means of transportation. The guides are in the form of profiled rails, along which the rocket slides on special "shoes."

The firing is controlled from within the cab of the vehicle. The controller has a panel on which he can set any order of firing: one, two, three shots or salvo firing. The ignition of the rocket charge is done by means of an electrical squib.

Minethrowers are also transported by automotive means. One of these is the German six-barrel minethrower of the Second World War. The mine used with this minethrower was shown in Fig. 2.4. Another minethrower is characterized by the use of crates as launchers of its jet spin stabilized rockets. The crate is attached to the body of the tank. The missiles are related to a class of rockets, one of which was shown in Fig. 2.7.

For aiming powder rockets, as is necessary, for instance, in anti-tank warfare, long tubes are used as launchers (bazookas). The rocket is inserted into the tube from the rear.

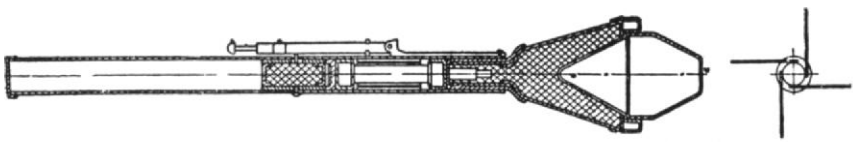


FIG. 10.1. Construction of an anti-tank gun.

In a short barrel design, the powder charge does not have time to burn completely in the tube, and completes combustion during free flight. For the protection of the gunner's face from the gas jet, a jet deflector, in the form of a conical mushroom made of fine mesh, which allows seeing the target but at the same time impedes the jet stream, is installed on the forward part of the barrel. In some cases the protection of the gunner is provided by a special mask worn over the face.

When the gun barrel is somewhat longer, the round is so designed that the moving powder charge burns out while the rocket is still in the launching tube. The gas jet escapes through the rear end of the tube and the necessity for protection of the gunner's face is obviated.

Figure 10.1 shows a transverse section of an anti-tank gun with an inserted round. This gun is characterized by the fact that the caliber of the head section of the round considerably exceeds the caliber of the barrel. This allows considerable increase in the power of the round.

The shell is inserted into the barrel from the front. The empinage, as is seen in Fig. 10.1, is made of four thin, flexible plates which are bent by hand around the tail of the rocket at the time of loading the gun.

The launchers of powder aircraft rockets differ very little from launchers mounted on surface means of transportation.

Figure 10.2 shows a typical launcher installation located under the wings of an aircraft.

A more contemporary design is one in which the cell with the powder rockets is extended prior to firing from the belly of the jet aircraft.

Large aircraft rocket missiles and large guided torpedoes are suspended from the aircraft in a manner similar to that used for ordinary bombs.

Let us pause briefly on the subject of launchers for anti-aircraft rockets.

The launching installations for the unguided anti-aircraft rockets differ little from the launching installations for firing against ground targets.

FIG. 10.2. Aircraft rockets mounted on their launchers. (Official U.S. Navy photograph)

If the firing is done by relatively small rounds, the tubular and rail guides are assembled into cells and the firing is conducted by salvos. As opposed to field installations, the anti-aircraft installations have a greater rotational freedom to facilitate aiming at the target.

The large unguided rockets are launched singly from installations similar to the one shown in Fig. 10.3 (the structure of the rocket on the launcher is shown in Fig. 2.24). Prior to launching, the operator adjusts the azimuth and elevation angle to correspond to the motion of the target.

It is not necessary for the launching installations of guided rockets to be trainable. The launching of a guided rocket is done, in the majority of

cases, in a vertical or almost vertical direction with further direction to the target by radio beam.

In the majority of cases the simplest guide rails are used as launchers.

The launching of an anti-aircraft barrage rocket is somewhat unique. The problem consists of lifting a cable to an altitude of ½ to 1 mile and its suspension at the highest point from a parachute. The cable is packed in

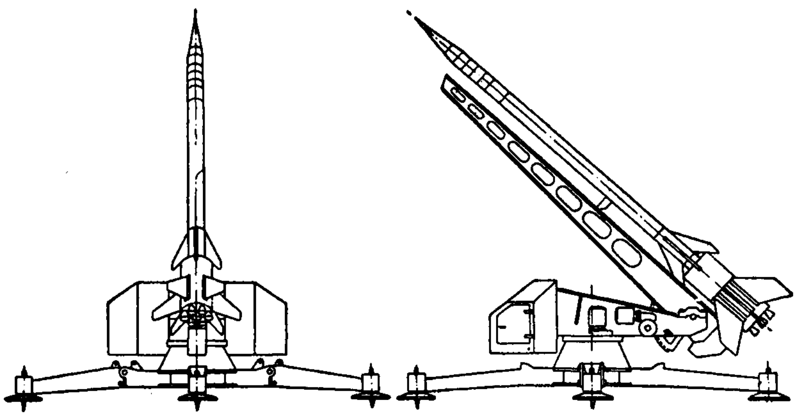


FIG. 10.3. Anti-aircraft launching installation.

a box which is placed on the ground, and the rocket, during its climb, takes up the cable and pulls it. To prevent entanglement of the cable, the launching installation is made extremely simple. In one case it is a rod with the rocket mounted on one end.

B. Hoisting, Transporting, Fueling, and Launching Equipment of Long Range Rockets

The organization for the launching of a long range, especially a long range liquid, rocket presents incomparably more complicated problems than the launching of short range rockets.

First of all, long range rockets in their physical dimensions and weights are such that they require special transportation and hoisting equipment. They are equipped with delicate, sensitive instrumentation whose more delicate components require a special environment for storage and transportation.

The weight of a long range rocket fully prepared for launching is measured in tens of tons. To transport and hoist such a rocket would be extremely difficult. Therefore, it is clear that a long range rocket must be fueled immediately prior to launching, in take-off attitude. But even if the weight of the fuel is not considered, there still remain the several tons of

the rocket structure itself and of its warhead. In this case, understandably, it is not possible to avoid special hoisting and transporting equipment, since this is required by the dimensions of the rocket, and especially by its handling as a structure quite sensitive to accidental damage.

For this reason, in some cases, the rocket is delivered to the launching site in parts. There the rocket is assembled in the vertical position.

After the body of the rocket is elevated, the nose section will be installed with the aid of a crane, and the stabilizer will be attached to the lower section. Then, the fueling of the rocket will begin. In the majority of cases, however, the rocket is delivered to the launching area in an assembled form as is done, for example, with the V-2 rocket.

The rocket is transported by means of a surface carriage (Figs. 10.4–

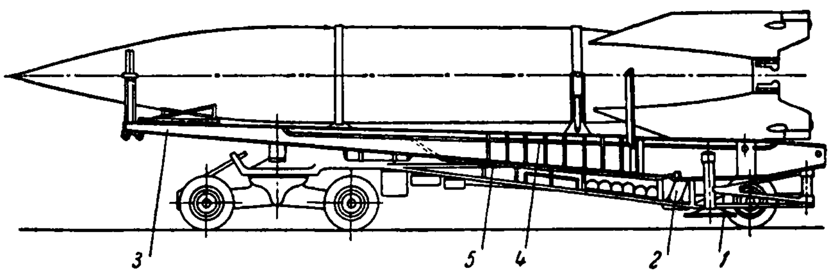


FIG. 10.4. A rocket on its carriage (V-2 long range ballistic rocket): *1*, brace; *2*, jack; *3*, folding platform; *4*, erecting rail; and *5*, ladder.

10.6). At the launching site, the launching platform is brought to the carriage on a dolly. This platform is mated to the carriage, after which the rocket is elevated.

The carriage rail, with the aid of hydraulic jacks, may be elevated to a vertical position together with the rocket (Fig. 10.5). In this manner the rocket is placed on the launching platform on its four fins. Stabilizing fins and the nozzle exit plane are specially reinforced to take the total weight of the rocket and to transmit these forces to the main lower bulkhead of the airframe, which is butt-jointed to the tail section. During rocket flight, the thrust force is transmitted to the same bulkhead.

A hydraulic telescoping jack, *2*, serves to elevate the carriage rail, *4* (Figs. 10.4 and 10.5). Since, during elevation, the center of gravity of the rocket is displaced relative to the points of support (wheels) backward, two pressure pads, *1*, are provided in the design of the carriage. They may be rotated about the vertical axis, thereby moving the support points backward. The support pad, *1*, is shown in Fig. 10.5 in contact with the ground.

After completion of rocket installation on the platform, the rocket is

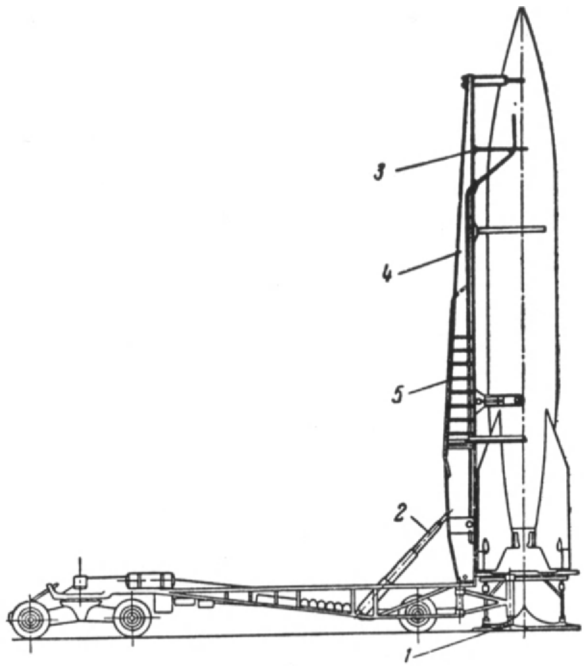


FIG. 10.5. Vertically erected rocket. Numbering is the same as in Fig. 10.4.

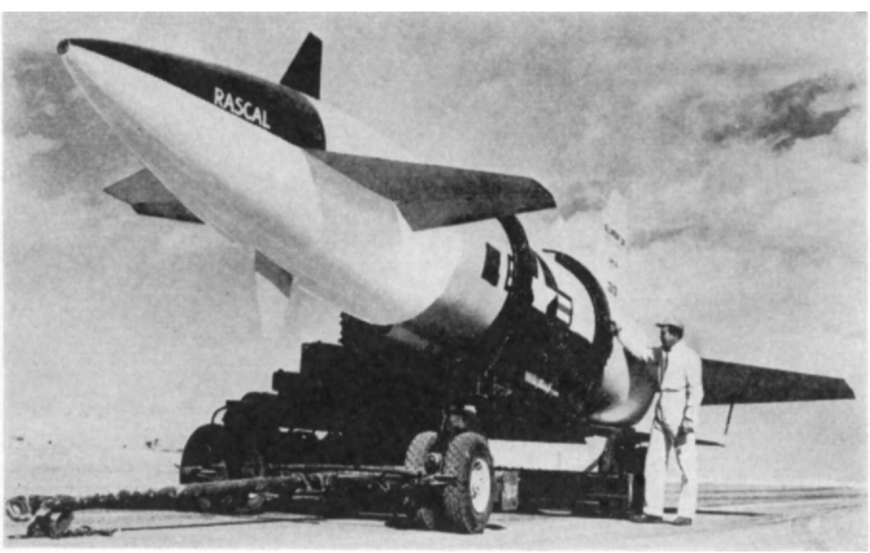


FIG. 10.6. Rocket ready for transportation. (Official U.S. Air Force photograph)

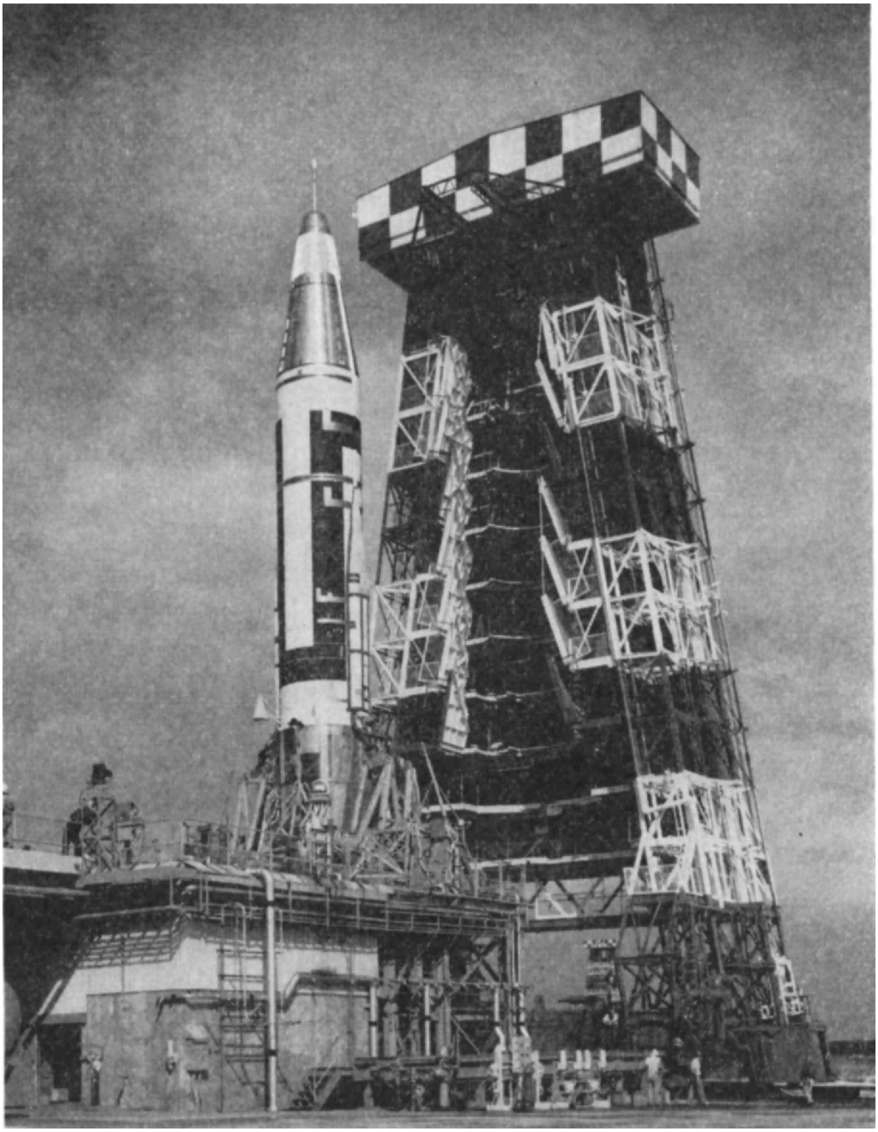


FIG. 10.7. U.S. Air Force Atlas on launching pad. (Official U.S. Air Force photograph)

freed of the straps tying it down to the rail. Later the, rail is moved slightly to the side and, during further preparation for take-off, it serves as a ladder along which the servicing personnel may reach the instrumentation and warhead sections. Servicing platforms are extended for convenience. One

of the platforms is at the level of the motor installation and the other at the level of the instrumentation section.

Figure 10.7 shows a much larger rocket (Convair Atlas) on its permanent launching pad. A series of servicing platforms is clearly visible on the mobile Gantry crane.

Figure 10.8 shows a rocket on the launching platform. A supply cable from a mobile electric station for ground testing the guidance system is supported by an arm attached to the pad. During launching the cable is released and hangs from the arm, while the guidance system is switched to the internal power.

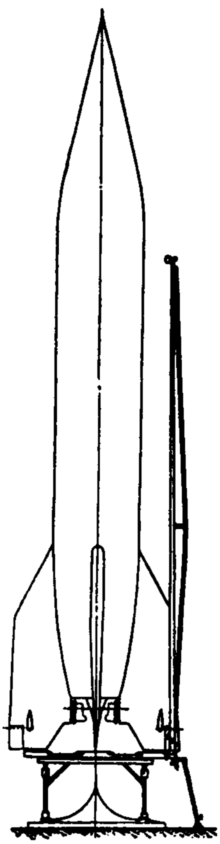


Fig. 10.8. Rocket on a launching platform.

Figure 10.9 shows a drawing of the launching platform.

The upper part of the platform consists of two rings, *1* and *2*. The lower ring rests on four supports and has a groove on top filled with ball bearings which support the upper ring. Therefore, both rings form a structure similar to a thrust ball bearing. The upper ring, with an installed rocket, may easily be turned to the required angle and the rocket plane I–III (see Fig. 9.17) may be made to coincide exactly with the plane of the trajectory. The rotation of the ring is done by means of a ratchet mechanism, *4*, connected to the chain, *10*.

Each of the fins is located between supports, *9*.

A flame deflector, *3*, in the shape of a conical plate is located below the launching platform. The rocket jet, during launching of the rocket, impinges against the plate and is dispersed in all directions below the platform.

Tubes *5* and *6*, attached to the upper ring, are used for installing the ignition system. The cable arm is attached to the bracket, *8*.

After the rocket is installed on the pad, its fueling begins. The alcohol is delivered in tank trucks. The liquid oxygen is brought up in large Dewar flasks mounted on truck trailers.

The oxygen is supplied through a flexible hose which is covered with a layer of white frost. The oxygen vaporizing in the tank escapes through a vent tube under the platform.

After fueling with alcohol and oxygen, the motor installation is charged

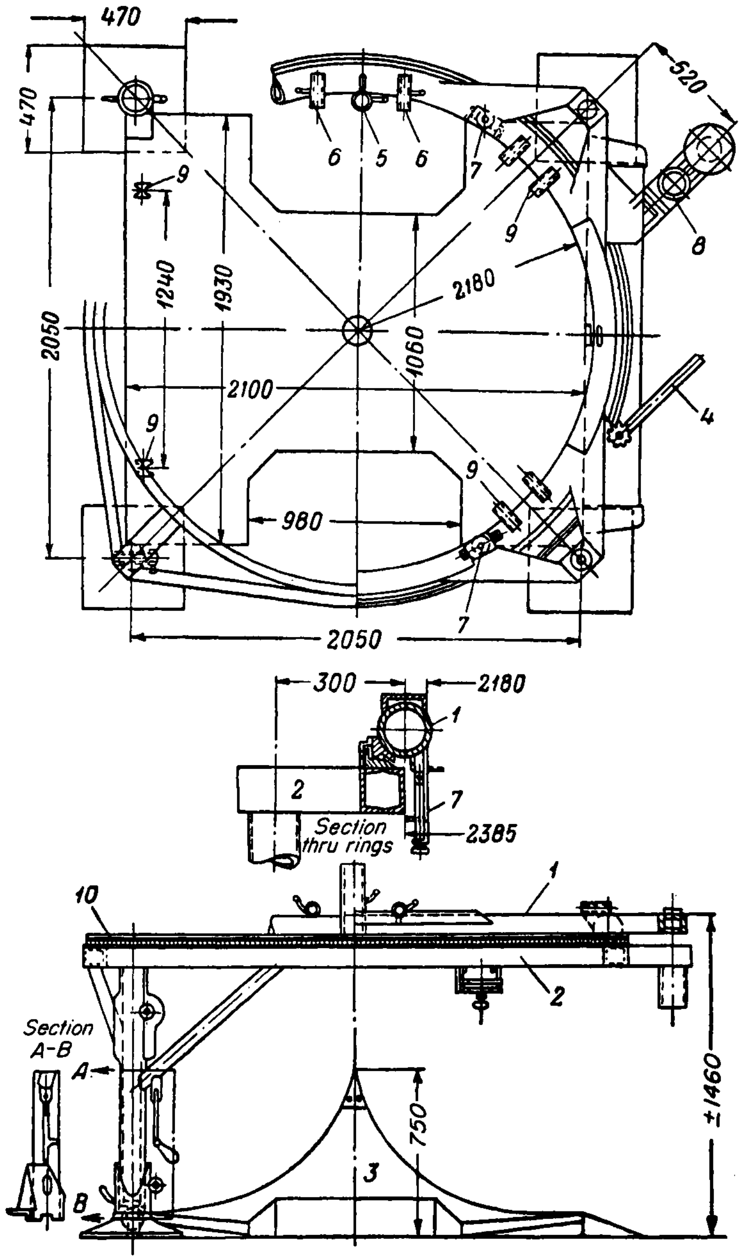


FIG. 10.9. Launching platform (dimensions in millimeters): *1*, upper ring; *2*, lower ring with four legs; *3*, flame deflector; *4*, ratchet mechanism for rotation of the upper ring; *5*, *6*, fittings; *7*, friction lock; *8*, mast support; *9*, fin rests, and *10*, chain.

with hydrogen peroxide, also brought up in a special cistern. Before fueling, the hydrogen peroxide is preheated to a temperature of 100° to 105°F. This is necessary for the proper generation of steam and assurance of motor operation in a given regime.

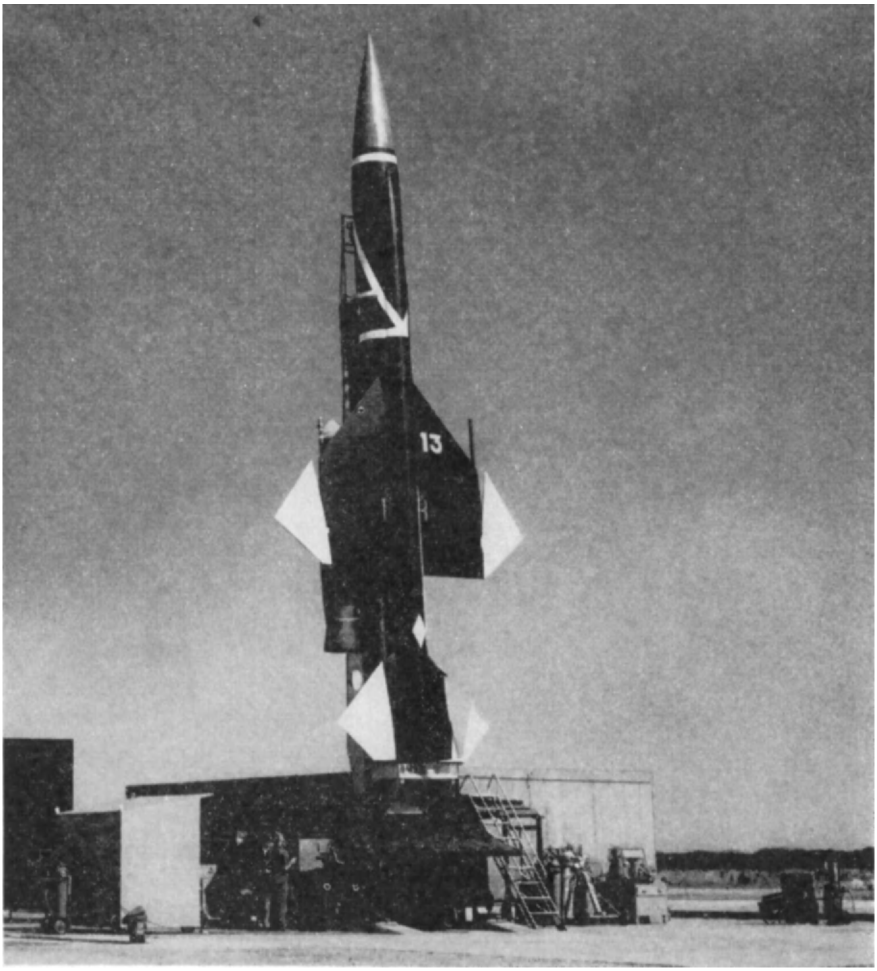


FIG. 10.10. Over-all view of the launching platform with the rocket in firing position. (Official U.S. Air Force photograph)

Potassium permanganate is poured from portable cans, also in a preheated state.

The over-all view of the launching platform, with the Bomarc rocket in firing position, is shown in Fig. 10.10. A typical schematic of the location of servicing machinery during fueling is shown in Fig. 10.11.

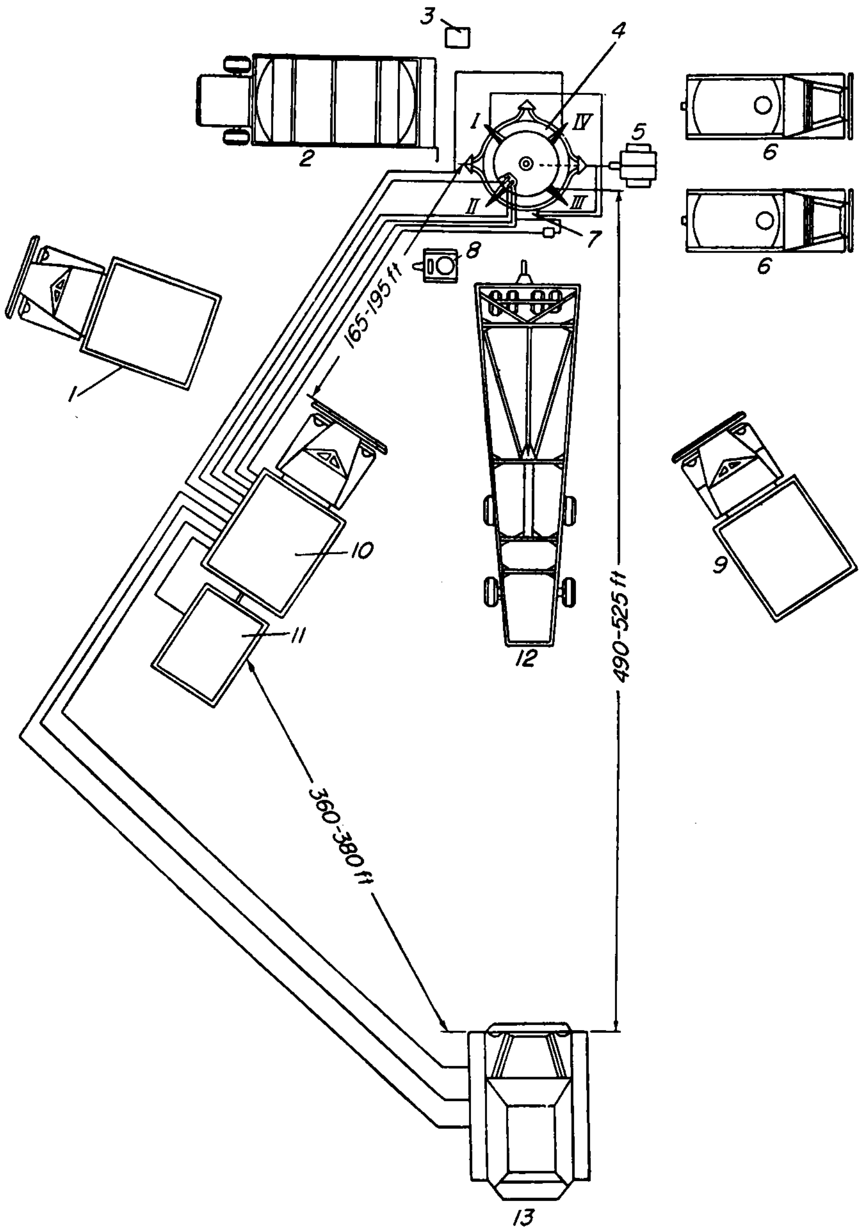


FIG. 10.11. Schematic of the servicing equipment location in the launching area during fueling: *1*, cable truck; *2*, tank truck with oxygen; *3*, oxygen pump; *4*, launching platform; *5*, pumping station; *6*, tank trucks with alcohol; *7*, pneumatic panel; *8*, peroxide heater; *9*, vehicle with additional equipment and spare parts; *10*, electric supply vehicle; *11*, motor-generator trailer; *12*, carriage; and *13*, armored truck.

As is seen from the presented schematic, during servicing of a long range ballistic rocket many auxiliary pieces of equipment are required. Besides the above-mentioned surface carriage, launching platform, and cisterns with alcohol, oxygen, and hydrogen peroxide, the launching equipment includes a pumping station for oxygen, a coupling with a pump for pumping alcohol, a heater for hydrogen peroxide, cans for potassium permanganate, a heater for potassium permanganate, a compressor capable of delivering up to 3300 psi, apparatus for warming up the motor with hot air, an armored vehicle for directing launching, a vehicle with a console for horizontal testing, a vehicle with a console for verticle testing, a vehicle with spare parts, a mobile electrogenerator, a cable coupler, a transformer coupler, an accumulator station, a caterpillar tractor for the carriage, a tractor for the launching platform and crew, a tractor for the oxygen cistern, a bus for the crew, a vehicle for the crew making measurements, liaison vehicles, and a supply truck. Therefore, from the above listing for a V-2 rocket, it is seen that the aggregate of ground equipment is quite complicated.

The purpose of the various units comprising the system is understandable without explanations. It is necessary only to point out, in addition, that the complicated system of steering and atuomation aboard the rocket is subjected, prior to launching, to a thorough checkout. The first stage of the checkout is the so-called horizontal tests, i.e., tests conducted while the rocket is still on the dolly. The vertical, or general, tests of the rocket already poised on the platform are more thorough. The operation of valves, fuel supply system, steering mechanisms and, in general, the entire automatic system, are checked out at this point.

During launching, the electricity and pneumatics are supplied by a transportable electric generator and compressor.

Immediately prior to launching, all auxiliary apparatus, with the exception of an armored car which is located at a distance of 500 ft from the platform for a V-2 launching, are removed. The launching command is given from the armored car.

The launching of a long range rocket from a surface platform is not the only possible method. During the Second World War, attempts were made to launch a rocket directly from a railroad flat car.

The railroad flat car is equipped with elevatable service platforms and a transportable launching platform. During elevation, the platform is placed on an erected rail and thus prepared for launching.

The described method of rocket launching from a platform may also be used for large anti-aircraft rockets.

Figure 10.12 shows a launching area of an anti-aircraft rocket with a servicing and guidance system.

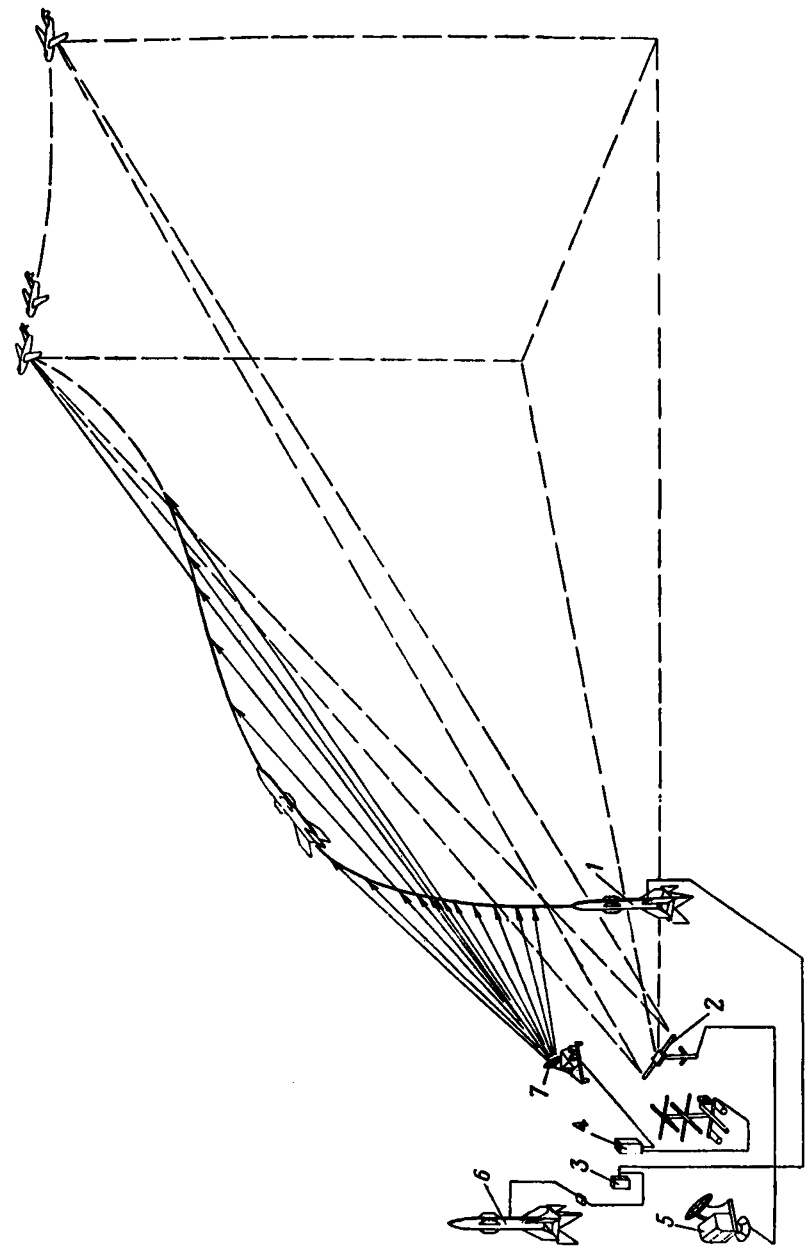


Fig. 10.12. Control system for a battery of anti-aircraft rockets; *1*, launching platform; *2*, range finder; *3*, control panel; *4*, computer; *5*, search radar; *6*, launching platform; and *7*, guidance radar.

The target is located by radar, which transmits a signal to the range finder. When the range to the target is determined the rocket is launched.

During the initial phase, in the first 6 to 8 sec, the flight is vertical with

the internal stabilizing system. Then begins the steering of the rocket toward the target, whose range is constantly being determined by the range finder. Signals are transmitted from the range finder to the computer, which determines the necessary direction of the rocket flight for target intercept. The results obtained by the computer are transmitted to the steering mechanism and, depending on the location of the rocket and the computer data, the necessary signal is transmitted to the antenna. In this manner the rocket is given the required direction of flight.

In conclusion let us pause briefly on the launching equipment of pilotless aircraft.

One of the methods of launching pilotless aircraft is the use of special catapults (Fig. 10.13). The pilotless aircraft is accelerated by means of a moving piston located underneath the rails of the catapult and actuated by the products of hydrogen peroxide decomposition.

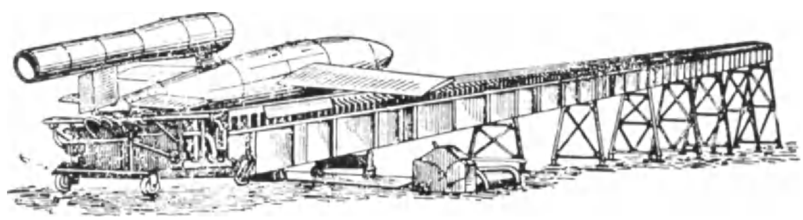


Fig. 10.13. Catapult for launching pilotless aircraft.

This method of solving the problem of launching pilotless aircraft cannot be regarded as successful. The catapult becomes complex and unwieldly and, most important, not maneuverable, which, under war conditions, is its most serious defect. Much more suitable is a light launching installation with guide rails and acceleration by means of powder rockets.

As was mentioned in Chapter II, the pilotless aircraft may be launched from mother planes.

C. Testing of Rocket Motors and Rockets

Each rocket, especially a long range rocket, during its development passes through an extensive program of tests of individual components, as well as in its entirely. A long range ballistic rocket is too complex and expensive a piece of apparatus to be built without exhaustive experimental development of components and assemblies.

As in aircraft design, the development of a new prototype is accompanied, first of all, by the testing of a rocket model in the wind tunnel in order to determine all of the aerodynamic coefficients.

In fabricating an experimental prototype, the more critical components are fabricated in larger numbers than required and part of them are struc-

turally tested, resulting in an experimental determination of their limiting loadings.

The steering system is subjected to a detailed check-out. The steering mechanisms and gyroscopes are tested on special stands. The operation of components under conditions of high vibration levels, which develop in the body of the rocket during flight, are especially tested. These vibrations are, usually, duplicated on special shake tables.

A separate, and more responsible, problem is the testing and development of motors.

As with all other details, the components of the motor, first of all, are subjected to thorough control during fabrication and assembly. This control consists of dimensional inspection and then testing of connections, piping, chamber components, and fuel tanks for pressure tightness.

An assembled motor, with the delivery system, is subjected to pneumatic and hydrostatic tests. During pneumatic tests, the system is filled with an excess pressure of 14–28 psi of air. All connections and welded joints are covered by brush with a solution of soap. Air leaks through the seams are immediately spotted by the formation of soap bubbles. During hydrostatic testing, the system is subjected to the full operating, and sometimes exceeding the operating, pressures. The appearance of leaks is monitored by the pressure drop within the system and by leakage at the joints and seams.

The motor, as a unit, is subjected (just as all the other rocket assemblies are) to functional tests. The proper functioning of the separate sections of the motor, the degree of proper functioning of the automatic system, the performance of the valves, and the performance and reliability of the liquid components supply are determined during this test.

After a full, thorough check-out, static firing tests are begun. The motor is installed on a special stand and is started. The performance of the operating motor is measured and the results compared with the calculations. Design changes are made if required.

The firing stands are not used solely for the design development of the motors. Investigation of processes within the liquid fueled rockets requires extensive experimentation with models (small motors), and with full size motors as well. The same stands, with corresponding changes in the program of measurements, are used for this purpose.

Motors of already developed production rockets are also subjected to static tests for the purpose of determining (or refining) the basic rated characteristics of the motors.

The experience of many years shows that, contrary to a widespread belief, the testing of rocket motors does not present serious hazards to the servicing personnel. However, in conducting static tests, a whole series of safety measures must be observed.

First of all, caution is necessary in handling fuel components, many of which are caustic and are harmful to the human organism. Among these are nitric acid and hydrogen peroxide. The handling of liquid oxygen also presents a certain danger. Therefore, the preparation for the test itself requires observance of definite safety rules such as, for instance, the use of special garb for personnel taking part in the charging of fuel and auxiliary tanks.

During improper operation of the motor there is a possibility of combustion chamber explosion. Such explosions are very powerful, are accompanied by extensive destruction, and quite often cause fire. Therefore, the test bay, i.e., the location of the motor during tests, is usually made semi-enclosed, surrounded on three sides by concrete walls in order to reduce the effect of fragmentation and to localize the center of the fire resulting from the explosion. The servicing personnel are located in shelters some distance from the motor. The starting and stopping of the motor are remotely controlled.

The administration of static tests also requires the introduction of safety measures. The servicing personnel are thoroughly indoctrinated in rules of conduct and in the sequence of operations. Immediately prior to start, a general signal is given (usually a siren), warning of the beginning of the static firing.

Test stands for large rocket motors, as a rule, are remotely located with respect to populated areas. This is necessary if only because the jet exhaust of a large rocket motor is accompanied by loud noise. A full description of the noise of an operating rocket motor is difficult to convey in words. Suffice it to say that the noise of an operating 25-ton liquid rocket motor many times exceeds the noise of the propellers of a powerful airplane and at a short distance is tolerated with difficulty by man.

The following basic parameters of an operating motor are determined and measured on the stand:

1. Thrust.
2. Consumption of: (a) combustible; (b) oxidizer; (c) auxiliary components.
3. Pressures: (a) in the combustion chamber; (b) ahead of injectors; (c) at the various points of the intake and delivery lines of the fuel system.
4. Temperatures: (a) of the gas in the combustion chamber and in the nozzle; (b) of chamber walls; (c) of the cooling liquid.
5. Velocity of exhaust gases.
6. Chemical composition of the combustion products.
7. Various parameters of the tubropump assembly or of the displacement delivery system.

The thrust, component consumption, and pressures are determined with a satisfactory degree of accuracy. The thrust is usually measured by a

hydraulic dynamometer; the consumption, with the aid of flowmeters. The pressure is measured by manometers.

The gas and wall temperatures are determined with a considerably lesser degree of accuracy. The chamber wall temperature is usually measured by means of thermocouples imbedded in the wall in advance. The accuracy of these measurements is not great, inasmuch as a considerable temperature gradient is established across the wall and the dimensions of the thermocouple are close to the thickness of the wall. Even greater difficulties are present in measuring the temperature in the chamber and the temperature of the high velocity jet in the motor nozzle. In these cases, approximate temperature measurements obtained with the aid of an optical pyrometer must be satisfactory.

The exhaust velocity of gases varies across the nozzle exit plane and its measurement at various points with the existing instrumentation is extremely difficult. Usually, one must be satisfied with the determination of the average effective exhaust velocity obtained from thrust and fuel consumption data.

The static tests yield the most extensive and fruitful data for the final development of the motor.

After the motor has been developed it is possible to begin the testing of the rocket as a unit. These tests consist, first of all, in static tests of a rocket with an operating motor, for the development of the automatic system in the environment approximating operating conditions.

The next step is the flight tests of the rocket. This stage is more serious.

The first launchings of military rockets are conducted, naturally, without a warhead. Instead, recording and radio transmitting instrumentation are installed in the prototypes, and a system of end instruments for the measuring of the most important flight parameters is located at various points on the rocket.

The measurements made in flight are:

1. Operating parameters of the motor: (a) chamber pressure; (b) component consumption; (c) parameters of the delivery system (number of turbine revolutions, tank pressure above the component surface).
2. Component level in tanks.
3. Temperature at various points on the rocket.
4. Axial and lateral accelerations.
5. Aerodynamic pressures at various points on the fuselage.
6. Loads and stresses at the more critical structural components.
7. Amplitude and frequency of developed vibrations.
8. Guidance system parameters: (a) angle of rocket rotation about gyroscopic axes; (b) vane rotation angles; (c) the magnitude of steering signals at certain intermediate sections of the automatic stabilizer; etc.

For the measurement of the indicated quantities a multitude of end instruments are used; specifically, pressure transducers with different ranges; level indicators; temperature transducers whose operation is based on change of electrical resistance with temperature. Slide wire potentiometers which generate voltage proportional to the angles of rotation are used as end instruments for the angle of vane rotation and of the rocket as a whole. Accelerometers (of the type consisting of a mass supported by a flexible element) are used for recording of accelerations and vibrations.

All mechanical end instruments (pressure, acceleration, level) are equipped with slide wire or potentiometer devices, which transform end instrument indications into electrical signals which are transmitted to the ground receiving station through radio channels.

In this manner, as a result of flight test, it becomes possible to determine the variation of the rocket basic flight parameters with time.

A measurement system of this type is called a telemetering system, and represents one of the most important branches of contemporary rocket technology. At the present time it is impossible to conceive of a development of a large rocket without the use of a telemetering system. On the basis of telemetered data the experimenters obtain refined information on flight conditions, and in the case of an unsuccessful launching, by analyzing the received data, they can determine the reasons for this failure and eliminate them in the future.

During flight tests the observation of flight may also be done by means of a cine-theodolite. The rocket is observed up to the limit of visibility and its coordinates with respect to time are registered simultaneously with photographing the rocket on a movie film. The data supplied by cine-theodolite photography, supplementing the telemetering data, are quite important for the design development of a rocket. In the absence of direct visibility the observation of the rocket along the trajectory is performed with the aid of electronic tracking devices.

The launching of large rockets is a very important undertaking. We already know that the rocket, because of its high velocity, even though not equipped with a warhead, can cause a great deal of damage on impact. During experimental launchings, failures are always possible. The malfunctioning of the motor or guidance system will result in rocket impact either in the vicinity of the launching area or with considerable deviation from the target. Finally, it is possible that the rocket will leave the predetermined course completely. In such cases populated areas may suffer. Not only experimental but operational launchings of the V-2 rocket in Germany serve as an example of this. It is known, for instance, that during the bombardment of London by the already developed at that time V-2 rockets, from the suburbs of Hague, out of a thousand launchings 80 were unsuccess-

ful, while five of the rocket fell in Hague, causing great destruction in the city.

From everything that has been said above it must be concluded that the testing of long range and sounding rockets must be done in deserted areas, and the basic path of the rocket must lay over unpopulated regions. Finally, rocket launchings themselves may be undertaken only after a long preparation and a thorough and most careful check-out of the equipment.

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